The interplay of symplectic and projective geometry in the context of plane wave spacetimes

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(photos of the blackboard notes from the talk given at JurekFest)

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Dilation of spacetime $\longrightarrow$ preserves a null geodesic

Alexseevski $\longrightarrow$ plane waves.

Rosen

$$\boxed{G_R = 2du\,dv - g(u)(dx,dx)} \quad n+2$$

$u \in \mathbb{R}$
$v \in \mathbb{R}$
$x \in V$ real dim n

$$G_B = 2du\,dv - p(u)(x,x)\,du^2 - g(dx,dx)$$

all the conf symm

$$\left\{\begin{array}{lll} (1) & D = & x(\partial_x) + 2v\partial_v. \\ (n) & P = & \partial_x. \\ (n) & Q = & h(\partial_x) + x\partial_v. \\ (1) & H = & \partial_v. \end{array}\right\}$$

conformal Heisenberg alg for n degrees of freedom.

$$\boxed{h'(u) = g^{-1}(u)}$$

$$D \otimes du = 0$$

$$D \otimes dv = -2^{-1} g'(u)(dx \otimes dx)$$

$$D \otimes dx = -2^{-1} g^{-1}(u)\, g'(u)(du \otimes dx + dx \otimes du)$$

$$D^2 \otimes \alpha = -R(\alpha) \qquad \alpha \text{ covector field}$$

$$Rg = s(u)\left((du)(dx) \otimes (dx)(du)\right)$$

$$\boxed{s(u) = g'' - 2^{-1} g' g^{-1} g'}$$

$S(u) = 0$.

$$g'' = 2^{-1} g' g^{-1} g'$$ back

$$g''' = 0 \qquad g = A + 2Bu + Cu^2$$

$$C = (Cu+B)\, g^{-1} (Cu+B) \qquad C = BA^{-1}B$$

$$g = A\left[I + A^{-1}Bu\right]^2$$

$$h' = \left(I + A^{-1} B u\right)^2 A^{-1}$$

$$h = D + u (A + Bu)^{-1}.$$

<u>FRACT. LINEAR</u>

$$\mathcal{D}^2 \otimes \alpha = -R(\alpha) \qquad \alpha \text{ covector field}$$

$$Rg = S(u)\left((du)(dx) \otimes (dx)(du)\right)$$

$$\boxed{S(u) = g'' - 2^{-1} g' g^{-1} g'}$$

$$\boxed{h' = g^{-1}} \qquad h'''$$

$$\boxed{\text{"Schwarzian"}}$$

(2N)-dimensional

$[AB]$ endomorphism of a v.s. s.t. $AB|_A = I|_A$

$AB|_B = -I|_B$

$\boxed{\dim A = \dim B}$

$$\frac{1}{4}\left([AC]+[BD]\right)^2$$

$|_A$

$|_B$

$|_C$

$|_D$

agrees with standard X-ratio

v.s. is 2-D.

$\alpha,\beta,\gamma,\delta \in \mathbb{C}^2$.

$$\frac{(\alpha\wedge\gamma)(\beta\wedge\delta)}{(\alpha\wedge\delta)(\beta\wedge\gamma)}$$

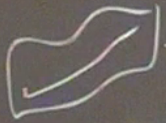
p_1, q_1, p_2, q_2.

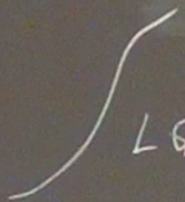
$p \to H_p$:

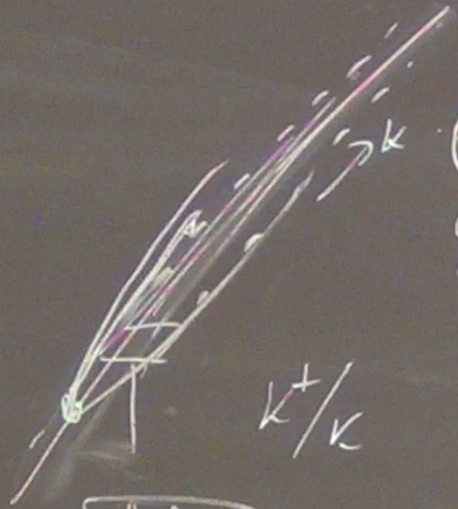
curve in $Gr(n, 2n)$.

$$\underbrace{\left[\overset{A}{H(p_1)}\;\overset{B}{H(q_1)} : \overset{C}{H(p_2)}\;\overset{D}{H(q_2)}\right]}_{[p_1, q_1; p_2, q_2]} \Bigg|_{H(p_1)} = I + (p_1 - q_2)(p_2 - q_1)\, S + \text{higher order}.$$

Penrose limit $\longleftrightarrow$ nbd of a null geodesic.







∂_k (affine)

geodesic deviation

$k^\perp / k$ umbral space

$$\boxed{k^c k^a R_{abc}{}^d}$$

Symplectic $\longleftarrow$ $\boxed{\text{data } 2n \text{ dim}}$ $\longrightarrow$ $\ddot{J} = R(J)$

Sachs eqn evolve L-subspaces

Sachs $\dot{J} = S(J)$

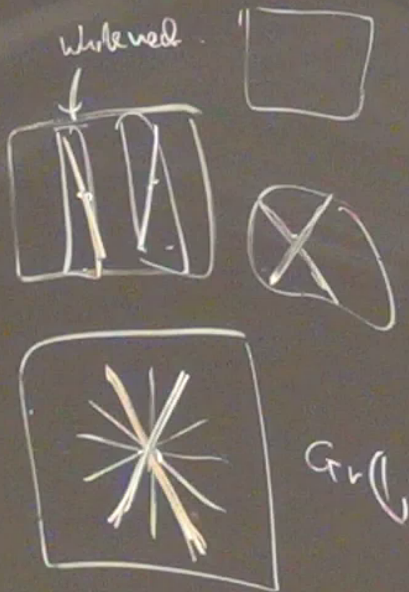
$$\boxed{\dot{S} + S^2 = R}$$

$S_b{}^a$ $\quad S_{ab} = S_{ba}$

$k^a S_{ab} = 0$

Roger $\longleftrightarrow$ 4-D.

Lionel $\longleftrightarrow$ any-dim.



$Gr(1,2)$

$$\int P_a \bar{\partial} X^a + \ldots \rightarrow \bar{\partial} \text{ replaced by Hilbert transform} \quad \boxed{H^2 = -I}$$

$$(v \cdot \partial)^{n+1} f_n(x^a) = (v \wedge x)^{n+1} (\partial_n f)_{-n-2}$$

$$\{ f_n(x^a) : 2 \text{ dof freedom, H rep} \}$$

Riemann surface world sheet

Square root of basic twistor

Square root of basic string theory $\rightarrow$ 2 dof

$\rightarrow$ 1-D manifold $\rightarrow$ proj structure