# A WALK IN JUREK'S GARDEN

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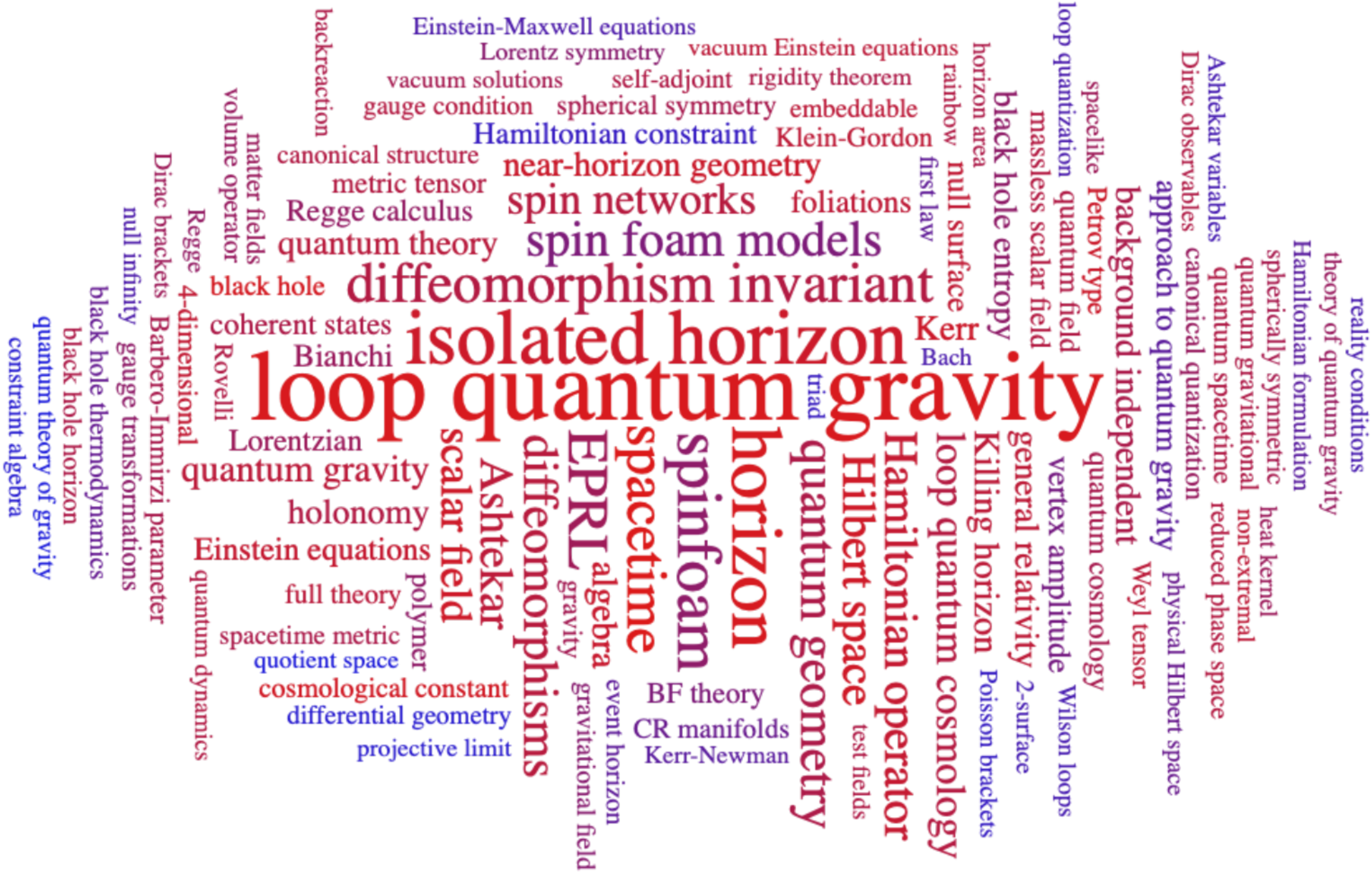
FAU


loop quantum gravity
isolated horizon
diffeomorphism invariant
spin foam models
spin networks
near-horizon geometry
horizon
spinfoam
spacetime
EPRL
diffeomorphisms
Ashtekar
scalar field
quantum geometry
Hilbert space
Hamiltonian operator
loop quantum cosmology
Killing horizon
general relativity
vertex amplitude
quantum cosmology
background independent
approach to quantum gravity
canonical quantization
quantum spacetime
quantum gravitational
spherically symmetric
Hamiltonian formulation
theory of quantum gravity
reality conditions
heat kernel
non-extremal
reduced phase space
physical Hilbert space
Weyl tensor
Wilson loops
2-surface
Poisson brackets
test fields
Einstein-Maxwell equations
Lorentz symmetry
vacuum Einstein equations
vacuum solutions
self-adjoint
rigidity theorem
gauge condition
spherical symmetry
embeddable
Hamiltonian constraint
Klein-Gordon
canonical structure
metric tensor
Regge calculus
quantum theory
foliations
black hole
coherent states
Bianchi
Kerr
Bach
triad
Lorentzian
quantum gravity
holonomy
Einstein equations
full theory
spacetime metric
quotient space
cosmological constant
differential geometry
projective limit
polymer
gravity
algebra
gravitational field
event horizon
BF theory
CR manifolds
Kerr-Newman
backreaction
volume operator
matter fields
Dirac brackets
Regge
4-dimensional
Rovelli
null infinity
Barbero-Immirzi parameter
quantum dynamics
gauge transformations
black hole thermodynamics
black hole horizon
quantum theory of gravity
constraint algebra
rainbow
horizon area
first law
null surface
black hole entropy
massless scalar field
loop quantization
quantum field
spacelike
Petrov type
Dirac observables
Ashtekar variables

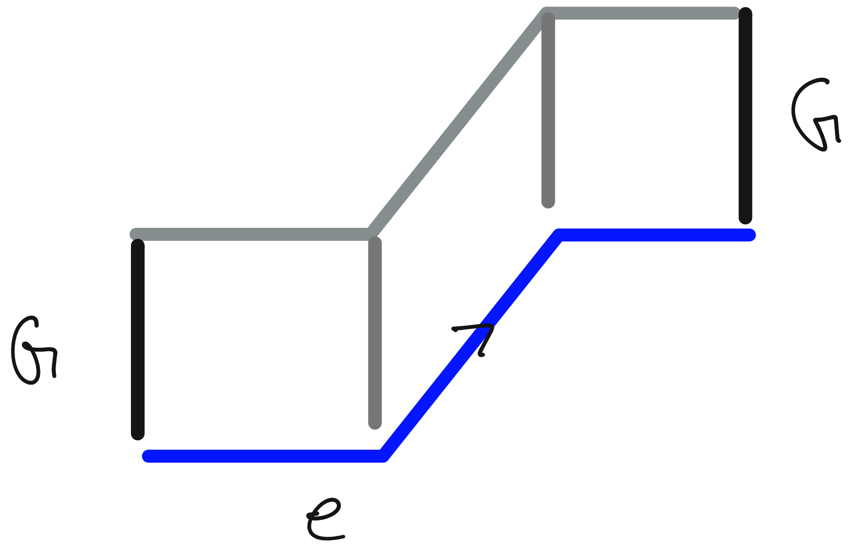
$$\mathcal{H} = \mathcal{L}^2(\overline{\mathcal{A}}/\overline{\mathcal{G}}, d\mu_{AL})$$

ASHTEKAR + LEWANDOWSKI

1993 + 94

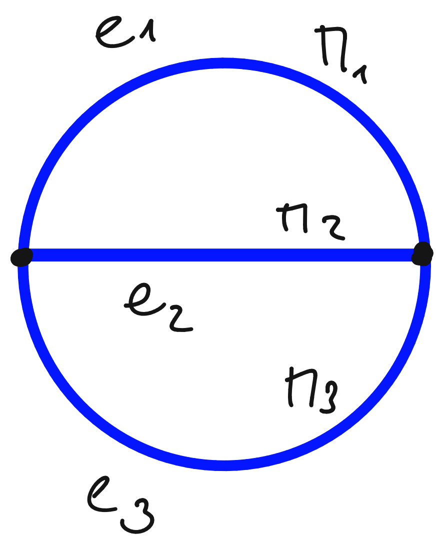
$$\overline{\mathcal{A}} \ni A : e \longmapsto h_e \in G$$

GROUPOID MORPHISMS



$$\mathcal{H} = L^2(\bar{\mathcal{A}}/\bar{\mathcal{G}}, d\mu_{AL})$$

ORTHONORMAL BASIS:

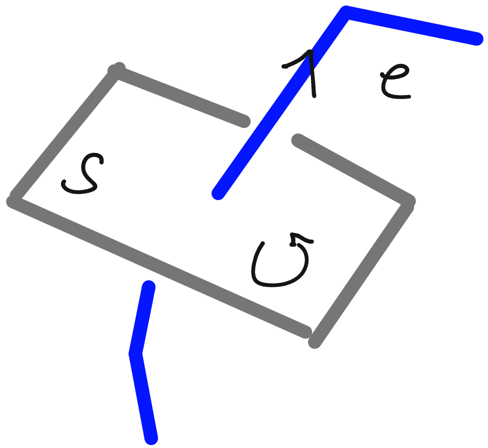



SU(2): SPIN NETS

PENROSE

$$\pi(h_e)\Psi[A] = h_e[A]\,\Psi[A]$$

$$\pi(E_S)\Psi[A] = \kappa(e,S)\,\left(h_{e_2}\,\tau\,h_{e_1}\right)$$




ASHTEKAR + LEWANDOWSKI

1993 + 1994

→ LQG

# 1. DIFFERENT REPS OF THE LQG COMMUTATION RELATIONS*

R. SEEGER + HS

# GENERAL STRUCTURE

HS 2011

$$\mathcal{H} = \bigoplus_{2} \mathcal{L}^2(\bar{\mathcal{A}}, d\mu_2)$$

$$\pi(h_e) = \begin{bmatrix} h_e & 0 & 0 & \cdots \\ 0 & h_e & 0 & \\ 0 & 0 & \ddots & \\ \vdots & & & \end{bmatrix}$$

$$\pi(E_S) = \begin{bmatrix} X_S + R_1^S + iI_1^S & E_{12} & E_{13} & \cdots \\ E_{21} & X_S + R_2^S + iI_2^S & E_{23} & \\ E_{31} & E_{32} & X_S + R_3^S + iI_3^S & \\ & \vdots & & \ddots \end{bmatrix}$$

## MORE GENERAL MEASURES :

$$\mathcal{H} = \mathcal{L}^2(\bar{\mathcal{A}}, d\mu)$$

KNOW EXAMPLES FOR $d\mu \neq d\mu_{AL}$

BAEZ VARADARAJAN

ASHTEKAR + LEWANDOWSKI OKOLOW

DO NOT SUPPORT SELFADJOINT $E_S$.

# MORE GENERAL FLUXES

R. SEEGER + HS 2018

$$\mathcal{H} = \bigoplus_{n=1}^{\infty} \mathcal{L}^2(\bar{\mathcal{A}}, d\mu_{AL})$$

$$\pi(h_e) = \begin{bmatrix} h_e & 0 & 0 & & \\ 0 & h_e & 0 & \cdots & \\ 0 & 0 & & \ddots & \\ & \vdots & & & \end{bmatrix} \qquad \pi(E_S) = \begin{bmatrix} X_S & \sqrt{1}\,\mathcal{E}_S & 0 & \cdots \\ \sqrt{1}\,\mathcal{E}_S & X_S & \sqrt{2}\,\mathcal{E}_S & 0 \\ 0 & \sqrt{2}\,\mathcal{E}_S & X_S & \ddots \\ \vdots & & \ddots & \ddots \end{bmatrix}$$

$$\mathcal{E}_S = \int_S * \mathcal{E}$$ CLASSICAL FLUX

INTERPRETATION: EXCITATIONS OVER VACUUM WITH

- $h_e$ AS BEFORE

- $E_s$ HAS GAUSSIAN FLUCTUATIONS AROUND $\langle E_s \rangle = 0$. COVARIANCE

$$\langle E^i_s \, E^j_{s'} \rangle = \mathcal{E}^i_s \, \mathcal{E}^j_{s'}$$

- STATES IN $\mathcal{H}$ ARE MIXED FOR $h$

- GEOMETRIC OPERATORS HAVE CHANGED SPECTRUM

## GENERALIZATIONS:

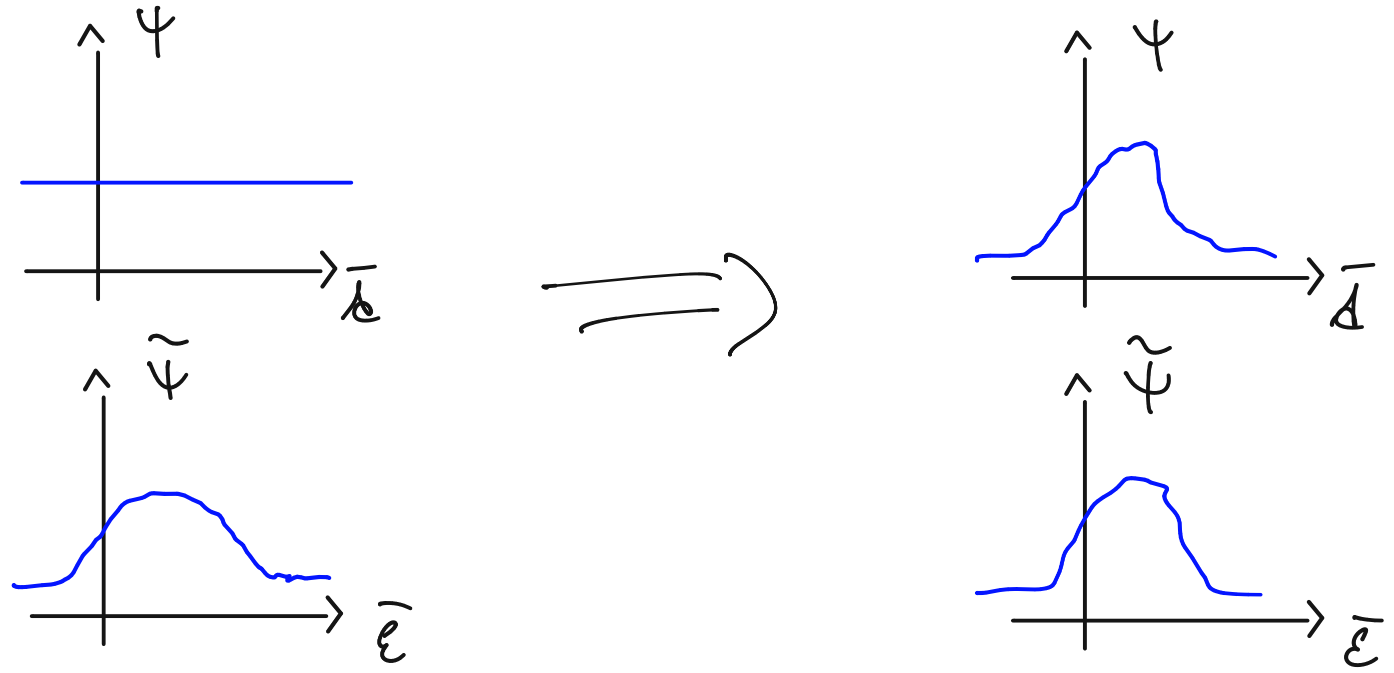
- MORE GENERAL COVARIANCES
- CONDENSATE $\langle E_S \rangle \neq 0$

## POSSIBLE APPLICATION:

FLUCTUATING GEOMETRY IN EARLY UNIVERSE

# OPEN QUESTIONS:

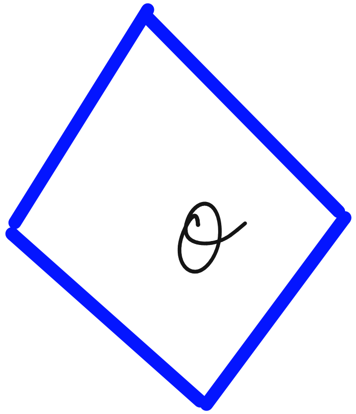
- HOW TO RESTRICT FLUCTUATIONS ?
- IMPLEMENTATION OF CONSTRAINTS ?
- FOOT IN THE DOOR FOR „COHERENT VACUUM" ?



# 2. HOLONOMIES, FLUXES AND DHR CHARGES

B. LOTTER + HS

# ALGEBRAIC QFT




- NET

$$\mathcal{O} \longmapsto \mathfrak{A}(\mathcal{O})$$

OF LOCAL OBSERVABLE ALGEBRAS

- LOCALITY

$$[\mathfrak{A}(\mathcal{O}_1), \mathfrak{A}(\mathcal{O}_2)] = 0 \text{ for } \mathcal{O}_1, \mathcal{O}_2 \text{ spacelike separated}$$

- ...

LOOKING FOR REPS OF THE NET

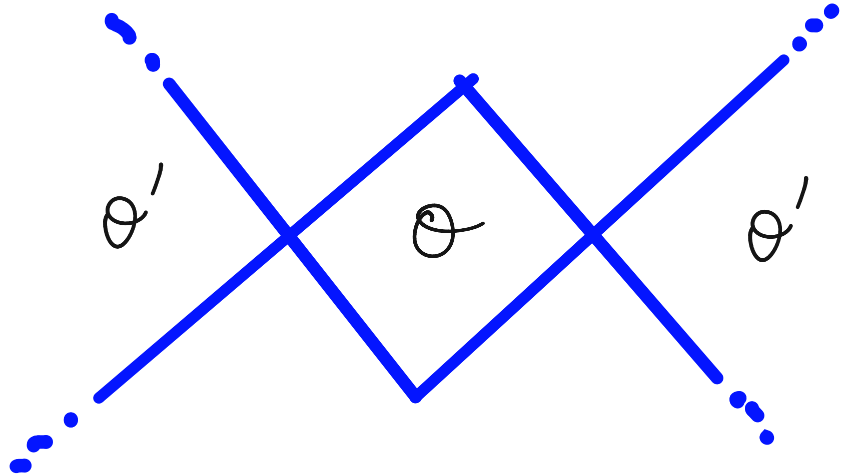
# DHR CHARGES (DOPLICHER, HAAG, ROBERTS)

RELATIVE TO REP $\pi_0$ („VACUUM") :

DEFINITION: REP $\pi$ SUCH THAT THERE IS DIAMOND $\mathcal{O}$ WITH

$$\pi(\mathfrak{A}(\mathcal{O}')) \simeq \pi_0(\mathfrak{A}(\mathcal{O}'))$$

EQUIVALENCE CLASS $\hat{\pi}$ MIGHT DESCRIBE LOCALIZED CHARGE IN $\mathcal{O}$



FROM AQFT AXIOMS IT FOLLOWS:

$\exists$ algebra morphism $\rho$ : $\pi = \pi_0 \circ \rho$

$\longrightarrow$ ADDITION OF CHARGES $\hat{=}$ COMPOSITION OF MORPHISMS

$\longrightarrow$ MORPHISMS HAVE MORE STRUCTURE

Braided monoidal category + ...

$\longrightarrow$ STATISTICS OF CHARGES

AND ...

## DHR EMBEDDING THEOREM:

CATEGORY OF MORPHISMS $\cong$ CATEGORY OF UNITARY IRREPS OF COMPACT GROUP $\mathcal{G}$

+ SOME ADD. STRUCTURE

## DHR RECONSTRUCTION THEOREM

$\exists$ FIELD ALGEBRAS $\mathcal{F}(\mathcal{O})$, REP $\pi$ OF $\mathcal{F}(\mathcal{O})$:

$$\pi\Big|_{\mathfrak{A}} \supset \bigoplus \left(\text{DHR-sectors}\right)$$

# TRANSLATION TO LQG?

- $\mathfrak{A}(\mathcal{O}) \longleftrightarrow$ gauge invariant part of HF-algebra in $\mathcal{O}$

- $\mathcal{O}_1, \mathcal{O}_2$ spacelike separation $\Leftrightarrow$ $\mathcal{O}_1 \cap \mathcal{O}_2 = \emptyset$

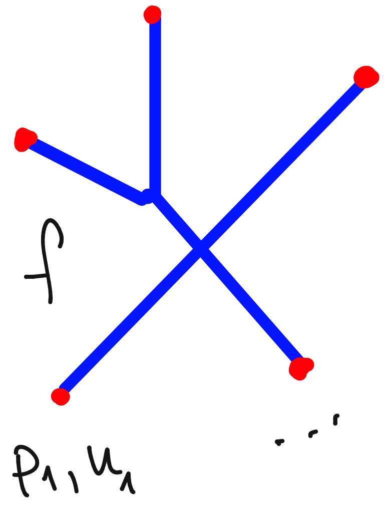
  $\Rightarrow [\mathfrak{A}(\mathcal{O}_1), \mathfrak{A}(\mathcal{O}_2)] = 0$

- $\pi_0 := \pi_{AL}$

# EMBEDDING:

$$\mathcal{f}(a) = f\, a\, f^{-1}$$

WITH $f$ : CHARGE NET WITH OPEN ENDS




CHARGED SECTORS LABELED

$$\left( (p_1, n_1)\, (p_2, n_2), \dots \right), \quad \sum_i n_i = 0$$

THESE ARE LABELS OF UNITARY IRREPS OF

$$\mathcal{G} \ni g = \left( x \longrightarrow g(x) \in U(1) \right)$$

[ CHARGES WITH $Q \equiv \sum_i u_i \neq 0$

NOT LOCALIZABLE : ABELIAN GAUGE FIELD ]

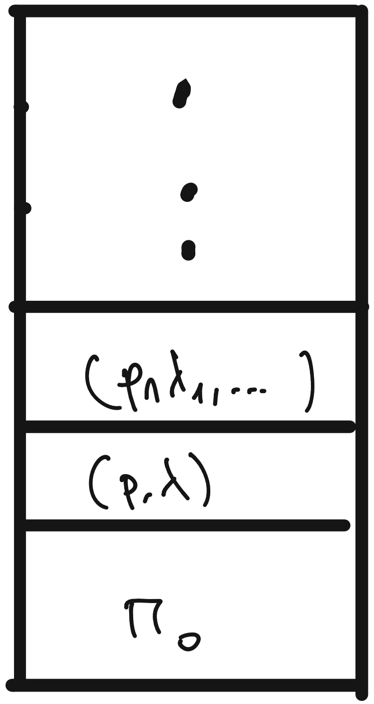
# RECONSTRUCTION: FOR ANY COMPACT $G$: WITH

$$\mathcal{F}(\mathcal{O}) = \mathcal{HF}\text{-algebra in } \mathcal{O}$$

$$\pi = \pi_{AC}$$

FIND

$$\pi|_{\mathcal{O}} = \bigoplus \pi_{((p_1,\lambda_1),(p_2,\lambda_2),\dots)}$$




WITH

$\lambda_i$ : HIGHEST WEIGHT LABELS OF $G$

$((p_1,\lambda_1),(p_2,\lambda_2),\dots)$ : IRREPS OF $\mathfrak{g}_G$

WHAT CAN THIS BE GOOD FOR ?

$\longrightarrow$ PARTICLE-LIKE PROPERTIES OF OPEN ENDS !

$\longrightarrow$ MORE GENERAL CHARGED SECTORS ?

$\longrightarrow$ INTERESTING FOR DHR : $\mathcal{G}$ CAN BE LOCAL GAUGE GROUP