Dark matter from quantum gravity

Carlo Rovelli

'I Wasn't Asked to Do Anything Illegal,' Lewandowski Says in Hearing

Variations of the parallel propagator and holonomy operator and the Gauss law constraint

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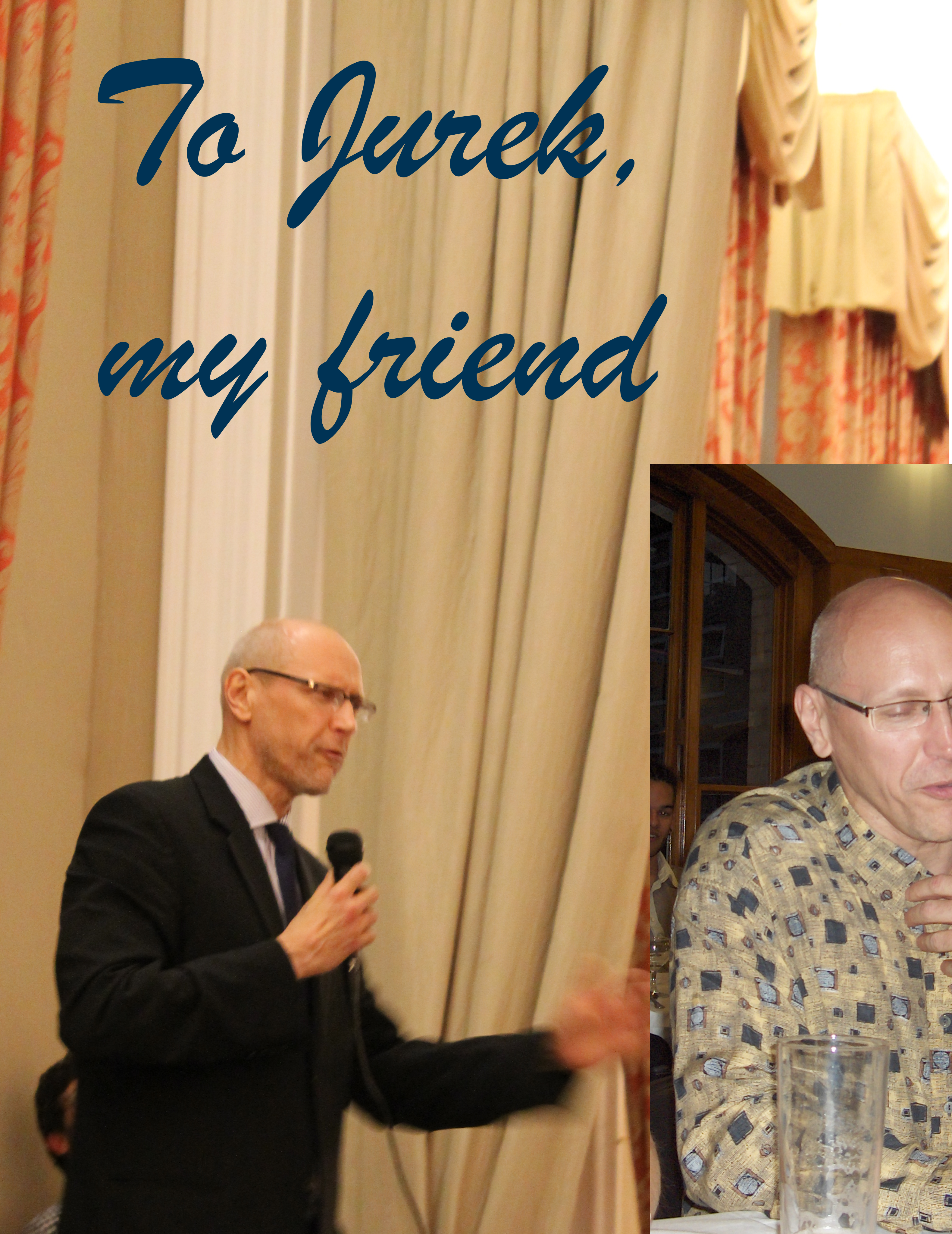
A simple derivation is presented of the equations for the variation of the parallel propagator and the holonomy operators of Yang–Mills (YM) connections caused by variations of both the connection and the path. The derivation does not make any direct use of functional derivatives and is based on the solution of the varied parallel transport equation. In particular, the different forms that these equations take for a two parameter family of curves in E^3 are discussed. As an example of this formalism, it is shown how any congruence defines a solution of the Hamilton–Jacobi version of the Gauss law constraint of YM theories, or equivalently, of the Dirac quantum-Gauss law constraint.

I. INTRODUCTION

A commonly used concept in gauge theories (both classically and quantum mechanically) is that of the parallel propagator (PP); a linear mapping from the fibers over a point x to the fibers over the point x' that depends on the connection along a path $\mathscr{C}$ from x to x'. If the path closes, then the mapping is from the fiber at x to itself and is then referred to as the holonomy operator. Often one is interested in the behavior of the parallel propagator (or holonomy operator) under variations of either the path or of the connection itself, i.e., the variational derivative of the PP (or holonomy) with respect to the path or the connection. The interest in these variations arises in a variety of physical situations as, for example, in the work on loop space quantization of both gauge theories and general relativity;[1] in the Hamilton–Jacobi treatment of constraints[2] in Yang–Mills (YM) and general relativity (GR); and in a study[3] of a non-Abelian generalization of quantum mechanics. This variational calculation, at least for special cases, is most often done (and occasionally with errors) in a rather straightforward but cumbersome fashion using the fact that the PP can be written as the path-ordered exponential

To Jurek, my friend



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Dark matter as white holes ?

- **Dark matter is the only truly completely unexplained observed phenomenon in fundamental physics.**
  [Dark *energy* is far less mysterious. Bianchi C.R, *Is dark energy really a mystery?*, Nature, 2010.]

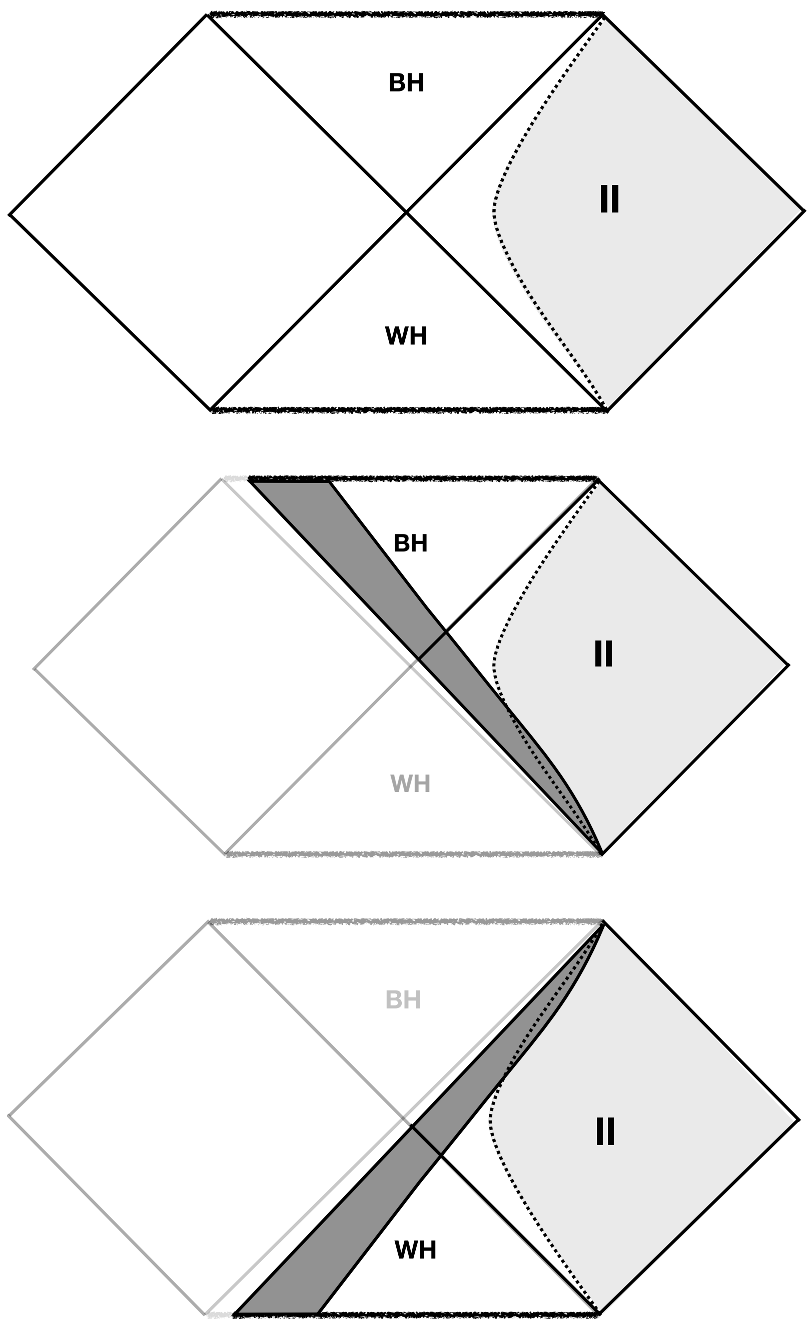
- **Old idea: dark matter is formed by Planck size remnants of Black hole evaporation.**
  - Aharonov Casher Nussinov *The Unitarity puzzle and Planck mass stable particles*, 1987.
  - MacGibbon, *Can Planck-mass relics of evaporating black holes close the Universe?* Nature, 1987.
  - Barrow Copeland Liddle 1992. Carr Gilbert Lidsey 1994. Liddle Green, 1997. Alexeyev Barrau G. Boudoul Khovanskaya Sazhin, 2002, Chen Adler, 2003, Barrau Blais Boudoul Polarski, 2004,Chen, 2004, Nozari Mehdipour, 2008.

- **➜ ➜ ➜ Remnants can be white holes**
  Haggard CR, *Black hole fireworks: quantum-gravity effects outside the horizon spark black to white hole tunneling,* 2015.
  Bianchi Christodoulou D'Ambrosio Haggard CR, *White Holes as Remnants: A Surprising Scenario for the End of a Black Hole*. 2018.

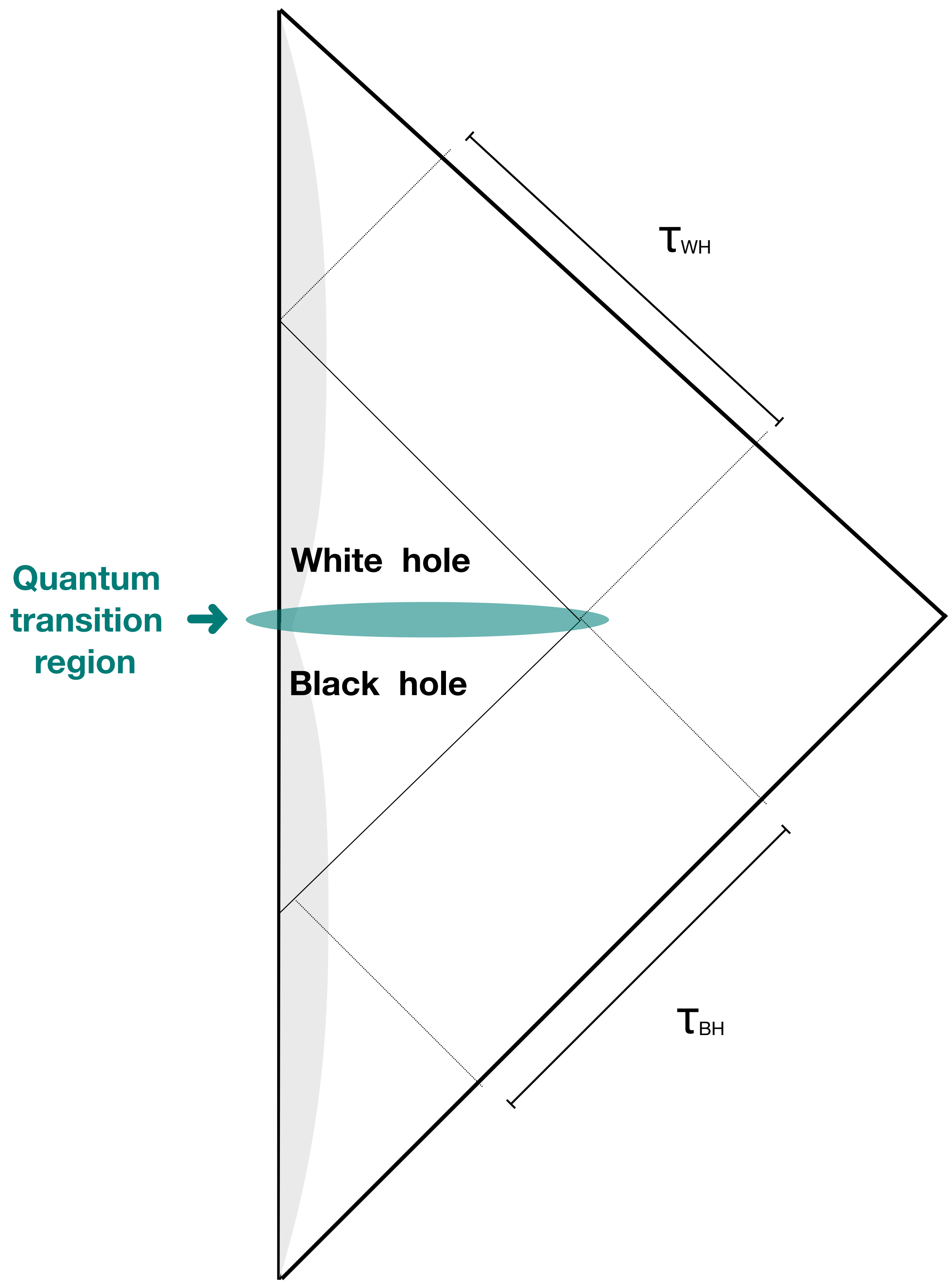
- **➜ ➜ ➜ Microgram (Planck scale) white holes are stabilised by quantum mechanics.**
  - Vidotto, C.R., *Small black/white hole stability and dark matter 2018*.

- **Microgram (Planck scale) white holes can be produced at the end of the evaporation of primordial black holes.**

White holes



- **White holes are undistinguishable from black holes from the exterior, unless matter enters or exit the horizon.**



Much growing evidence for a black-to-white hole scenario, and much recent work on this possibility in LQG

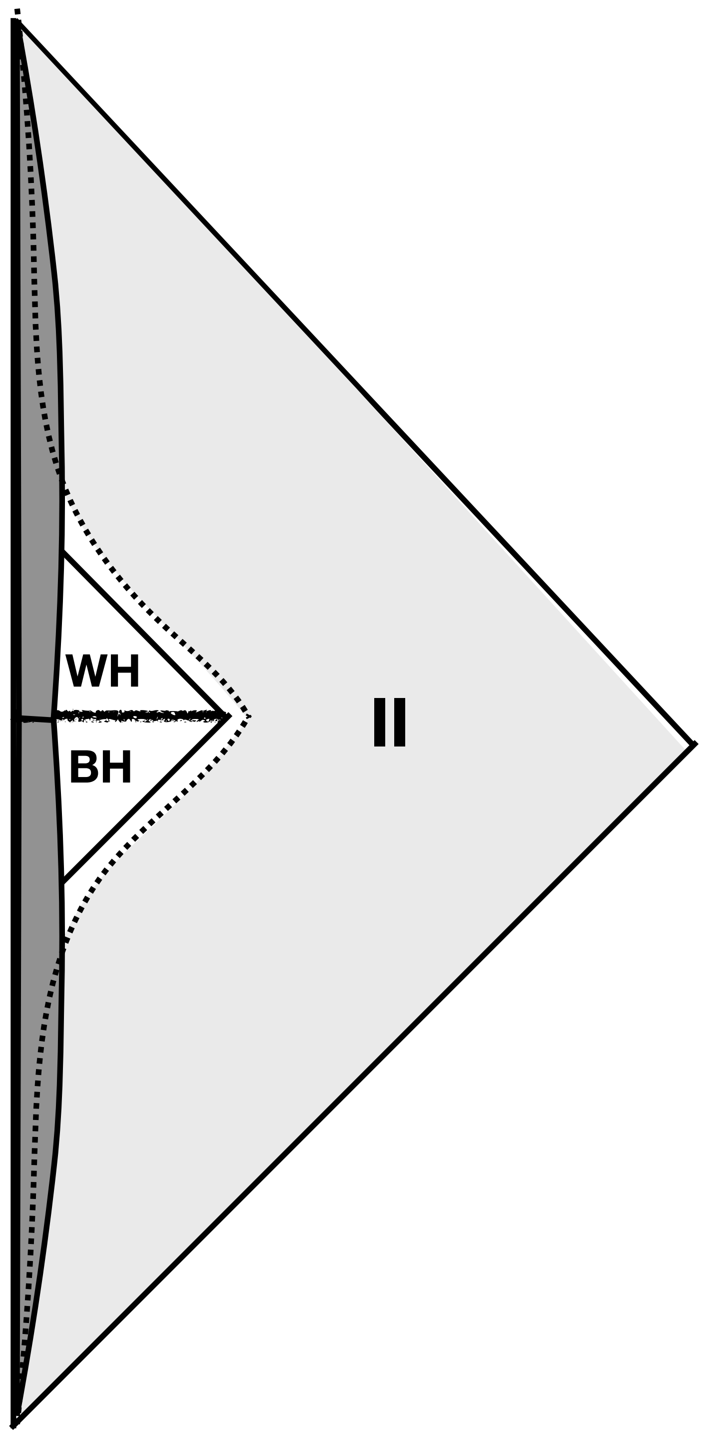
τ_{BH} τ_{WH} m

No event horizon

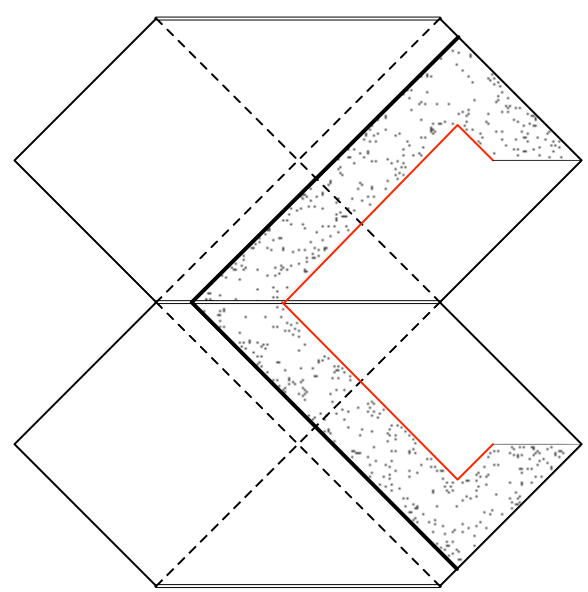
Information that enters the black hole escapes via the white hole.

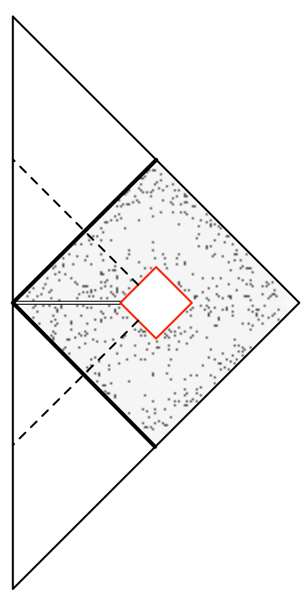
Modesto '04
Ashtekar Bojowald '05
Gambini Pullin '13
Vidotto CR '14
Corichi Singh '16
Speziale, Christodoulou, Vilensky CR '16
D'Ambrosio '18
Christodoulou D'Ambrosio '17
Ashtekar, Olmedo, Singh PRL'18
Bodendorfer, Mele, Münch '19
Martin-Dussaud, CR '19

The black to white hole transition



- **A black to white hole transition is allowed by a rapid quantum tunneling around the classical singularity**
  Haggard CR, 2015,

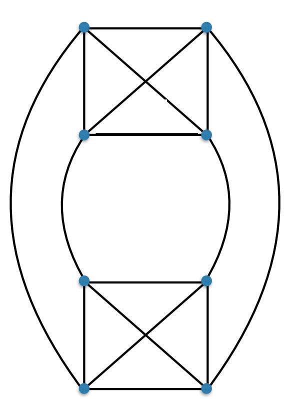


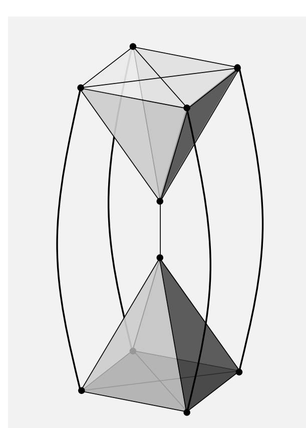


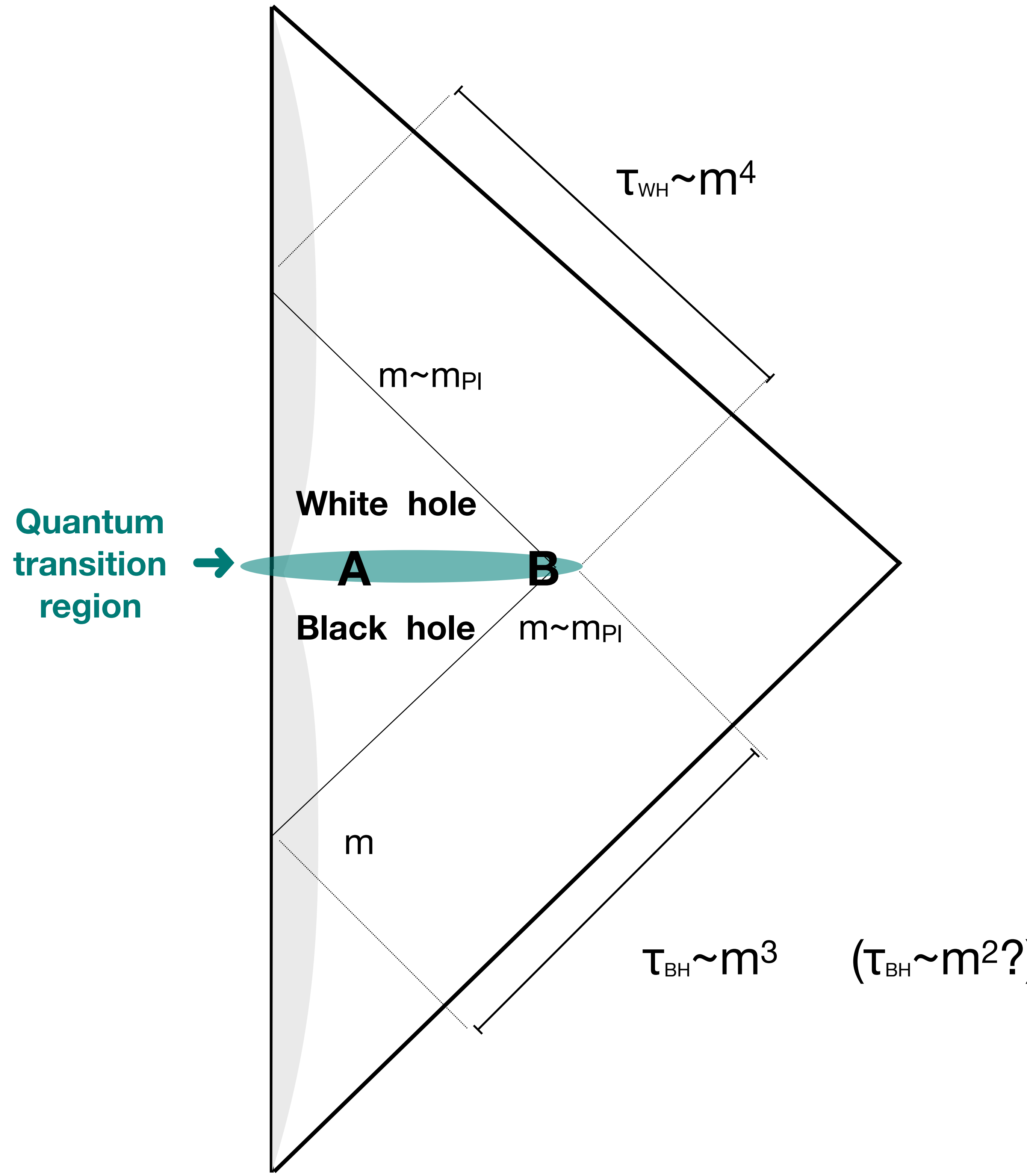
Time scales

- $\tau_{WH} \neq \tau_{BH}$: De Lorenzo, Perez, 2015

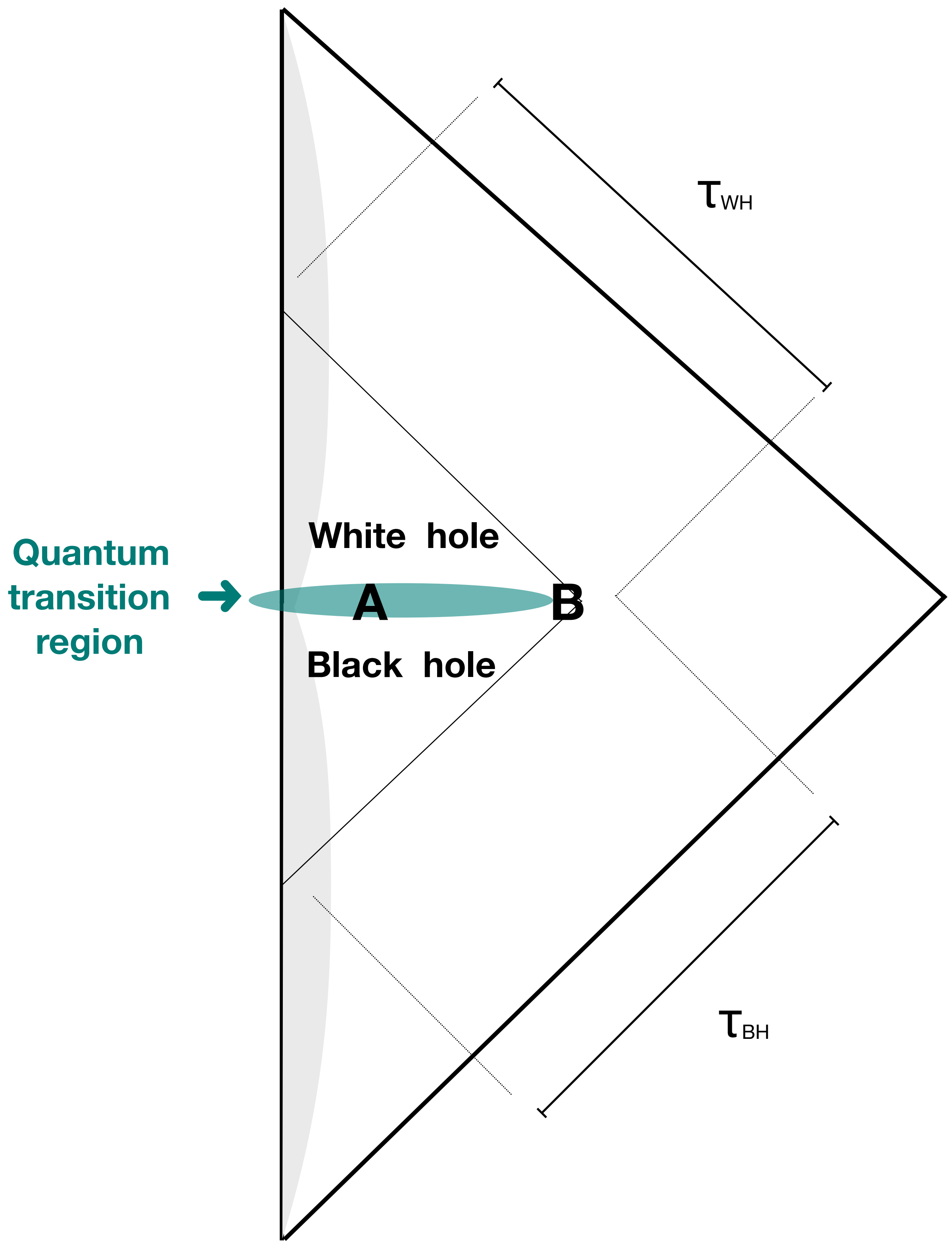
- Christodoulou Speziale Vilensky, C.R., PRD ‘16 *Planck star tunneling time: An astrophysically relevant observable from background-free quantum gravity*.
  - Christodoulou D’Ambrosio, '18, *Characteristic Time Scales for the Geometry Transition of a Black Hole to a White Hole from Spinfoams,* arXiv:1801.03027.

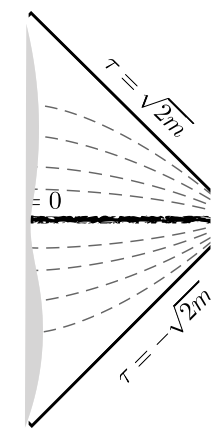


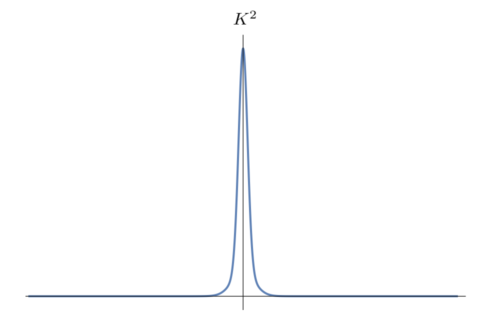




Region A transition





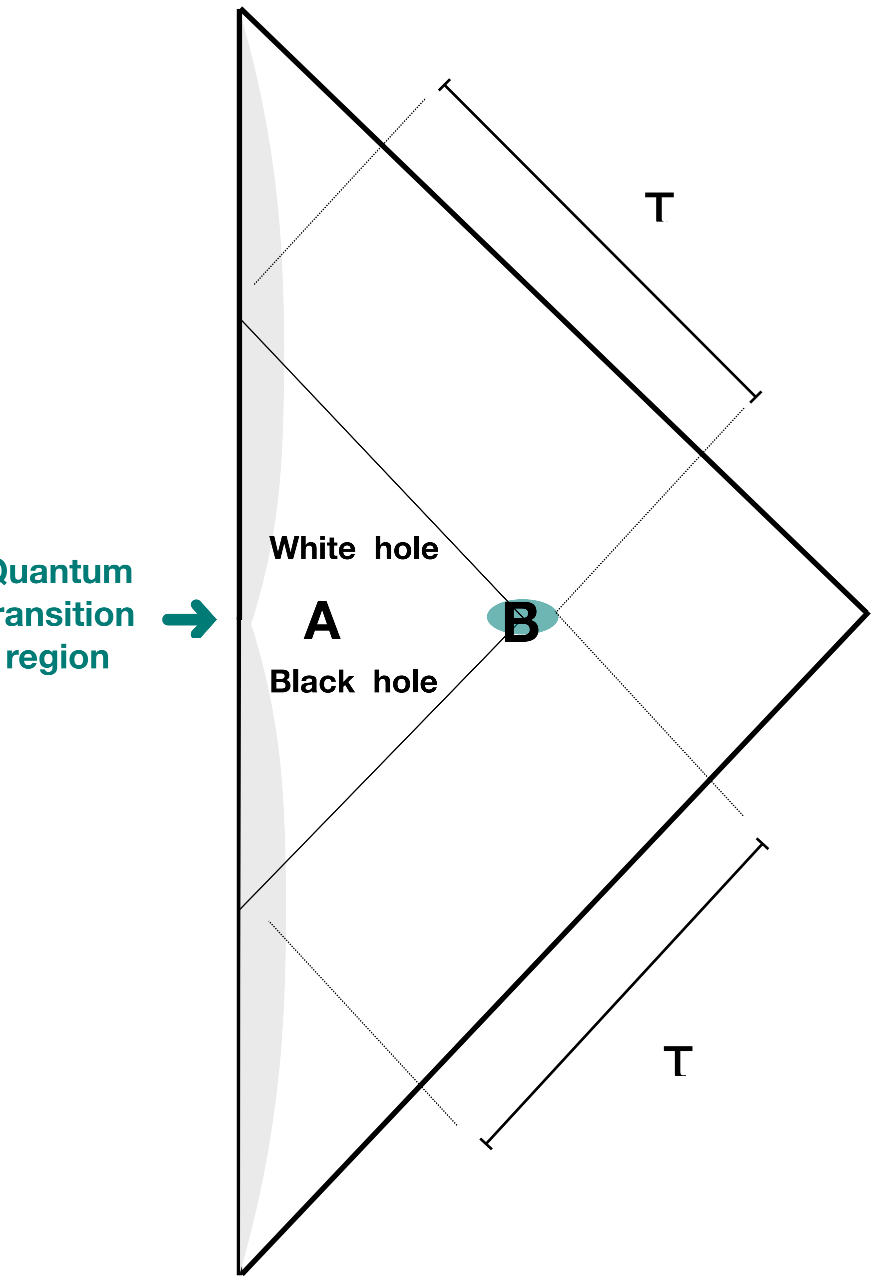


- - Ashtekar, Olmedo, Singh PRL'18, PRD '18, *Quantum Extension of the Kruskal Space-time*.
- Bodendorfer, Mele, Münch '19 *Effective Quantum Extended Spacetime of Polymer Schwarzschild Black Hole*.
- - Development from: Modesto PRD '04, Gambini Pullin PRL '13, …

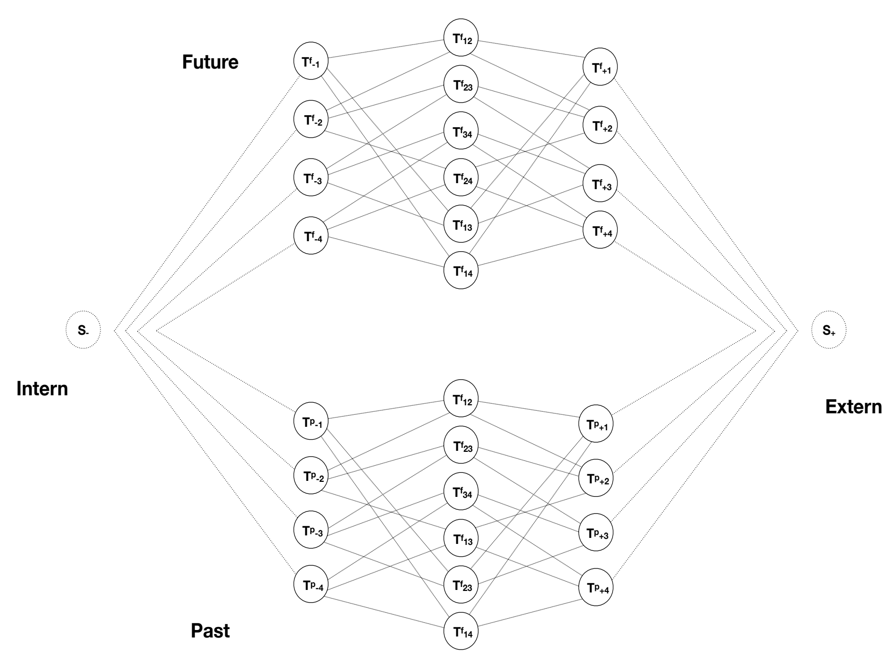
- **Approximate internal metric around the transition**

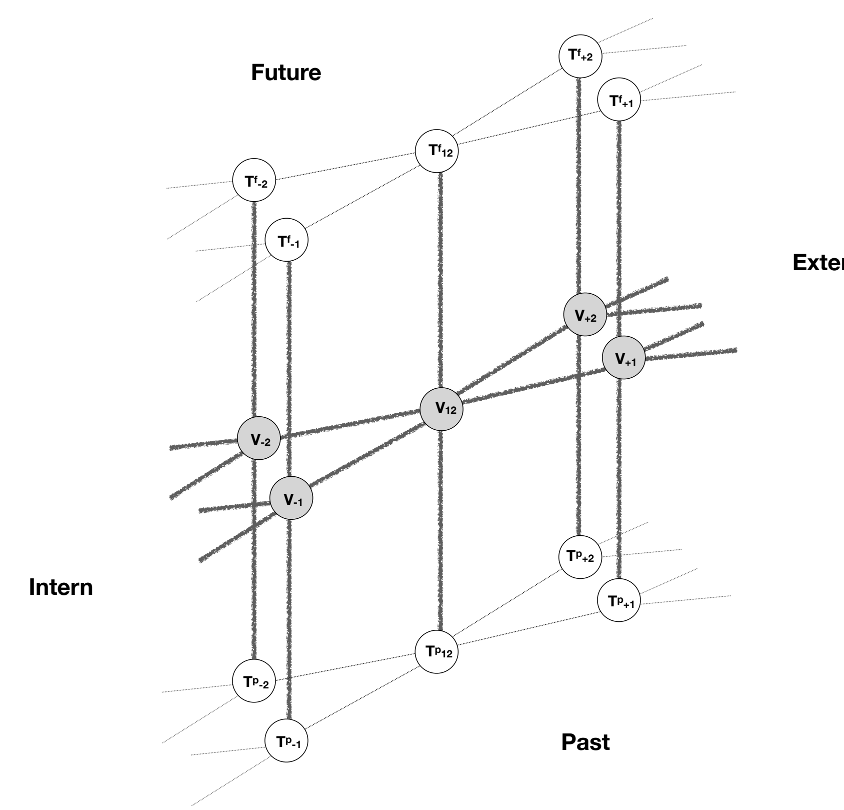
$$ds_l^2 = \frac{4(\tau^2+l)^2}{2m-\tau^2}d\tau^2 - \frac{2m-\tau^2}{\tau^2+l}dx^2 - (\tau^2+l)^2 d\Omega^2.$$

Region B transition



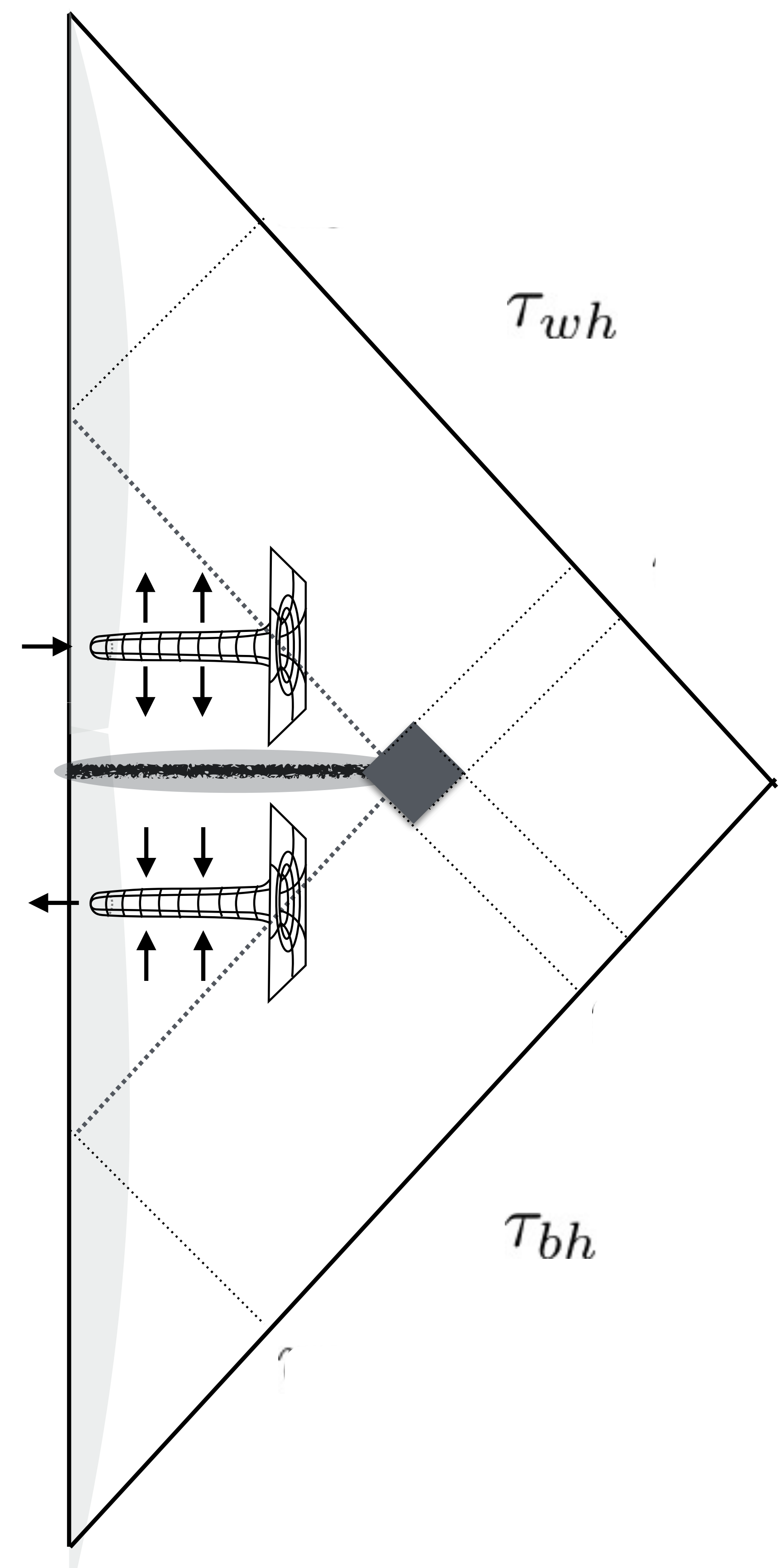
- **Spinfoam calculation** In course
- **Topology S2** X [0,1] X [0,1] **too hard for a triangulation**
- **➜ Use the Warsaw version of the spinfoam amplitude** **Kamiński Kisielowski Lewandowski 2009**





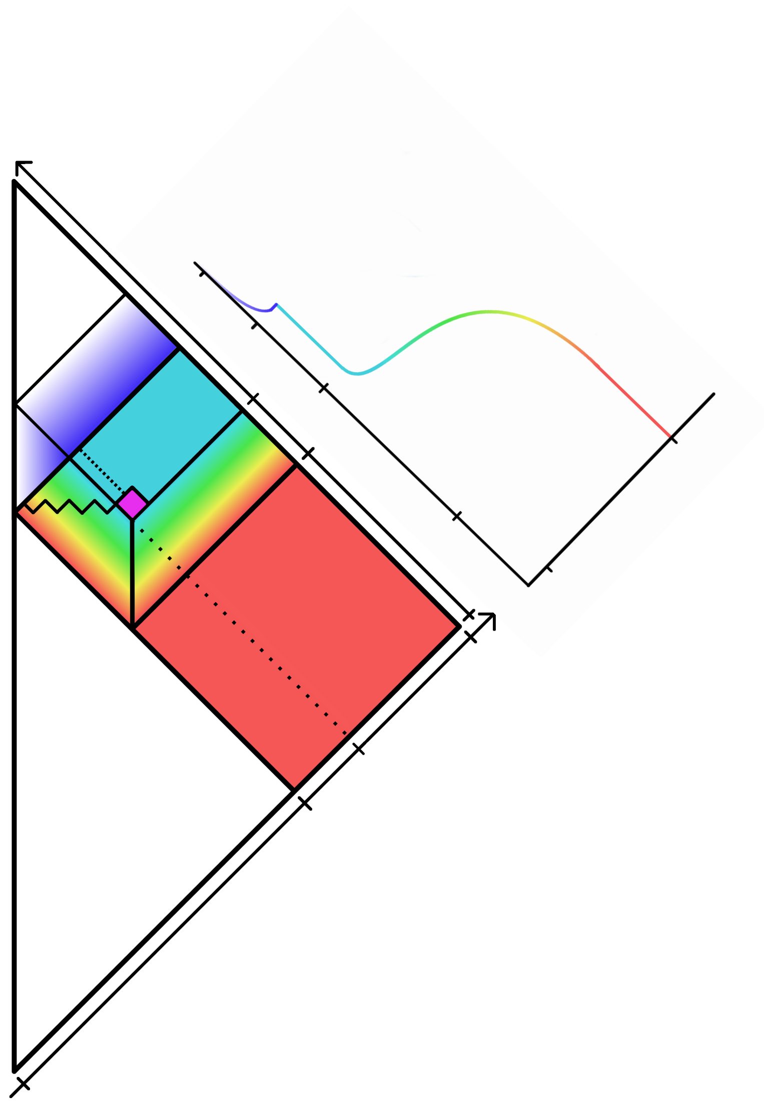
Internal geometry

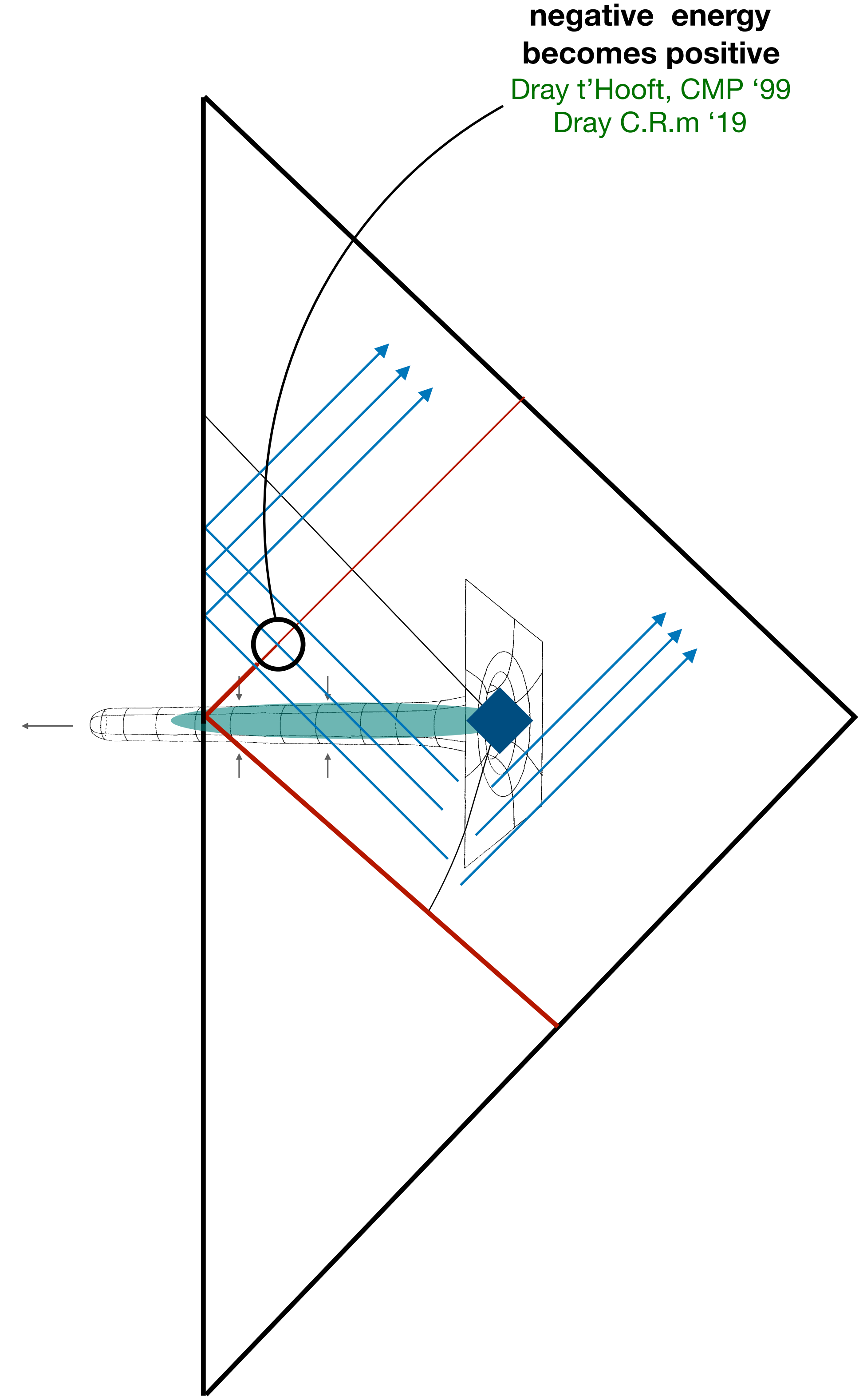
$$ds_l^2 = \frac{4(\tau^2+l)^2}{2m-\tau^2}d\tau^2 - \frac{2m-\tau^2}{\tau^2+l}dx^2 - (\tau^2+l)^2 d\Omega^2.$$



Internal geometry 2 (with Hawking radiation)

- The interior metric including the back reaction of the Hawking radiation has been modelled.
[Martin-Dussaud, C.R. '18, *Evaporating black-to-white hole*, arXiv: 1905.07251]





$$(I)\left[\begin{array}{l} ds^2=-du\,dv+r^2d\Omega^2 \\ r=\frac{1}{2}(v-u) \end{array}\right.$$

$$(II)\left[\begin{array}{l} ds^2=-\left(1-\frac{2m}{r}\right)du\,dv+r^2d\Omega^2 \\ r=2m\left(1+W\left(e^{\frac{v-u}{4m}-1}\right)\right) \end{array}\right.$$

$$(III)\left[\ ds^2=-\left(1-\frac{2N(v)}{r}\right)dv^2+2dvdr+r^2d\Omega^2\right.$$

$$(IV)\left[\ ds^2=-\left(1-\frac{2M(u)}{r}\right)du^2-2dudr+r^2d\Omega^2\right.$$

$$(V)\left[\begin{array}{l} ds^2=-du\,dv+r^2d\Omega^2 \\ r=\frac{1}{2}(v-u) \end{array}\right.$$

$$(V)\left[\begin{array}{l} ds^2=-\left(1-\frac{2m_1}{r}\right)du\,dv+r^2d\Omega^2 \\ r=2m_1\left(1+W\left(e^{\frac{v-u}{4m_1}-1}\right)\right) \end{array}\right.$$

$$(VIa)\left[\begin{array}{l} ds^2=\left(1-\frac{2m_1}{r}\right)du\,dv+r^2d\Omega^2 \\ r=2m_1\left(1+W\left(-e^{-\frac{v+u}{4m_1}-1}\right)\right) \end{array}\right.$$

$$(VIb)\left[\ ds^2=-\left(1-\frac{2N(v)}{r}\right)dv^2+2dvdr+r^2d\Omega^2\right.$$

$$(VII)\left[\ ds^2=-\left(1-\frac{2\mu(u,v)}{r}\right)du\,dv+r^2d\Omega^2\right.$$

$$(VIII)\left[\ ds^2=-\left(1-\frac{2P(u)}{r}\right)du^2-2dudr+r^2d\Omega^2\right.$$

$$(IX)\left[\begin{array}{l} ds^2=-du\,dv+r^2d\Omega^2 \\ r=\frac{1}{2}(v-u) \end{array}\right.$$

$$(Ia)\left[\begin{array}{l} u=-4m\left[1+W\left(-\frac{\tan U}{e}\right)\right] \\ v=-4m\left[1+W\left(-\frac{\tan(V+2V_0-\pi)}{e}\right)\right] \end{array}\right.$$

$$(Ib)\left[\begin{array}{l} u=-4m\left[1+W\left(-\frac{\tan U}{e}\right)\right] \\ v=f_1(V)\text{ increasing, such that} \\ \begin{cases} f_1(-2V_0+3\pi/4)=-4m(1+W(1/e)) \\ f_1(\pi/4)=0 \end{cases} \end{array}\right.$$

$$(Ic)\left[\begin{array}{l} u=c_1+f_1(U-2V_0+\pi) \\ v=c_1+f_1(V) \end{array}\right.$$

$$(II)\left[\begin{array}{l} u=-4m\log(-\tan U) \\ v=4m\log\tan V \end{array}\right.$$

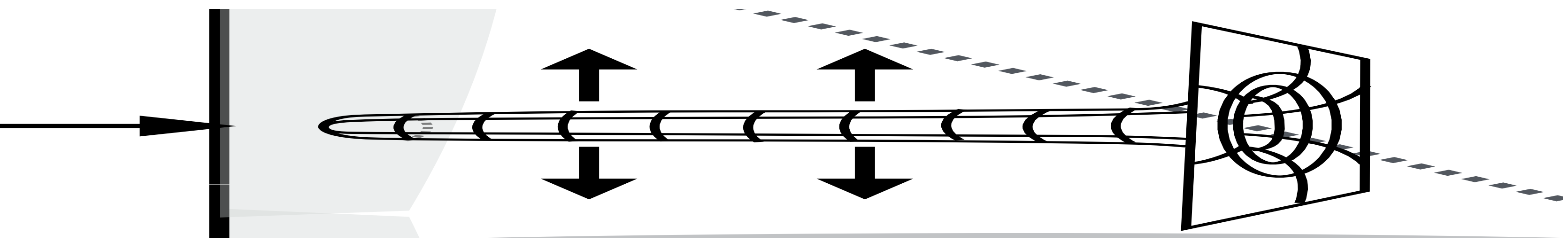
$$(III)\left[\begin{array}{l} v=f_2(V)\text{ increasing, such that} \\ \qquad\qquad f_2(\pi/4)=N^{-1}(M(0)) \\ r=g(U,V)\text{ such that} \\ \begin{cases} \frac{\partial g}{\partial V}=\frac{f_2'(V)}{2}\left(1-\frac{2N(f_2(V))}{g(U,V)}\right) \\ g(U,\pi/4)=-\frac{1}{2}f_1(U-2V_0+\pi) \\ g(2V_0-\pi/2-V,V)=0 \end{cases} \end{array}\right.$$

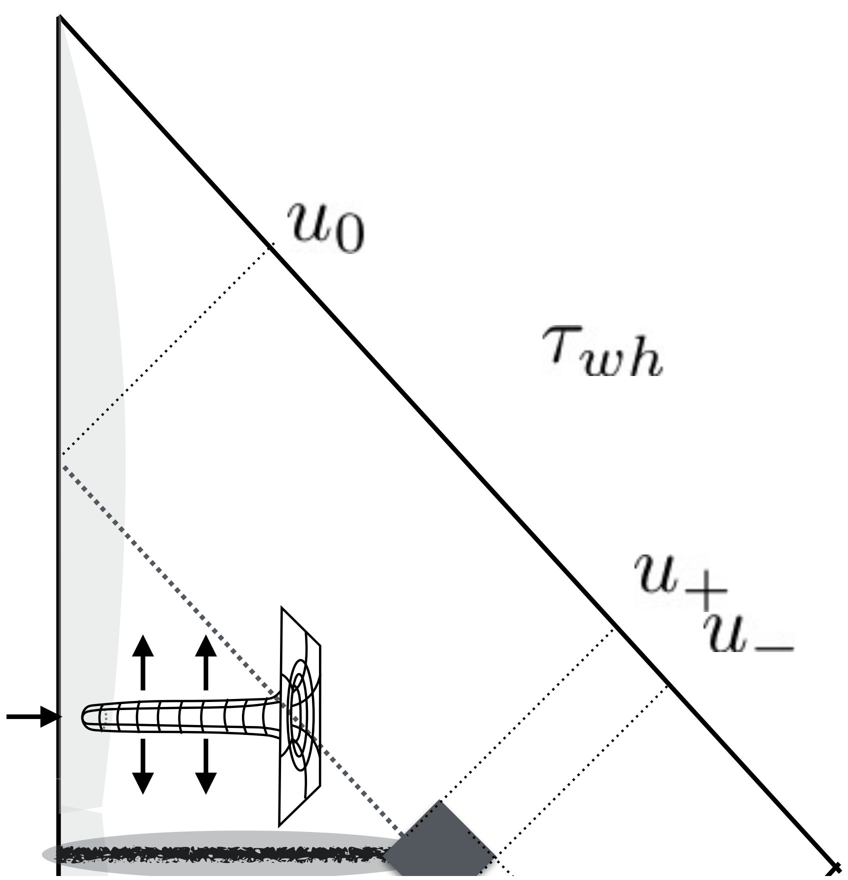
$$(IV)\left[\begin{array}{l} u=M^{-1}(N(f_2(U+\pi/2))) \\ r=h(U,V)\text{ such that} \\ \begin{cases} \frac{\partial h}{\partial U}=-\frac{u'(U)}{2}\left(1-\frac{2M(u(U))}{h(U,V)}\right) \\ h(-\pi/4,V)=2m\left(1+W\left(\frac{\tan V}{e}\right)\right) \\ h(U,\pi/2)=\infty \\ h(U,U+\pi/2)=g(U,U+\pi/2) \end{cases} \end{array}\right.$$

$$(V)\left[\begin{array}{l} v=M^{-1}(N(f_2(V_0)))+2h(V_0-\pi/2,V) \\ u=M^{-1}(N(f_2(V_0)))+2h(V_0-\pi/2,U+\pi/2) \end{array}\right.$$

- Martin-Dussaud, C.R. '18, *Evaporating black-to-white hole*, arXiv:1905.07251

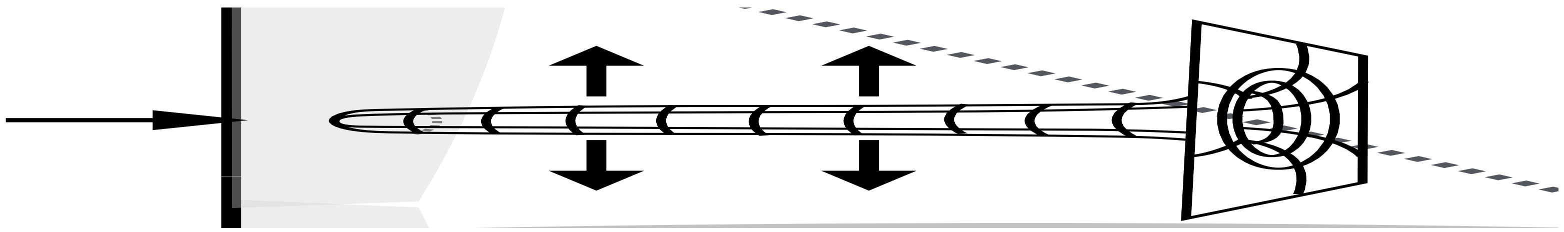
Remnant





- **Long wavelength modes trapped inside**
- **Planck scale particle with a large number of internal states, that can decay into low energy quanta only in a very long time.**

Stability



- Long wavelength modes trapped inside
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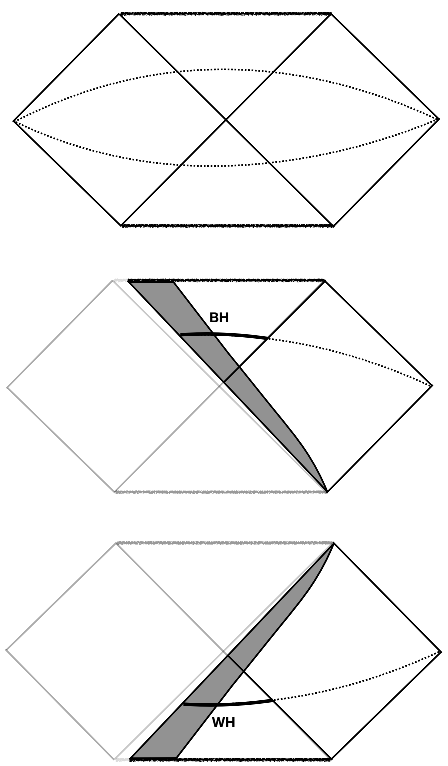

$$\text{States: } |H, m, v\rangle \qquad H = B,W$$

Processes

- **Black hole volume increase and white hole volume decrease.** Christodoulou CR, *How big is a black hole* 2015

$$|B, m, v\rangle \to |B, m, v+\delta v\rangle,$$
$$|W, m, v\rangle \to |W, m, v-\delta v\rangle.$$

$$\frac{dv}{dt} = \pm 3\sqrt{3}\pi m_o^2.$$

- **White to black instability.** Frolov and I.~Novikov, 2012

$$|W, m, v\rangle \to |B, m, v\rangle.$$

$$\tau_{W\to B} \sim m.$$

- **Hawking evaporation** Hawking 1973

$$|B, m, v\rangle \to |B, m-\delta m, v\rangle.$$

$$\frac{dm}{dt} = \frac{\hbar}{m^2}.$$

- **Black to white tunnelling** Christodoulou D'Ambrosio '17

$$|B, m, v\rangle \to |W, m, v\rangle.$$

$$p \sim e^{-\frac{m^2}{\hbar}}/m$$

Dynamics

- **Dynamics Microgram (Planck scale) white holes are stabilised by quantum mechanics.**
Vidotto CR., *2018*.

$$|\psi\rangle = \begin{pmatrix} B(m,v) \\ W(m,v) \end{pmatrix} \qquad i\hbar\,\partial_t|\psi\rangle = H|\psi\rangle$$

$$H = \begin{pmatrix} m + 3\sqrt{3}\ i\pi m_o^2 \frac{\partial}{\partial v} - i\,\frac{\hbar^2}{m^2}\frac{\partial}{\partial m} & b\frac{\hbar}{m} \\ c\frac{\hbar}{m}e^{-m^2/\hbar} & m - 3\sqrt{3}\ i\pi m_o^2 \frac{\partial}{\partial v} \end{pmatrix}$$

- **Area gap** ➜ $$\mu \equiv \frac{3^{\frac{1}{4}}}{2}\sqrt{\frac{\hbar}{G}}.$$
- **Reduced H on minimum mass eigenspace** $$H = \begin{pmatrix} \mu & \frac{b\hbar}{\mu} \\ \frac{a\hbar}{\mu} & \mu \end{pmatrix}$$
- **Lowest eigenstate** $$|R\rangle = \frac{\sqrt{\frac{a}{b}}|B,\mu\rangle - |W,\mu\rangle}{\sqrt{1+\frac{a}{b}}}$$

Final picture

- **Primordial black holes**

$$10^{-18}cm \leq R_o < 10^{-13}cm.$$
$$10^{10}gr \leq m_o^3 < 10^{15}gr.$$

↓

- **Hawking evaporation**

$$m_o^3 < T_H.$$

↓

- **Black to White quantum transition**

↓

- **White hole instability**

↓

- **Remnant = quasi stable quantum superposition of B&W hole**

$$\tau_{BH} \sim m_o^4.$$
$$m_o^4 \geq T_H.$$

Other possibility:
Remnants from
before the bounce

Vidotto, *Quantum insights on Primordial Black Holes as Dark Matter* 2018

- What happens at the end of the evaporation?

They tunnel int white holes

- What happens at r~0?

Spacetime continues
into a white hole

- B to W explosion and remnants can play a role in astrophysics and cosmology as Dark Matter.
- (Also
- Sources of cosmic rays
- Sources of FRB)