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Recent results in spherically symmetric loop quantum gravity

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Plan

- Quantum theory of spherically symmetric vacuum space-times.
  Applications:
- Hawking radiation.
- Casimir effect.
- Self gravitating shells.
- Bonus: axial symmetry.

Summary:

We have recently found, in closed form, the space of physical states corresponding to spherically symmetric vacuum space-times in loop quantum gravity.

We studied the quantization of test fields in such quantum space-times.
“Quantum field theory on a quantum space-time”.

Main message: the quantum background space-time acts as a lattice discretization of the field theory, naturally regulating it and eliminating infinities, but otherwise changing in only small ways the traditional picture of QFT on CST.

Spherically symmetric LQG Kastrup, Thiemann, mid 90's.

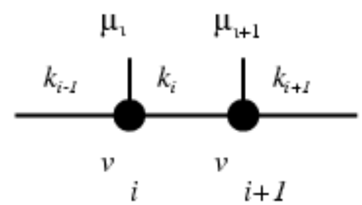
We use the variables adapted to spherical symmetry developed by Bojowald and Swiderski (CQG23, 2129 (2006)). One ends up with two canonical pairs, E^x, E^φ, K_x, K_φ.

$$g_{xx} = \frac{(E^\varphi)^2}{|E^x|}, \qquad g_{\theta\theta} = |E^x|,$$

$$K_{xx} = -\mathrm{sign}(E^x)\frac{(E^\varphi)^2}{\sqrt{|E^x|}} K_x \qquad K_{\theta\theta} = -\sqrt{|E^x|}\frac{A_\varphi}{2\gamma},$$

Kinematical states are given by one dimensional spin networks,

$$T_{g,\vec{k},\vec{\mu}}(K_x, K_\varphi) = \langle K_x, K_\varphi \Big|$$



$$\Big\rangle$$

$$= \prod_{e_j \in g} \exp\left(\frac{i}{2}k_j \int_{e_j} K_x(x)dx\right) \prod_{v_j \in g} \exp\left(\frac{i}{2}\mu_j \gamma K_\varphi(v_j)\right)$$

After a rescaling and combination of the constraints that turns their algebra into a Lie algebra, we were able to solve in closed form for the space of physical states of spherically symmetric vacuum LQG (RG, JP PRL 110, 211301)

A basis of he physical states are given by $|\tilde{g},\vec{k},M>$ with $\tilde{g}$ a diffeo equivalent class of one dimensional graphs, the k’s are proportional to the eigenvalues of the areas of symmetry and M is the ADM mass.

More details in: RG, J. Olmedo, JP, CQG 31 095009 (2014) arXiv:1310.5996

We were able to find in closed form the solution to the Hamiltonian constraint. This constitutes the physical space of states for pure gravity. Among other features, one can choose states that approximate Schwarzschild very well in the regions of low curvature, but the singularity is eliminated.

We will study quantum fields living on this quantum state.
For the combined system we assume the states have the form of a direct product between the gravity and the matter states.

We will represent the matter part of the Hamiltonian constraint as a Dirac observable of the gravitational degrees of freedom. This will allow to promote it to an operator that is well defined on the physical space of states.

Putting a field to live on the quantum space-time.

The main effect of considering the quantum vacuum is that the equations for the scalar field become similar to those of a scalar field discretized on a lattice and with a “dressed” metric. The lattice in this case is provided by the (one dimensional) spin network state of the background space-time.

$$\hat{H}_i = \hat{A}_i P_{\phi,i}^2 + \hat{B}_i \left(\phi_{i+1} - \phi_i\right)^2 + \hat{C}_i P_{\phi,i} \left(\phi_{i+1} - \phi_i\right)$$

For states with equally spaced nodes and $z(x)=x/x_{max}$,

$$\begin{aligned}
\hat{A}_i|\psi,\tilde{g},\vec{k}\rangle_{\rm grav} &= \frac{1}{2}\frac{\left[\sqrt{|\Delta k_i+1|}-\sqrt{|\Delta k_i-1|}\right]^2}{\ell_{\rm Planck}^2}\frac{\left(1+\mathcal{K}_{\varphi,i}^2-\frac{2GM}{\sqrt{\ell_{\rm Planck}^2 k_i}}\right)}{\sqrt{\ell_{\rm Planck}^2 k_i}}|\psi,\tilde{g},\vec{k}\rangle_{\rm grav},\\
\hat{B}_i|\psi,\tilde{g},\vec{k}\rangle_{\rm grav} &= 2\frac{\left[\sqrt{|\Delta k_i+1|}-\sqrt{|\Delta k_i-1|}\right]^2}{\ell_{\rm Planck}^2}\left(1+\mathcal{K}_{\varphi,i}^2-\frac{2GM}{\sqrt{\ell_{\rm Planck}^2 k_i}}\right)k_i^{3/2}\ell_{\rm Planck}^3|\psi,\tilde{g},\vec{k}\rangle_{\rm grav},\\
\hat{C}_i|\psi,\tilde{g},\vec{k}\rangle_{\rm grav} &= 2\frac{\left[\sqrt{|\Delta k_i+1|}-\sqrt{|\Delta k_i-1|}\right]^2}{\ell_{\rm Planck}^2}\sqrt{1+\mathcal{K}_{\varphi,i}^2-\frac{2GM}{\sqrt{\ell_{\rm Planck}^2 k_i}}}\mathcal{K}_{\varphi,i}\sqrt{\ell_{\rm Planck}^2 k_i}|\psi,\tilde{g},\vec{k}\rangle_{\rm grav}.
\end{aligned}$$

The spacing in the lattice is bounded by the condition of the quantization of the area of the surfaces of spherical symmetry. That condition implies that the points are separated a distance at least $L^2_{Planck}/(4GM)$ in the exterior of the black hole.

As a consequence, the discrete equations can be excellent approximations of the continuum equations at energies lower than the Planck energy, and most calculations are essentially those in the continuum.

One can proceed to define modes and in terms of them creation and annihilation operators and compute the vacua. The calculations of the Unruh, Boulware and Hartle-Hawking vacua resemble those of the continuum with very small corrections.

The main change is that certain trans Planckian modes that would have wavelengths smaller than the lattice spacing are suppressed. This implies that physical quantities that may diverge at horizons, like the stress energy tensor, remain finite. This may have implications for future attempts to do back reaction calculations.

The canonical equations for the scalar field correspond to a spatially discretized version of the Klein-Gordon equation in curved space time.

$$(\sqrt{-g}\,g^{ab}\phi_{,a})_{,b} = 0$$

The construction of quantum vacua is carried out considering modes that solve the wave equation and creation and anhilation operators for these modes. We will only sketch the properties fo the Boulware modes

$$z(x^*_j) = \frac{x(x^*_j)}{N_V \Delta} \qquad x^*_j = x_j + 2GM \ln\left(\frac{x_j}{2GM} - 1\right)$$

$$\partial_0^{\,2}\phi(x^*_j,t) - \left[\frac{\phi(x^*_{j+1},t) - \phi(x^*_j,t)}{\Delta_j^{\,2}} - \frac{\phi(x^*_j,t) - \phi(x^*_{j-1},t)}{\Delta_j \Delta_{j-1}}\right] = 0$$

$$\Delta_j = \Delta + \frac{2GM}{j}$$

Asymptotically $\Delta_j \to \Delta$ and one recovers an excellent aproximation to the Boulware vacuum.

Asymptotically, near scri$_-$ and scri$_+$ the modes are:

$$f_\omega = \frac{1}{\sqrt{2\pi\omega}} \exp(-i\omega_n t - ik_n x^*) \qquad g_\omega = \frac{1}{\sqrt{2\pi\omega}} \exp(-i\omega_n t + ik_n x^*)$$

with

$$\omega_n = \sqrt{\frac{2 - 2\cos(k_n \Delta)}{\Delta^2}} \text{ and } k_n = \frac{2\pi n}{(N_V - i_H)\Delta}$$

Near to the horizon trans Planckian modes are heavily suppressed due to the discreteness of the spin network state. In our treatment there are no arbitrary frequency trans-Planckian modes, the dispersion relation is modified in a sub-luminal way (it does not affect the horizon structure) and there are no singularities from physical quantities, like the expectation value of the stress energy tensor.

One can perform similar analyses for the Unruh and Hartle-Hawking vacua.

Calculation of Hawking radiation

R. Gambini, JP CQG 31, 115003 (2014)
arXiv: 1312.3595

The calculation of the Hawking radiation proceeds in the usual way, through the computation of the Bogoliubov coefficients.

$$\beta_{i_1,k} = \left(u_{i_1}^{\text{out}}, u_k^{\text{in}*}\right)$$

With the states

$$u_{\omega,\ell,m}^{\text{in}} = \frac{1}{\sqrt{4\pi\omega}} \frac{\exp\left(-i\omega U\right)}{r} Y_\ell^m\left(\theta,\varphi\right)$$

$$u_{\omega,\ell,m}^{\text{out}} = \frac{1}{\sqrt{4\pi\omega}} \frac{\exp\left(-i\omega u\right)}{r} Y_\ell^m\left(\theta,\varphi\right)$$

and u=t-x*, and U=-exp(u/(4GM)).

The usual calculation yields an expression for the number operator of the out photons in terms of the in states of the form,

$$\langle \mathrm{in}|N^{\mathrm{out}}_{i_1,i_2}|\mathrm{in}\rangle = \frac{t_{\ell_1}(\omega_1)t^*_{\ell_2}(\omega_2)\delta(\omega_1-\omega_2)}{2\pi\sqrt{\omega_1\omega_2}}\int_{-\infty}^{\infty} dz \exp\left(-i\frac{(\omega_1-\omega_2)}{2}z\right)\left(\frac{\kappa}{2}\right)^2 \frac{\delta_{\ell_1,\ell_2}\delta_{m_1,m_2}}{\sinh^2\left(\frac{\kappa}{2}\left(z-i\epsilon\right)\right)},$$

With $z=u_2-u_1$ and i_1,i_2 the labels associated with the in and out states.

This expression has problems at z=0, hence the addition of the $i\varepsilon$ term. In our approach, discreteness leads to $|z|> L_{\mathrm{Planck}}$, so that problem is eliminated. This is exactly the heuristic cutoff that had been proposed by Agulló, Navarro-Salas, Olmo and Parker PRD80, 047503 (2009)! The corrected expression for the Hawking radiation is,

$$\langle \mathrm{in}|N^{\mathrm{out}}_{i_1,i_2}|\mathrm{in}\rangle = \frac{|t_\ell(\omega)|^2}{\exp\left(2\pi\omega/\kappa\right)-1} - \frac{\kappa^2\ell_{\mathrm{Planck}}}{96\pi^3\omega}$$

Notice that it is remarkable that the cutoff that arises naturally does not interfere with the Hawking radiation.

The Casimir effect on a quantum geometry

R. Gambini, J. Olmedo, JP arXiv: 1410.4479
CQG 32, 11502 (2015)

To compute the Casimir force we will need to compute the integral of the expectation value of the T00 component of the stress energy in the region between the two shells, integrate it, and compute its derivative with respect to the separation of the shells. We will assume the shells are very far away from the origin (or black hole) to be able to ignore the centrifugal potential.

We consider a conformally coupled massless scalar field. The relevant component of the (improved) energy momentum tensor is,

$$T_{00} = \frac{1}{2}\dot{\phi}^2 + \frac{1}{6}\left(\phi'\right)^2 - \frac{1}{3}\phi\ddot{\phi}.$$

We begin by considering the modes for a scalar field,

$$u_{n,l,m} = \exp(-i\omega(t+r))Y_{\ell,m}(\theta,\varphi)/(\sqrt{4\pi\omega}r),$$

And imposing Dirichlet boundary conditions at the shells, the fields take the form,

$$\phi\left(t,r_j\right) = \sqrt{\frac{2\pi}{N_L\Delta}}\sum_{n=1}^{N_L-1}\sum_{\ell=0}^{\frac{2r_0}{\Delta}}\sum_{m=-\ell}^{\ell}\left[\frac{a_{n,\ell,m}e^{-i\omega_{n,\ell}t}}{\sqrt{2\pi\omega_{n,\ell}}}\frac{\sin\left(\frac{\pi nj\Delta}{N_L\Delta}\right)}{r_j}Y_{\ell,m}\left(\theta,\varphi\right) + \frac{a^\dagger_{n,\ell,m}e^{i\omega_{n,\ell}t}}{\sqrt{2\pi\omega_{n,\ell}}}\frac{\sin\left(\frac{\pi nj\Delta}{N_L\Delta}\right)}{r_j}Y^*_{\ell,m}\left(\theta,\varphi\right)\right]$$

Which corresponds to a spherical sector from r_0 to r_0+L with $N_l\,\Delta$=L

The dispersion relation is typical of a lattice $\omega_n^2 = (2 - 2\cos(k_n\Delta))/\Delta^2$ and $k_n = (2\pi n)/(N_I\Delta)$.
and the creation and annihilation operators have the usual commutation relations.

To compute the expectation value of the stress tensor we need to compute radial and time derivatives of the field. We start from Green's function,

$$G_+^L(x, x') = \langle 0_L|\phi(x)\phi(x')|0_L\rangle.$$

Which can be readily computed with the fields of the previous slide.

From there we then compute,

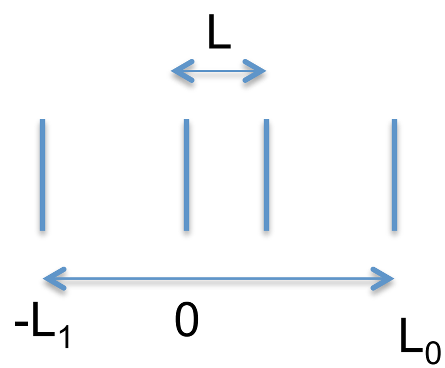
$$\begin{aligned}\langle 0_L|\dot{\phi}^2|0_L\rangle &= \frac{\partial^2}{\partial t\,\partial t'} G_+^L\Big|_{x=x'} = \frac{1}{4\pi L r_0^2}\sum_{n=1}^{N_L-1} \frac{2r_0^2}{3}\sin^2(k_n z)\left[\frac{8}{\Delta^3} + \frac{12}{\Delta^3}\sin^2\left(\frac{\Delta k_n}{2}\right)\right. \\ &\left. -\frac{8}{\Delta^3}\sin^3\left(\frac{\Delta k_n}{2}\right) + \frac{3}{\Delta^3}\sin^4\left(\frac{\Delta k_n}{2}\right) + \ldots\right].\end{aligned}$$

$$\langle 0_L| - \phi\ddot{\phi}|0_L\rangle\Big| = -\frac{\partial^2}{\partial^2 t'}\, G_+^L\big|_{x=x'} = \langle 0_L|\dot{\phi}^2|0_L\rangle.$$

And the stress energy tensor $\langle 0_L|T_{00}|0_L\rangle = \langle 0_L|\dot{\phi}^2|0_L\rangle,$

With higher order terms all finite in the limit Δ->0

To compute the Casimir force, we conduct the previous calculation for a slab of width $L_0 > L$ and add up all the energies inside the slab L and in the regions between L and L_0, and differentiate with respect to L,



$$\begin{aligned}
\text{Force} &= -\frac{d}{dL}\left(\int_{-L_1}^{0} dz \langle 0_{L_1}|T_{00}|0_{L_1}\rangle + \int_0^L dz \langle 0_L|T_{00}|0_L\rangle + \int_L^{L_0} dz \langle 0_{\tilde{L}_0}|T_{00}|0_{\tilde{L}_0}\rangle\right) \\
&= -\frac{\pi^2}{480 L^4} + \mathcal{O}(L_0^{-1}) + \mathcal{O}(\Delta).
\end{aligned}$$

This is the correct result, including the numerical coefficient. If one repeats the calculation for the s mode only, one gets the right 1+1D result.

No regularization nor renormalization are needed!

Self gravitating shells M. Campiglia, R. Gambini, J. Olmedo, JP arXiv:1601:05688

$$
\begin{aligned}
H_T \; = & \int dx \Big[- N' \Big(- \sqrt{|E^x|} \left(1 + K_\varphi^2\right) \\
& + \frac{\left[(E^x)'\right]^2 \sqrt{|E^x|}}{4 \left(E^\varphi\right)^2} + F(r) p \, \Theta(x - r) + 2M \Big) \\
& + N^x \left[-(E^x)' K_x + E^\varphi K_\varphi' - p \, \delta\left(x - r\right) \right] \Big] \\
& + 2N_- M + N_+ \left[F(r) p + 2M \right] ,
\end{aligned}
$$

$$
F(r) \; = \; \sqrt{|E^x|} \left(\eta \left(E^x\right)' \left(E^\varphi\right)^{-2} + 2K_\varphi \left(E^\varphi\right)^{-1} \right) |_{x=r}
$$

There exist new Dirac observables associated with the mass of the shell and the starting point at scri- from where it is launched.

A lengthy calculation shows that a redefinition like the one in vacuum yields an Abelian Hamiltonian and that it survives polymerization and quantization.

One can carry out a quantization in the same space of states considered before, tensor a space of square integrable functions for the shell variables.

We at the moment do not know how to find the physical space of states in closed form, as we were able to do in the vacuum case.

Although we cannot solve for the self-gravitating shell in a closed form, we use the partial results we have to motivate an approximate calculation. We will keep that the shell has canonical variables related to its mass and initial position at scri- and ignore the other details of loop quantum gravity based quantum geometry.

We then study quantum fields living on these shells using the **geometric optics approximation.**

The Bogoliubov coefficients now become operators on the quantum geometry.

Traditional geometric optics calculation of the Hawking radiation of a collapsing shell (Hawking 1975)

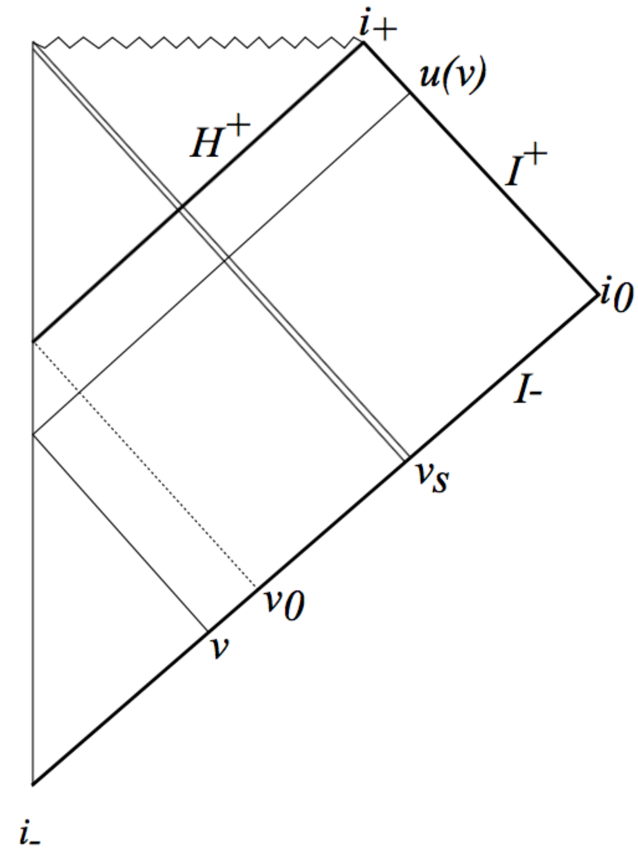
Metric: $$ds^2 = -\left(1 - \frac{2M\theta(v - v_s)}{r}\right) dv^2 + 2dvdr + r^2 d\Omega^2,$$

Geometric optics: $$u(v) = v - 4M \ln\left(\frac{v_0 - v}{4M_0}\right),$$

In modes: $$\psi_{lm\omega'}(r, v, \theta, \phi) = \frac{e^{-i\omega' v}}{4\pi r\sqrt{\omega'}} Y_{lm}(\theta, \phi)$$

Out modes: $$\chi_{lm\omega}(r, u, \theta, \phi) = \frac{e^{-i\omega u(v)}}{4\pi r\sqrt{\omega}} Y_{lm}(\theta, \phi),$$

Bogoliubov coefficients: $$\beta_{\omega\omega'} = -\langle \chi_{lm\omega}, \psi^*_{lm\omega'} \rangle$$



Particles produced:

$$\langle N^H_\omega \rangle = \int_0^\infty d\omega' \beta_{\omega\omega'} \beta^*_{\omega\omega'}$$ = some calculations= $$\frac{1}{e^{8M\omega\pi} - 1}$$

Hawking radiation in a collapsing quantum shell:

We keep from the full model that: $\left[\widehat{M}, \widehat{v}_s\right] = i\hbar\widehat{I},$ e.g. $\psi(M)$

The quantum time of arrival:

$$\hat{u}\left(v, \widehat{v}_0, \widehat{M}\right) = v\widehat{I} - 2\left[\widehat{M}\ln\left(\frac{\widehat{v}_0 - v\widehat{I}}{4M_0}\right) + \ln\left(\frac{\widehat{v}_0 - v\widehat{I}}{4M_0}\right)\widehat{M}\right]$$

Bogoliubov coefficients become operators

$$\hat{\beta}_{\omega\omega'} = -\frac{1}{2\pi}\sqrt{\frac{\omega'}{\omega}}\lim_{\epsilon\to 0}\int_{-\infty}^{+\infty} dv\theta\left(\widehat{v}_0 - v\widehat{I}\right) e^{-i\omega\hat{u}_\epsilon(v) - i\omega' v}\theta\left(\widehat{v}_0 - v\widehat{I}\right)$$

One has to be a bit careful about the domain of the quantum time of arrival, since the expression is valid for $v_0 > v$. It turns out that suitable extensions can be found and one can show that the results are independent of them.

$$\hat{u}_\epsilon\left(v, \widehat{v}_0, \widehat{M}\right) = v\widehat{I} - 2\left[\widehat{M} f_\epsilon\left(\frac{\widehat{v}_0 - v\widehat{I}}{4M_0}\right) + f_\epsilon\left(\frac{\widehat{v}_0 - v\widehat{I}}{4M_0}\right)\widehat{M}\right] \qquad f_\epsilon(x) = \begin{cases} \ln(x), & x \geq \epsilon \\ \ln(\epsilon), & x < \epsilon \end{cases}$$

We were able to compute the number operator expectation value.,

$$\left\langle N^{QS}_{\omega_1\omega_2}\right\rangle = \int_0^\infty d\omega' \left\langle \hat{\beta}_{\omega_1\omega'}\hat{\beta}^*_{\omega_2\omega'}\right\rangle$$

For a squeezed state,

$$\langle N^{QS}_{\omega_j}\rangle \sim \frac{1}{e^{8M\omega_j\pi}-1}\left[\frac{1}{2}+\frac{i}{2\pi}\int_{-\epsilon}^{\epsilon} d\left(\Delta\omega\right)\frac{\epsilon-|\Delta\omega|}{\epsilon\Delta\omega}e^{-\alpha\Delta\omega i}e^{-\Delta\omega^2\frac{\sigma^2}{4}}\right] \quad \alpha = u_n - v_0 - 4M\ln\left(4M_0\omega_j\right)$$

It again has the correct classical limit up to a factor,

$$\left\langle N^{QS}_{\omega_1\omega_2}\right\rangle = e^{-[\omega_1-\omega_2]^2\frac{\sigma^2}{4}}\left\langle N^{CS}_{\omega_1\omega_2}\right\rangle$$

Although we have done the explicit calculations for a Gaussian squeezed state, one can see that for any peaked state in M, v_0, similar results would hold. We get Hawking radiation.

However, the radiation obtained differs from the one obtained on a classical Background space-time.

For instance, information about the initial quantum state persists in correlations like,

$$\langle N_{\omega_1} N_{\omega_2} \rangle - \langle N_{\omega_1} \rangle \langle N_{\omega_2} \rangle$$

Which vanish for the calculation on a classical space-time.

More generally, the quantum density matrix has non-vanishing off-diagonal ρ_{ω_1,ω_2} elements.

The general expressions are complicated. To illustrate the issue of information loss it is good to consider the semiclassical limit, $\hbar \to 0, \Delta v_H \to 0 \quad \text{with} \quad \Delta M = \hbar/\Delta v_H$

$$N_\omega = \frac{1}{e^{8\pi \bar{M}\omega} - 1} + \frac{1}{32\pi \bar{M}\omega} \int_{-\infty}^{+\infty} dy e^{-i4\bar{M}\omega y} \frac{1 - e^{-4\Delta M^2 \omega^2 y^2}}{\sinh^2(y/2)},$$

Which correctly vanishes for Δ->0.

The message seems to be that the new correlations that arise in the Hawking radiation due to the fuzziness of the quantum geometry (and the horizon) allow to retrieve the information of the state that collapsed to form the black hole (the extra term in the expression is essentially the Fourier transform of the initial state of the shell).

R. Eyheralde, M. Campiglia, R. Gambini, JP, CQG 34 (2017) no.23, 235015
also arXiv:1908.04270

Bonus: axial symmetry

We choose adapted coordinates

$$\{x, y, \phi\} \qquad \phi \in S^1 \qquad x, y \in \mathbb{R}$$

The Killing vector is $K^a = (\partial_\phi)^a$

Invariance under the symmetry it generates is given by

$$\mathcal{L}_{\tilde{K}} A_a^i = \epsilon_{ijk} \lambda^j A_a^k \quad \text{with} \quad \tilde{K} = \lambda_i K^i = \lambda_3 \partial_\phi \qquad \lambda_1 = \lambda_2 = 0$$

Which reduces to $\partial_\phi A_a^i = \epsilon_{i3k} A_a^k$ with a similar equation for the triads.

Notice that we choose the Lie derivative proportional to an *O(2)* rotation.
This had not been done in previous treatments,
Husain, JP, MPLA 5, 733 (1990).
If one does not do this not all degrees of freedom are recovered
Benguria, Cordero, Teitelboim NPB122, 61 (1977).

The most general solution to the symmetry equation is,

$$A = A_a^i \tau_i \mathrm{d}x^a = \left((\cos(\phi)\tau_1 + \sin(\phi)\tau_2)\mathbf{a}_a^1 + (-\sin(\phi)\tau_1 + \cos(\phi)\tau_2)\mathbf{a}_a^2 + \mathbf{a}_a^3\tau_3\right)\mathrm{d}x^a$$
$$E = E_i^a \tau^i \partial_a = \left((\cos(\phi)\tau^1 + \sin(\phi)\tau^2)\mathbf{e}_1^a + (-\sin(\phi)\tau^1 + \cos(\phi)\tau^2)\mathbf{e}_2^a + \mathbf{e}_3^a\tau^3\right)\partial_a,$$

The non-trivial Poisson brackets reduce to $\{\mathbf{a}_a^i(\vec{x}), \mathbf{e}_j^b(\vec{x}')\} = 4G\beta\, \delta_j^i \delta_a^b \delta^{(2)}(\vec{x} - \vec{x}')$

The determinant of the triad becomes

$$E = \det(E) = \frac{1}{3!}\varepsilon_{abc}\varepsilon^{ijk} E_i^a E_j^b E_k^c = \frac{1}{3!}\varepsilon_{abc}\varepsilon^{ijk} \mathbf{e}_i^a \mathbf{e}_j^b \mathbf{e}_k^c = \det(\mathbf{e}) = \mathbf{e}.$$

And the spatial metric can be written as $q_{ab} = E E_a^i E_b^i = \mathbf{e}\, \mathbf{e}_a^i \mathbf{e}_b^i$

And it only depends on the reduced triads.

We showed that the Kerr solution solves all the constraints and equations of motion that result.

We studied the boundary terms needed to make the action differentiable in the asymptotically flat context.

Details in CQG (2019) no.12, 125009

Hints at quantization:

$$U = \exp\left(i\int_\sigma dx^a a_a^j \tau^j\right)\exp\left(i a_\phi^3 c_\phi^3 \tau^3\right)$$

A genuine holonomy in the σ plane (r,θ) and a point holonomy in the ϕ direction.

One would have two dimensional spin networks with insertions corresponding to the ϕ direction.

There is the issue of the domain of σ, (0,infinity)x(0,π).

A preliminary analysis of the Hamiltonian constraint shows that all added vertices are exceptional, which suggests that an implementation of Thiemann's Hamiltonian will close the constraint algebra.

This becomes an ideal arena to test the new ideas for the Hamiltonian that Madhavan Varadarajan is proposing.

To be continued⋯

Summary:

- We can solve LQG with spherical symmetry in closed form.
- We can formulate quantum field theory in quantum space-times for fields on spherically symmetric gravity backgrounds.
- It approximates quantum field theory in curved space time very well.
- Discreteness naturally regularizes physical quantities, opening the possibility of back reaction calculations.
- Hawking radiation can be computed and the result coincides with previous heuristic results.
- The Casimir effect can be computed and the right dependence on separation is obtained.
- Self gravitating shells studied on the quantum space-time, opening the possibility of back reaction calculations.
- Geometrical optics calculations on the quantum geometry of a shell suggest that information may leak through the Hawking radiation.
- Generalization to the axisymmetric case is on its way. New arena to test the dynamics.

- Happy birthday Jurek!