On the dynamics of loop quantum Universe

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Principles/properties of LQG

- Main features:
  - Manifest background independence of fudamental components.
  - Quantization og GR.
  - Geometry data revovered thorugh quantum observables.
  - Quantization of extended objects (holonomies and fluxes).
  - Systematic application of Dirac program for constraint systems.
  - Quantization program completed via coupling to matter (matter clocks, explicit action of the Hamiltonian)
- Limitations:
  - Enormous technical complication
  - No physically plausible genuine dynamical predictions yet.

Loop Quantum Cosmology

LQC - application of LQG methods and components to simpler, highly symmetric scenarios

- (Early) History
  - First idea: Bojowald, ∼2000
  - Mathematical structure: Ashtekar, Bojowald, Lewandowski, 2003
  - Genuine quantum dynamics: Ashtekar, TP, Singh, 2006
  - Hybrid quantization of inhomogeneous models: Garaym Martín-Benito, Mena-Marugán, 2009
  - Perturbations and CMB imprint: Agulo, Ashtekar, Nelson, ..., 2012
- Main achievements
  - Control over (genuine) quantum dynamics for homogeneous models
  - Cornucopia of results using (phenomenological) classical effective approach
  - Evolution of perturbations via semiclassical/dressed metric approach

Jurek's involvement in LQC (very incomplete list)

- Mathematical structure (kinematics) of LQC.
  - Ashtekar, Bojowald, Lewandowski, 2003, "Mathematical structure of loop quantum cosmology".
  - Kamiński, Lewandowski, Szulc, 2006, "Closed FRW model in LQC"
- Selfadjointness of evolution generators.
  - Kamiński, Lewandowski, 2007 "The Flat FRW model in LQC: The Self-adjointness".
- The role of time and the notion of evolution.
  - Kamiński, Lewandowski, TP, 2009, "Quantum constraints, Dirac observables and evolution: Group averaging versus Schrodinger picture in LQC".
- First studies of elements of field theory on LQC background.
  - Ashtekar, Kamiński, Lewandowski, 2009, "Quantum field theory on a cosmological, quantum space-time".

Model and Dirac quantization program

- Fix a model: FRW flat universe with massless scalar field.
- Variables: fluxes of triads and holonomies of Ashtekar connections $\to$ functions of coeffcients $(v \propto V, b \propto H_r)$.
- Action: Einstein-Hilbert + canonical $3+1$ splitting. $->$ algebra of constraints
- Kinematical level quantization:
  - Hilbert space: $\mathcal{H}_{\mathrm{kin}} = L^2(\bar{\mathbb{R}}, \mathrm{d}\mu_v) \otimes L^2(\mathbb{R}, \mathrm{d}\phi)$, basis of eigen. flux operator.
  - Basic operators: volume $\hat{V}|v\rangle = \alpha|v||v\rangle$ and $U(1)$ components of holonomies $N_\lambda|v\rangle = |v+\lambda\rangle$.
- Rewriting the sole nontrivial constraint (Hamiltonian) in terms of basic operators –Thiemann regularization.
- Finding the kernel of the constaint by group averaging (spectral decomposition).

The Hamiltonian constraint

- (Gravitational) Hamiltonian constraint splits into two parts:
  - Euclidean: depending on the curvature of Ashtekar connection.
  - Lorentzian: depending on extrinsic curvature.
- The Euclidean part is regularized via approximating the curvature by holonomies along square loop.
  - Area gap: “borrowing” the lowest eigenvalue of area operator from full LQG and setting loop area to it fixes $\lambda = 1/2$.
- The Lorentzian part can be regularized in several different ways: for the full theory two proposals.

Thiemann regularization: Lorentzian part

- The Lorentzian part

$$C_L = -2(1+\gamma^2)\int_{\mathcal{V}} \mathrm{d}^3x |\det E|^{-1/2} E^a_i E^b_j K^i_{[a} K^j_{b]}$$

- For Lorentzian part two schemes of regulatization
  - Spatial Ricci scalar extraction (traditional for LQC):
    - Express $K^i_a = \gamma^{-1}(A^i_a - \Gamma^i_a)$.
    - In flat FRW only the first term ($\propto C_E$) contributes

$$C_g = -\gamma^{-2} C_E$$

    - In full LQG spatial Ricci scalar quatized independently: Alesci, Assanioussi, Lewandowski 2014.
  - The Full Thiemann regularization (original proposal for LQG):

$$K^i_a \propto \lambda^{-1}\gamma^{-2} h^{(\lambda)}_i \{(h^{(\lambda)}_i)^{-1}, \{C_e, V\}\}$$

    - Qualitatively different structure than that of C_E.

Quantum Hamiltonian constraint

Ashtekat, TP, Singh, 2006

- Quantum Hamiltonian constraint built by replacing V and $h^{(\bar{\mu})}$ by operators.

$$\boxed{\hat{C} = \mathbb{I} \otimes \hat{p}_\phi^2 - \hat{\Theta} \otimes \mathbb{I}}$$

  where $\hat{\Theta}$ is a regular difference operator
- Form of $\hat{\Theta}$ depends on regularization scheme
  - Standard LQC: 2nd order operator

$$\boxed{\hat{\Theta} = -f_+(v)N^4 + f_o(v)\mathbb{I} - f_-(v)N^{-4}}$$

  - Thiemann reg. LQC: 4th order operator

$$\boxed{\hat{\Theta} = -g_+(v)N^8\gamma^2 f_+(v)N^4 + g_o(v)\mathbb{I} + \gamma^2 f_-(v)N^{-4} - g_-(v)N^{-8}}$$

  - In large v limit $f_{\pm,o}, g_{\pm,o} \propto v^2$.
- Separable superselection sectors of functions supported on (semi)lattices in v.

Physical states

- Generaized eigenfunctions found numerically via solving the difference equation (in v or differential one in b.
- Unitary evolution generation: identification of deficiency space (eigenspaces of purely imainary eigenvalues)
- Possible to preform spectral decomposition of $\hat{\Theta}$ explicitly. Physical states

$$\Psi(v,\phi) = \int_0^\infty \mathrm{d}k \tilde{\Psi}(k) e_k(v) e^{i\omega(k)\phi}, \ \omega(k) = \sqrt{12\pi G} k$$

- Schroedinger like evolution equation

$$-i\partial_\phi \Psi(v,\phi) = \sqrt{|\hat{\Theta}|}\Psi(v,\phi).$$

Observables

- Physical observables constructed out of gravitational (kinematical ones)

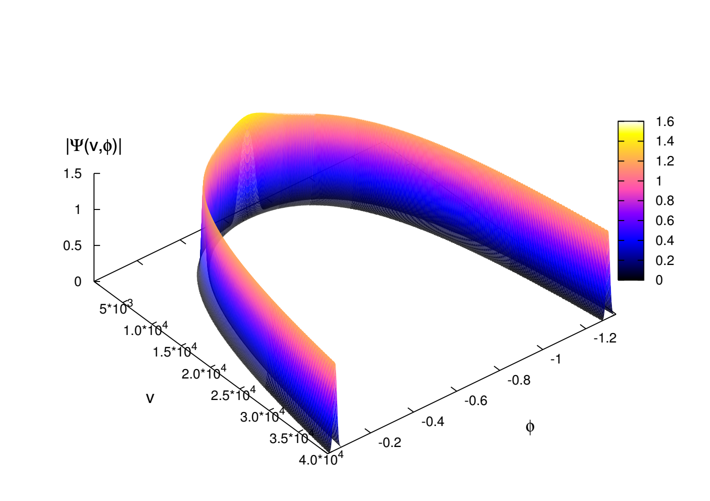
$$[\hat{O}_\phi]\Psi(v,\phi') = e^{i\sqrt{|\hat{\Theta}|}(\phi'-\phi)}\hat{O}_{\rm gr}\Psi(v,\phi)$$

- Standard selections for $\hat{O}_{\rm gr}$:
  - Volume: $\hat{V} = 2\pi\gamma\sqrt{\Delta}\ell_{\rm Pl}^2|v|$.
  - Compactified volume: $\hat{\theta}_K = \arctan(|v|/K)$.
  - Gravitational energy density: $\hat{\rho}_{\rm gr} = -1/2\hat{V}^{-1}\hat{\Theta}\hat{V}^{-1}$.
  - Hubble parameter: $H = i/6\hat{V}^{-1/2}[\hat{V}, \hat{V}^{-1/2}\hat{\Theta}\hat{V}^{-1/2}]\hat{V}^{-1/2}$

Dynamics in standard LQC

- $\hat{\Theta}$ selfadjoint – unique unitary evolution.
- Two epochs of large (classical) universe connected by a quantum bounce
- Bounce determined by a critical energy density $\rho_{\rm cr} \approx 0.42\rho_{\rm Pl}$.
- Semiclassicality preservation between epochs enforced by strong triangle inequalities on variations (Kamiński, TP,2010). Further semiclassicality results: Corichi, Singh, 2007, Corichi, Montoya

Standard LQC state evolution



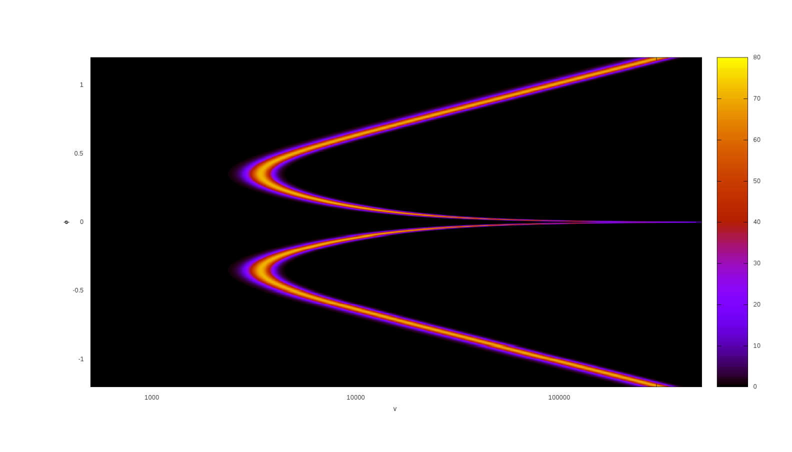
Rysunek: Gaussian wave packet $|\Psi(v,\phi)|$.

Thiemann-regularized model

Yang, Ding, Ma, 2009
Assanioussi, Dapor, Liegener, TP, 2018

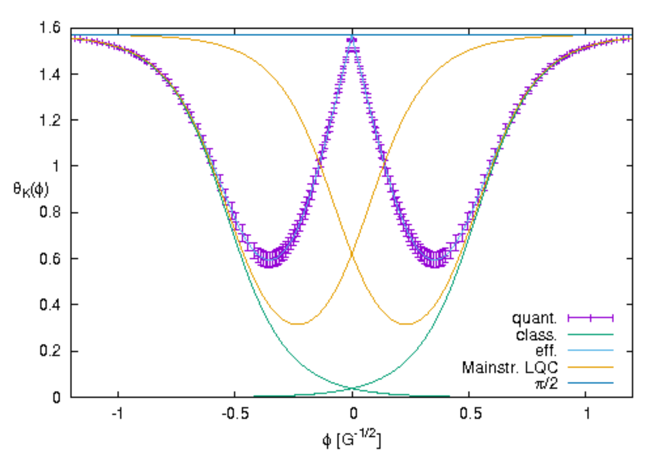
- Evolution operator 4th order but admitting a $U(1)$ family of selfadjoint extensions, each corresponding to a 2nd order system.
- State approaches from $v = \infty$ where its evolution mimicks that of GR.
- at Planckian energy densityit bounces, entering rapid expansion similar in nature to that of deSitter universe in LQC with cosmological constant of Planckian order,
- the state reaches infinite volume at finite ϕ, then undergoes transition through scri (guided by selfadjoint extension), then starts to recollapse
- it bounces for the second time at Planckian energy density, then quickly approaches GR trajectory of expanding universe.

Gaussian energy state



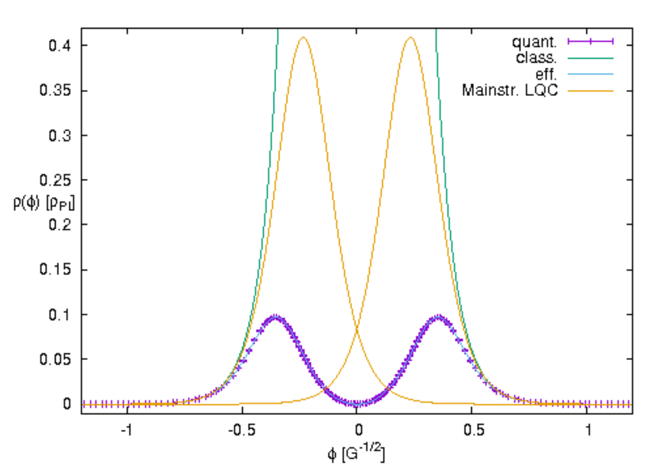
Rysunek: Gaussian wave packet $|v|^{1/2}|\Psi(v, \phi)|$.

Quantum trajectory



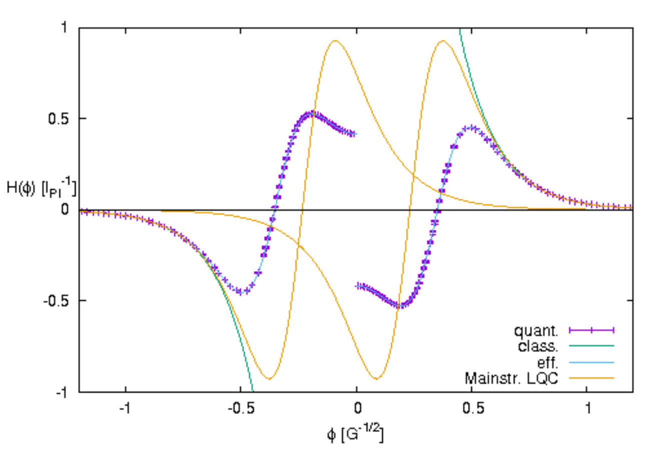
Rysunek: Observable $\theta_K(v)$ for $K = 5 \cdot 10^3$.

Quantum trajectory



Rysunek: Matter energy density.

Quantum trajectory



Rysunek: Hubble rate.

The conformal infinity transition

A peculiar point ot transition through infinite volume

- Well defined extension through future/past SCRI.
- Deterministic evolution of conformally invariant DOF.
- Allows for (almost) direct analog of Penrose CCC idea.
- Due to two infinities joining better control over transition.

The consequences of the evolution picture

- After implementing the Thiemann regularization scheme for the Lorentzian part of the Hamiltonian constraint the evolution of an FRW universe gets significantly enriched:
  - now between large semiclasscial branches it features two deSitter epochs with transitions through scri,
  - for most of the deSitter phase the Hubble horizon is of Planckian order, which can give significant effect on the structure of perturbations.
- Disturbing consequence:
  - Changing a detail (proposal of a regularization prescription) of a quantization procedure leads to significantly different dynamical prediction for the considered physical system: **Burden or opportunity?**

Thank you for your attention!

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