I.

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."Standard Classical Mechanics Sitting in Standard Classical GR"

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Comments:

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1. A lot of material - much must be left out - even a **few little dishonesties** to save time.

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2. Material has appeared in 4-5 published papers
Feb 2019,General Relativity and Gravitation
Aug 2018, Living Reviews

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3. We are dealing with **Einstein-Maxwell theory**

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No strings attached,
No non-Commuting VARIABLES,
No higher dimensions

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4. **Most of the discussion/action takes place in the far field region -**
In neighborhood of Future Null Infinity

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II. **I. PRELIMINARIES**

1. **Minkowski space-time**, a natural set of null surfaces (in neighborhood of null infinity) are the asymptotic **light cones with origins at arbitrary spacetime points, $\mathbf{x^a}$-** they label the surfaces.

ie. a four real parameter set of these asymptotic null surfaces.

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The **optical parameters of associated family of null generators (the null geodesics)** - **vanishing [TWIST] AND [SHEAR]**. .

To each one-parameter family of these asymptotic light cones we can construct a one parameter family of **associated null coordinate systems**.

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Now Asymptotically flat space-times.

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2. The **major development in gravitational radiation theory** (1960s) was Bondi's introduction of one-parameter families of null surfaces (Bondi surfaces) in the neighborhood of future null infinity.

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The null generators of the surfaces (the null geodesics) **HAVE VANISHING TWIST** but have **NON-VANISHING asymptotic SHEAR**.

[Aside; This **asymptotic shear**, $\sigma^0(u,\zeta,\overline{\zeta})$, is the **free data** for the description of gravitational radiation.]

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3. Transitioning from **Minkowski space to asymptotically flat space** - one could choose to use the null geodesics that are **[either] TWIST-FREE** OR **SHEAR FREE.**

Bondi (**naturally**) chose the **TWIST FREE path**.

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We suggest its preferable to chose the other path, **I.E. use asymptotically SHEAR FREE** null geodesics near null infinity - **RATHER then TWIST-FREE**.

This is THE MAJOR INNOVATION here

III.

4. **CLAIM**; **choosing the asymptotically SHEAR-FREE version leads to both Bondi's results and to a NEW set of remarkable results**.

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Initially the **asymptotically shear-free version is beset with problems, – they can be overcome.**

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5. **Back to asymptotically flat spacetimes:** Working with the **asymptotically shear free null geodesics** requires some technology — **discussed very superficially**.

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a. The **asymptotically shear free geodesics sets** are labeled by **four complex parameters**, z^a ... **Defining a complex 4-space i.e., H-space.** Each H-space point refers to a **SET** (**a bundle**) of null rays in the **physical spacetime**.

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The **imaginary part of z^a is a measure of the TWIST** - the **real part** determines the **average position of the set**. (**a generalization of light-cones.)** .

We have four real parameters to work with at INFINIT**Y - as IN the Minkowski case**

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b. A curve in H-space, $z^a = \xi^a(t) = \xi_R{}^a(t) + i\xi_I{}^a(t)$ corresponds in the physical spacetime to a null geodesic congruence with **TWIST.** Each congruence can be used to construct an **asymptotic coordinate & tetrad system** − **an ASYMPTOTICALLY SHEAR FREE** system.

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c. **A VERY special H-space-curve exists: The COMPLEX CENTER OF MASS System**

IV. **Quick Review** - HERE just to see what we are talking about

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II. - Asymptotic Einstein-Maxwell Eqs

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1. **Spin-coefficient description of asymptotic Weyl and Maxwell Tensor & Definitions** - Here just to be seen

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\begin{aligned}
\Psi_0 &= \Psi_0^0 r^{-5} + O(r^{-6}), \\
\Psi_1 &= \Psi_1^0 r^{-4} + O(r^{-5}), \\
\Psi_2 &= \Psi_2^0 r^{-3} + O(r^{-4}), \\
\Psi_3 &= \Psi_3^0 r^{-2} + O(r^{-3}), \\
\Psi_4 &= \Psi_4^0 r^{-1} + O(r^{-2}).
\end{aligned}
$$

$$
\begin{aligned}
\phi_0 &= \phi_0^0 r^{-3} + O(r^{-4}), \\
\phi_1 &= \phi_1^0 r^{-2} + O(r^{-3}), \\
\phi_2 &= \phi_2^0 r^{-1} + O(r^{-2}),
\end{aligned}
$$

with

$$
\begin{aligned}
\Psi_n^0 &= \Psi_n^0(u, \zeta, \bar{\zeta}), \\
\phi_n^0 &= \phi_n^0(u, \zeta, \bar{\zeta}).
\end{aligned}
$$

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The remaining (non-radial) **Bianchi Identities and Maxwell equations** yield the evolution equations:

$$\dot{\Psi}_2^0 = -\ \Psi_3^0 + \sigma^0 \Psi_4^0 + k\phi_2^0 \bar{\phi}_2^0, \tag{1}$$

$$\dot{\Psi}_1^0 = -\ \Psi_2^0 + 2\sigma^0 \Psi_3^0 + 2k\phi_1^0 \bar{\phi}_2^0, \tag{2}$$

$$\dot{\Psi}_0^0 = -\ \Psi_1^0 + 3\sigma^0 \Psi_2^0 + 3k\phi_0^0 \bar{\phi}_2^0, \tag{3}$$

$$k = 2Gc^{-4}, \tag{4}$$

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$$\dot{\phi}_1^0 = -\ \phi_2^0, \tag{5}$$

$$\dot{\phi}_0^0 = -\ \phi_1^0 + \sigma^0 \phi_2^0. \tag{6}$$

Overdot denotes u-derivative.

V.

DEFINITIONS OF PHYSICAL VARIABLES

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a. (a little lie) **Def 1**, **Bondi-Sachs mass and linear-Momentum** l=0&1 harmonics of Ψ_2^0 **(Classical)**

All constants taken as =1, i.e. c=h=G=k=1

$$\Psi_2^0 = M + P^i Y_{1i} \tag{7}$$

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b. **Def 2 Complex Mass Dipole**: **NEW**

Mass Dipole plus i Angular Momentum -

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$(D^i_{(complex)} = D^i_{(mass)} + ic^{-1}J^i)$,

$l = 1$ harmonic of Ψ_1^0;

$$\Psi_1^0 = \quad (D^i_{(mass)} + ic^{-1}J^i)Y^1_{1i} + \ldots . \tag{8}$$

.

Def 3 Complex E&M dipole, (electric and i magnetic dipoles, .

$D^i_{\text{complex}} = (D^i_{Elec} + iD_{Mag})$

the $l = 1$ harmonic component of ϕ_0^0 . **(STANDARD)**

$$\phi_0^0 = 2(D^i_{Elec} + iD_{Mag})Y^1_{1i}. \tag{9}$$

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VI.

III. **MAJOR STEP**

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Choose a complex world in H-Space, $z^a_{CCofM} = \xi^a(t) = \xi^a_R(t) + i\xi^a_I(t)$ with its associated ASYMPTOTICALLY SHEAR FREE system so that **COMPLEX MASS DIPOLE VANISHES - DEFINITION**

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$$D^i_{(complex)} = (D^i_{(mass)} + ic^{-1}J^i) = 0 \tag{10}$$

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This H-Space curve (and its physical space uniquely associated null geodesic congruence) **define the COMPLEX CENTER OF MASS** . .

and a unique **ASYMPTOTICALLY SHEAR FREE** system - **the CENTER OF MASS SYSTEM.**

VII.

IV. **RESULTS** - & ***no more definitions***

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BY transforming from the **CENTER OF MASS SYSTEM back** to a **Bondi system** we obtain a series surprising and remarkable results.

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Result #1 Expressions for Mass DIpole and Spin and Orbital Angular Momentum - **NOT DEFINITIONS** but derived .

$$D^i_{(mass)} = M_B \xi^i_R - c^{-1} P^k_B \xi^j_I \epsilon_{jki} + \ldots, \quad (11)$$

$$J^i = c M_B \xi^i_I + P^k_B \xi^j_R \epsilon_{jki} + \ldots. \quad (12)$$

or

$$\vec{D}_{(mass)} = M_B \vec{r} + c^{-2} M_B^{-1} \vec{P}_B \, x \vec{S}, \quad (13)$$

$$\vec{r} = \xi^i_R = (\xi^1_R, \xi^2_R, \xi^3_R), \quad (14)$$

$$\overrightarrow{S} = c M_B \xi^j_I = c M_B (\xi^1_I, \xi^2_I, \xi^3_I), \quad (15)$$

$$\vec{J} = \vec{S} + \vec{r} x \vec{P}. \quad (16)$$

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Result #2 From Bianchi Identities **the Kinematic Linear Momentum**

$$P^i_B = M_B \xi^{i\prime}_R - \frac{2q^2}{3c^3} \xi^{i\prime\prime}_R \quad (17)$$

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Result #3 in Bianchi Identities - **Angular Momentum Conservation**.

J$^{i\ \prime} = \frac{2q^2}{3c^3}(\xi^{j\prime}_R \xi^{k\prime\prime}_R + \xi^{k\prime}_I \xi^{k\prime\prime}_I)\epsilon_{kji}$, (18)

Exactly the same as L & L plus spin loss- no derivation -

JUST sitting in the BI.

VIII.

Result: V - Energy loss

$$M'_B = -\frac{G}{5c^7}(Q^{jk'''}_{Mass}Q^{jk'''}_{Mass}+Q^{jk'''}_{Spin}Q^{jk'''}_{Spin})-\frac{4q^2}{3c^5}(\xi^{i''}_R\xi^{i''}_R+\xi^{i''}_I\xi^{i''}_I)-\frac{4}{45c^7}(.. \quad (19)$$

. **Bondi Energy loss** & **E&M dipole & quadrupole energy loss**

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Result: 5 - Newton's 2nd Law

$$P^{i\prime}_B = F^i_{recoil} \quad (20)$$

F^i_{recoil} has many non-linear radiation terms – time derivatives of the gravitational quadrupole and the E&M dipole and quadrupole moments.

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Substitute **momentum expression**, Eq.($P^i_B = M_B\xi^{i\prime}_R - \frac{2q^2}{3c^3}\xi^{i''}_R$), into **Momentum lose** Eq.($P^{i\prime}_B = F^i_{recoil}$) leading to Newton's second law – with Rocket Force and Radiation Reaction Force.

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******Result: 6 - Rocket Force and Radiation Reaction Force********

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$$M_B\xi^{i''}_R = F^i \equiv M'_B\xi^{i\prime}_R + \frac{2q^2}{3c^3}\xi^{i'''}_R + F^i_{recoil}. \quad (21)$$

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WE believe: remarkable - exact Abraham-Lorentz-Dirac radiation reaction force - **NO mass renormalization. No derivation - just sitting there to be observed in the $l = 1$ part of a BI.**

IX.

Conclusions:

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These results of C.M. and E&M theory are clearly sitting in GR.

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It is not at all clear what are the implications or even the meaning?????

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Do they fit into a quantum theory? A Schrodinger Eq????
If so HOW?

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What happens with the runaway behavior resulting from the radiation reaction term in the EQS of motion - is there a term that suppresses the runaway behavior. ?????

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Is it possible to get two body eqs of motion from this type of analysis???? or other classical results????