# Isolated Horizon Entropy in Loop Quantum Gravity

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**To celebrate Jurek's 60th birthday**

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# Outline

1. **Introduction to Isolated Horizon and Loop Quantum Gravity**
2. **Chen-Simons Theory Description of Isolated Horizon Entropy in LQG**
3. **BF Theory Description of Isolated Horizon Entropy**
4. **Concluding Remarks**

# Limitation of the global notions in GR

- The event horizon definition of BH requires knowledge of the entire space-time all the way to future null infinity.
- The use of stationary space-times to derive black hole thermodynamics is not ideal.

# Limitation of the global notions in GR

- The event horizon definition of BH requires knowledge of the entire space-time all the way to future null infinity.
- The use of stationary space-times to derive black hole thermodynamics is not ideal.
- The global nature of event horizon makes it difficult to use in quantum theory. In order for a definition of the horizon of black hole to make sense, one needs to be able to formulate it in terms of phase space functions which can be quantized.
- The global notions of ADM energy and ADM angular momentum are of limited use, because they do not distinguish the mass of black holes from the energy of surrounding gravitational radiation.

# Quasi-local notion of Isolated Horizon

- The notion of isolated horizon is defined quasi-locally as a portion of the event horizon which is in equilibrium [Ashtekar, Beetle and Fairhurst, 1998].

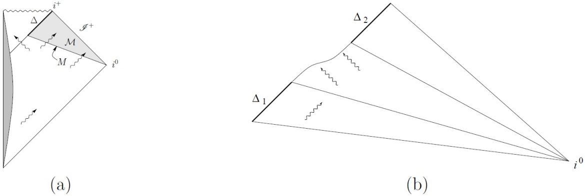


Figure 1: (a) A typical gravitational collapse. The portion $\Delta$ of the horizon at late times is isolated. The space-time $\mathcal{M}$ of interest is the triangular region bounded by $\Delta$, $\mathscr{I}^+$ and a partial Cauchy slice $M$. (b) Space-time diagram of a black hole which is initially in equilibrium, absorbs a small amount of radiation, and again settles down to equilibrium. Portions $\Delta_1$ and $\Delta_2$ of the horizon are isolated.

# Quasi-local notion of Isolated Horizon

- (Weakly) Isolated Horizon: A three-dimensional null hypersurface $\Delta$ of a space-time $(\mathcal{M}, g_{ab})$ is said to be a weakly isolated horizon if the following conditions hold:
  (1). $\Delta$ is topologically $\mathbb{R} \times S$ with $S$ a compact two-dimensional manifold;
  (2). The expansion $\theta_{(l)}$ of any null normal $l$ to $\Delta$ vanishes;
  (3). The field equations hold at $\Delta$, and the stress-energy tensor $T_{ab}$ of external matter fields is such that, at $\Delta$, $-T^a{}_b l^b$ is a future-directed and causal vector for any future-directed null normal $l^a$.
  (4). An equivalence class $[l]$ of future-directed null normals is equipped with $\Delta$, with $l' \sim l$ if $l' = cl$ ($c > 0$ a constant), such that $\mathcal{L}_l \omega_a \triangleq 0$ for all $l \in [l]$, where $\omega_a$ is related to the induced derivative operator $D_a$ on $\Delta$ by $D_a l_b \triangleq \omega_a l_b$.

# Thermodynamics of Isolated Horizon

- There are indeed nontrival solutions to Einstein's equations, which including IHs surrounded by radiations in spacetime [**Lewandowski** 1999; Ashtekar, Beetle, Dreyer, Fairhurst, Krishnan, **Lewandowski**, Wisniewski, 2000] .
- The definition of weakly isolated horizon implies automatically the zeroth law of IH mechanics as the surface gravity $\tilde{\kappa}_{(l)} \equiv \omega_a l^a$ is constant on $\Delta$ [Ashtekar, Beetle and Fairhurst, 1998].

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- Let us consider an 4-dimensional spacetime region $\mathcal{M}$ with an isolated horizon $\Delta$ as an inner boundary. The Hamiltonian framework for $\mathcal{M}$ provides an elegant way to define the quasi-local notions of energy $E_\Delta$ and angular momentum $J_\Delta$ associated to $\Delta$. Then the first law of IH mechanics holds as [Ashtekar, Beetle and **Lewandowski**, 2001]

$$\delta E_\Delta = \frac{\tilde{\kappa}_{(l)}}{8\pi G}\delta a_\Delta + \Phi_{(l)}\delta Q_\Delta + \Omega_{(l)}\delta J_\Delta .$$

# Kinematical structure of LQG

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- There is a unique gauge and diffeomorphism invariant cyclic representation of the holonomy-flux $*-$algebra, given by the Ashtekar-Lewandowski measure $\mu_{AL}$ [LOST, 2005].
- The kinematical Hilbert space of LQG is spanned by spin network states, $|\Gamma, \{j_e\}, \{i_v\} >$, over graphs in the spatial manifold $M$

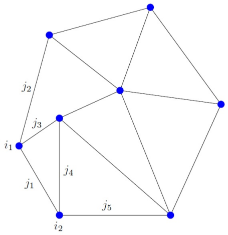


Figure: Dona and Speziale, arXiv:1007.0402.

# Quantum isolated horizon

- In the case when $M$ has a boundary $H$, some edges of spin networks in $M$ may intersect $H$ and endow it a quantum area at each intersection [Rovelli, 1996].

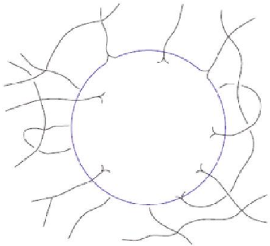

Figure: Ashtekar, Baez and Krasnov, gr-qc/0005126.

# Palatini formalism

Consider the Palatini action of GR on $\mathcal{M}$:

$$S[e, A] = -\frac{1}{4\kappa}\int_{\mathcal{M}} \varepsilon_{IJKL} e^I \wedge e^J \wedge F(A)^{KL} + \frac{1}{4\kappa}\int_{\tau_\infty} \varepsilon_{IJKL} e^I \wedge e^J \wedge A^{KL}$$

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- For later convenience, we define the solder form $\Sigma^{IJ} \equiv e^I \wedge e^J$.
- The second-order variation of the Palatini action leads to the conservation identity of the symplectic current as

$$\frac{1}{\kappa}\Big(\int_{M_1} \delta_{[1}(*\Sigma)_{IJ} \wedge \delta_{2]}A^{IJ} - \int_{M_2} \delta_{[1}(*\Sigma)_{IJ} \wedge \delta_{2]}A^{IJ} + \int_{\Delta} \delta_{[1}(*\Sigma)_{IJ} \wedge \delta_{2]}A^{IJ}\Big) = 0,$$

where $(*\Sigma)_{KL} = \frac{1}{2}\varepsilon_{IJKL}\Sigma^{IJ}$, and $M_1, M_2$ are spacelike boundary of $\mathcal{M}$.

# Basic variables in time gauge

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- Let the $so(3,1)$ connection $A^{IJ}$ and the cotetrad $e^I$ be in the time-gauge in which $e_0^a$ is normal to the partial Cauchy surface $M$, reducing the internal local gauge group from $SO(1,3)$ to $SO(3)$.
- The pull-back of the spacetime variables to $M$ can be written in terms of the Ashtekar-Barbero variables as

$$\mathcal{A}^i = \gamma A^{0i} - \frac{1}{2}\epsilon^i{}_{jk}A^{jk}; \qquad \Sigma^i = \epsilon^i{}_{jk}\Sigma^{jk}.$$

# Symplectic structure in time gauge

For spherically symmetric IHs with a given area $a_0$, the symplectic structure can be obtained on $M$ with the inner boundary $H = M \cap \Delta$ as [Engle, Noui, Perez, Pranzetti, 2009]

$$\Omega(\delta_1, \delta_2) = \frac{1}{2\kappa\gamma} \int_M 2\delta_{[1}\Sigma^i \wedge \delta_{2]}\mathcal{A}_i - \frac{1}{\kappa}\frac{a_0}{\pi(1-\gamma^2)} \oint_H 2\delta_{[1}\mathcal{A}_i \wedge \delta_{2]}\mathcal{A}^i.$$

- The symplectic structure consists of a bulk term, the standard symplectic structure used in LQG, and a surface term, the symplectic structure of an $SU(2)$ Chern-Simons theory on $H$.

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- The symplectic structure consists of a bulk term, the standard symplectic structure used in LQG, and a surface term, the symplectic structure of an $SU(2)$ Chern-Simons theory on $H$.
- In terms of the Ashtekar-Barbero variables, the isolated horizon boundary conditions take the form

$$\Sigma^i = -\frac{a_0}{\pi(1-\gamma^2)} F^i(\mathcal{A}).$$

# Approaches to the entropy calculation for IH

- Choices of the gauge group for the Chern-Simons theory
    - $U(1)$ group [Ashtekar, Baez, Krasnov, 2000]: Applicable to arbitrary axi-symmetric IH [Ashtekar, Engle, Broeck, 2004].
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  - $SU(2)$ group [Engle, Noui, Perez, Pranzetti, 2009]: Spherically symmetric IH.
- With either of the above choices, detail analysis can estimate the number of surface Chern-Simons states on the punctured horizon consistent with the given area.
- For the states counting, one usually uses the spectrum of the standard area operator in LQG [Rovelli, Smolin, 1995; Ashtekar, **Lewandowski**,1997]

$$a_S = 8\pi\gamma\ell_p^2 \sum_I \sqrt{j_I(j_I+1)}$$

# Calculation of the entropy for IH

- By the standard area operator
  - The leading order expression of the entropy agrees with the Hawking-Bekenstein formula by choosing the Barbero-Immirzi parameter $\gamma \approx 0.274$, with either $U(1)$ or $SU(2)$ Chern-Simons theories [Domagala, **Lewandowski**, 2004; Ghosh, Mitra, 2005; Agullo, Barbero, Borja, Diaz-Polo, Villasenor, 2009].
  - The sub-leading order expressions of the entropy are logarithmic terms for both gauge groups, but the front coefficients are different: $-\frac{1}{2}$ for $U(1)$ while $-\frac{3}{2}$ for $SU(2)$.

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  - The sub-leading order expressions of the entropy are logarithmic terms for both gauge groups, but the front coefficients are different: $-\frac{1}{2}$ for $U(1)$ while $-\frac{3}{2}$ for $SU(2)$.
- Alternative choice: the flux-area operator for IH [Barbero, **Lewandowski**, Villasenor, 2009]

$$a_H^{flux} = 8\pi\gamma\ell_p^2(\sum_p |m_p|)$$

Problem: The Barbero-Immirzi parameter takes different values for even and odd values of the horizon "area".

# Calculation of the entropy for IH

- The above isolated horizon framework can be generalized to arbitrary even-dimensional spacetimes [Bodendorfer, Thiemann, Thurn, 2013], where the horizon degrees of freedom are encoded in the $SO(2n)$-Chern-Simons theory.

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- Limitations:
  - The Chern-Simons theory description of IH degrees of freedom requires that the area of the horizons has to be fixed.
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- Is there any way out?
- BF theory approach:
  - $SO(1,1)$ BF theory description of IH [Wang, YM, Zhao, 2014; Wang, Huang, 2015]
  - $SU(2)$ BF theory description of spherically symmetric IH [Pranzetti, Sahlmann, 2015]

# Near horizon coordinates

- In the neighborhood of $\Delta$, we choose the Bondi-like coordinates given by $(v, r, x^i)$, $i = 1, 2$, where the horizon is given by $r = 0$ [Lewandowski, 2000].

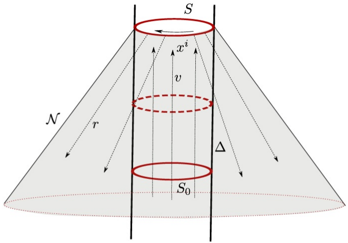


Figure 11: The near horizon coordinates. The isolated horizon is $\Delta$ and the transverse null surface is $\mathcal{N}$. The affine parameter along the outgoing null geodesics on $\mathcal{N}$ is $r$, and $v$ is a coordinate along the null generators on $\Delta$, and $x^1$ are coordinates on the cross-sections of $\Delta$.

Figure: Krishnan, arXiv:1303.4635.

# Gauge choice of the tetrad

- To describe the geometry near the isolated horizon $\Delta$, it is convenient to employ the Newman-Penrose formalism with the null tetrad $(l, n, m, \bar{m})$ adapted to $\Delta$, such that the real vectors $l$ and $n$ coincide with the outgoing and ingoing future directed null vectors at $\Delta$ respectively.

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- We choose an appropriate set of co-tetrad fields which are compatible with the metric as:

$$e^0 = \sqrt{\frac{1}{2}}(\alpha n + \frac{1}{\alpha} l),\ e^1 = \sqrt{\frac{1}{2}}(\alpha n - \frac{1}{\alpha} l),$$
$$e^2 = \sqrt{\frac{1}{2}}(m + \bar{m}),\ e^3 = i\sqrt{\frac{1}{2}}(m - \bar{m}),$$

where $\alpha(x)$ is an arbitrary function of the coordinates.
- Each choice of $\alpha(x)$ characterizes a local Lorentz frame in the plane $\mathcal{I}$ formed by $\{e^0, e^1\}$.

# Horizon degrees of freedom

- The horizon integral of the symplectic current is reduced to

$$\frac{1}{\kappa}\int_{\Delta}\delta_{[1}(*\Sigma)_{IJ}\wedge\delta_{2]}A^{IJ}=\frac{2}{\kappa}\int_{\Delta}\delta_{[1}\Sigma^{23}\wedge\delta_{2]}A^{01}.$$

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- It turns out that the 2-form $\Sigma^{23}$ is closed.
  Hence we can define an 1-form $B$ locally on $\Delta$ such that

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- In terms of Ashtekar-Barbero variables, the full symplectic structure can be obtained as

$$\Omega(\delta_1,\delta_2)=\frac{1}{2\kappa\gamma}\int_M 2\delta_{[1}\Sigma^i\wedge\delta_{2]}\mathcal{A}_i+\frac{1}{\kappa}\oint_H 2\delta_{[2}B\wedge\delta_{1]}A$$

# Quantum *BF* theory with sources

- To adapt the structure of LQG in the bulk, the boundary *BF* theory is intersected by the spin networks, and satisfies

$$F = dA = 0, \quad dB = \frac{\Sigma^1}{2\kappa}$$

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- Let's assume that the graph $\Gamma$ underling a spin network state intersects $H$ by $n$ intersections: $\mathcal{P} = \{p_i | i = 1, \cdots, n\}$. For every intersection $p_i$ we associated a small enough bounded neighborhood $s_i$. Then the physical degrees of freedom of our sourced BF theory are encoded in

$$f_i = \int_{s_i} dB = \oint_{\partial s_i} B$$

- We can obtain the quantum Hilbert space of the *BF* theory with $n$ intersections as: $\mathcal{H}_H^{\mathcal{P}} = L^2(\mathbb{R}^n)$.

# Quantum horizon boundary condition

- Consider the bulk kinematical Hilbert space $\mathcal{H}_M^{\mathcal{P}}$ defined on a graph $\Gamma \subset M$ with $\mathcal{P}$ as the set of its end points on $H$. $\mathcal{H}_M^{\mathcal{P}}$ can be spanned by the spin network states $|\mathcal{P}, \{j_p, m_p\}; \cdots >$, where $j_p$ and $m_p$ are respectively the spin labels and magnetic numbers of the edge $e_p$ with $p \in \mathcal{P}$.

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- The integral $\Sigma^1(H) = \int_H \Sigma^1$ can be promoted as an operator:

$$\hat{\Sigma}^1(H)|\mathcal{P}, \{j_p, m_p\}; \cdots >= 16\pi\gamma\ell_p^2 \sum_{p \in \Gamma \cap H} m_p|\mathcal{P}, \{j_p, m_p\}; \cdots > .$$

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- The topology of the horizon $H$ imposes an additional global constraint:

$$\sum_{p\in\Gamma\cap H} m_p \;=\; 0.$$

## Solving the quantum boundary condition

- The equations of the boundary BF theory motive us to input the quantum version of the horizon boundary condition as

$$(Id \otimes \hat{f}_i(s_i) - \frac{\hat{\Sigma}^1(s_i)}{2\kappa} \otimes Id)(\Psi_v \otimes \Psi_b) = 0,$$

where $\Psi_v \in \mathcal{H}_M^{\mathcal{P}}$ and $\Psi_b \in \mathcal{H}_H^{\mathcal{P}}$.

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where $\Psi_v \in \mathcal{H}_M^{\mathcal{P}}$ and $\Psi_b \in \mathcal{H}_H^{\mathcal{P}}$.

- The space of kinematical states on a fixed $\Gamma$, satisfying the boundary condition, can be written as

$$\mathcal{H}_\Gamma = \bigoplus_{\{j_p, m_p\}_{p \in \Gamma \cap H}} \mathcal{H}_M^{\mathcal{P}}(\{j_p, m_p\}) \otimes \mathcal{H}_H^{\mathcal{P}}(\{m_p\}),$$

where $\mathcal{H}_H^{\mathcal{P}}(\{m_p\})$ denotes the subspace corresponds to the spectrum $\{m_p\}$ in the spectral decomposition of $\mathcal{H}_H^{\mathcal{P}}$ with respect to the operators $\hat{f}_p$ on the boundary.

# Area constraint

- The imposition of the diffeomorphism constraint implies that one only needs to consider the diffeomorphism equivalence class of quantum states.
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# Area constraint

- The imposition of the diffeomorphism constraint implies that one only needs to consider the diffeomorphism equivalence class of quantum states.
  Hence, in the following states counting, we will only take account of the number of intersections on $H$, while the possible positions of intersections are irrelevant.
- We employ the flux-area operator $\hat{a}_H^{flux}$ in the bulk Hilbert space $\mathcal{H}_M^{\mathcal{P}}$ with a horizon boundary $H$, corresponding to the classical area $\int_H |dB|$ of $H$, and thus have the area constraint:

$$\sum_{p\in\mathcal{P}} |m_p| = a, \quad m_p \in \mathbb{N}/2,$$

where $a = \dfrac{a_H}{8\pi\gamma\ell_p^2}$.

# States counting

- For a given horizon area $a_H$, the horizon states satisfying the boundary condition are labelled by sequences $(v_1, \cdots, v_n)$ subject to the global constraint and area constraint, where $v_i = 2m_i$ are integers.
- We assume that for each given ordering sequence $(v_1, \cdots, v_n)$, there exists at least one state in the bulk Hilbert space of LQG, which is annihilated by the Hamiltonian constraint.

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- We assume that for each given ordering sequence $(v_1, \cdots, v_n)$, there exists at least one state in the bulk Hilbert space of LQG, which is annihilated by the Hamiltonian constraint.
- By the generating function method [Barbero, **Lewandowski**, Villasenor, 2009], the dimension $\mathcal{N}$ of the horizon Hilbert space compatible with the given macroscopic horizon area equals to the coefficient of the term $z^0 x^{2a}$ in the expansion of the generating function:

$$G(x, z) = \left(1 - \sum_{n=1}^{\infty}(z^n + z^{-n})x^n\right)^{-1}.$$

# Entropy of IH

- The entropy for such an isolated horizon is given by

$$S = \ln \mathcal{N} = \frac{\ln 3}{\pi\gamma}\frac{a_H}{4\ell_p^2} - \frac{1}{2}\ln\frac{a_H}{\ell_p^2} + \ln\frac{4\sqrt{\gamma}}{3}.$$

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- If we fix the value of the Barbero-Immirzi parameter as $\gamma = \frac{\ln 3}{\pi}$, which coincides with its value for the even $2a$ of flux-area in Chern-Simons theory, the Bekenstein-Hawking area law can be obtained by the leading term.
- The quantum correction to the Bekenstein-Hawking area law in our approach is the logarithmic term with coefficient $-\frac{1}{2}$, which coincides with its value in $U(1)$ Chern-Simons theory, and a constant term $\ln\frac{4\sqrt{\gamma}}{3}$.

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- One could also employ the spectrum of standard area operator in LQG for the states counting. Then the entropy formula would again be similar to that of $U(1)$ Chern-Simons theory.

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- The above BF theory approach admits extension to arbitrary dimensional horizons [Wang, Huang, 2014].
- While the boundary theory is still $SO(1,1)$ BF theory with sources, the bulk theory would be LQG based on $SO(D)$ connections [Bodendorfer, Thiemann, Thurn, 2011].
- The leading order expression of the entropy for an arbitrary dimensional IH reads

$$S = \ln \mathcal{N} = \frac{\ln 3}{2\pi\gamma} \frac{a_H}{4G\hbar}.$$

- The value of the Barbero-Immirzi parameter is fixed as $\gamma = \frac{\ln 3}{2\pi}$

# Summary and Remarks

- The quasilocal notion of isolated horizon lays down a suitable framework to study black hole entropy by quantum gravity.
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- Loop quantum gravity provides a fundamental scenario to account for the statistic origin of the isolated horizon entropy.
- In the Chern-Simons theory description of the horizon, the boundary degrees of freedom are encoded in the Chern-Simons connection,
  while in the BF theory description, the connection becomes pure gauge, and the non-trivial degrees of freedom of the horizon are all encoded in the $B$ field.
- The entropy formula of IH derived from the BF theory coincide with one case of the Chern-Simons theory up to the sub-leading term, indicating the same value of Barbero-Immirzi parameter and the same coefficient of the sub-leading logarithmic term.

# Summary and Remarks

- In the BF theory approach, the area of the horizons is not fixed, rather it is encoded in the dynamical $B$ field.
  Thus the covariant phase space of the system is enlarged so that all spacetime solutions with any isolated horizons as inner boundaries are included.
- The BF theory explanation of horizon entropy in LQG is applicable to general IHs in arbitrary dimensions.

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- In the BF theory approach, the area of the horizons is not fixed, rather it is encoded in the dynamical $B$ field.
  Thus the covariant phase space of the system is enlarged so that all spacetime solutions with any isolated horizons as inner boundaries are included.
- The BF theory explanation of horizon entropy in LQG is applicable to general IHs in arbitrary dimensions.
- In the generalization to arbitrary dimensional spacetimes based on $SO(D)$ connections, the value for the Barbero-Immirzi parameter, $\gamma = \frac{\ln 3}{2\pi}$, is dimension independent.

# Summary and Remarks

- In 4d spacetime, the different choices of connection formulations imply different values for the Barbero-Immirzi parameter by the entropy calculations. This provides the possibility to determine the internal gauge group of LQG from other considerations or experiments.

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- In 4d spacetime, the different choices of connection formulations imply different values for the Barbero-Immirzi parameter by the entropy calculations. This provides the possibility to determine the internal gauge group of LQG from other considerations or experiments.
- In the current entropy calculation, one calculates the dimension of the horizon Hilbert space compatible with the given macroscopic horizon area. This treatment is essentially to consider only spherically symmetric IHs.
- To further consider more general cases, one would need to introduce the notions of multipole moments of IHs. The related open issues are being studied [Ashtekar, Khera, **Lewandowski**, Song, YM,...].

# Happy Birthday! Jurek