Second law protection theorem for Lorentz-violating black holes

Jorma Louko

School of Mathematical Sciences, University of Nottingham

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P. Ezra, JL and W. Smith, in preparation
Plan

1. Einstein gravity Penrose process
   ▶ Splitting, collisions, tether...

2. Covariant Lorentz violation
   ▶ Einstein-Æther

3. Lorentz-violating Penrose process
   ▶ Spherical symmetry
   ▶ Splitting

4. Results
   ▶ Energy extraction admission theorem
   ▶ Energy extraction no-go theorem

5. Upshots
1. **Einstein gravity Penrose process**

*Rotating black hole*

**Splitting version**  Penrose and Floyd 1971

1. Drop in shuttle
   + payload (waste)
2. Eject payload in ergoregion,
   *against* the rotation
3. Collect shuttle, extract
   energy from velocity

**Extracted energy** $> m_{\text{waste}} c^2$

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Picture: Misner, Thorne and Wheeler 1973
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- Energy budget drawn at infinity
- Comes from rotational energy
  $\rightarrow$ Laws of BH mechanics...

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**Extracted energy** \( > m_{\text{waste}} c^2 \)

- Energy budget drawn at infinity
- Comes from rotational energy
  - Laws of BH mechanics...
- Exists for \( \frac{|J|}{M^2} > 2/(\sqrt{2} + 1) \)
  
  Fayos Valles and Llanta Salleras 1991
  (and only for?)

- Collision version more efficient
  
  Wald 1974,...
Einstein gravity Penrose process (cont’d)

**Tether version**  Penrose 1969

1. Lower payload (waste) to ergoregion by a tether
2. Extract energy from pull on the tether

**Extracted energy** $> m_{\text{waste}}c^2$

Picture: Penrose 1969
Einstein gravity Penrose process (cont’d)

Tether version  Penrose 1969

1. Lower payload (waste) to ergoregion by a tether
2. Extract energy from pull on the tether

Extracted energy $> m_{\text{waste}} c^2$

- Tether’s net contribution to energy budget assumed negligible
- → Ongoing debate... Marolf and Sorkin 2002
  A. R. Brown 2013

Today: no tethers!
2. Covariant Lorentz violation: Einstein-æther

Fundamental Jacobson and Mattingly 2001,… or effective Hořava 2009,…

Dynamical fields:

- $g^{(A)}_{ab}$ ($-++++$)
- $u^a$ with $u_a u^a = -1$ (æther)

$\Rightarrow$ Distinguished timelike direction at each point
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Dynamical fields:
- $g_{ab}^{(A)} (−+++)$
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Build second metric:

$$g_{ab}^{(B)} = −u_a u_b + c^{-2} (g_{ab}^{(A)} + u_a u_b) (−+++), \text{ but faster!}$$

$c > 1$
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g^{(B)}_{ab} = -u_a u_b + c^{-2} \left( g^{(A)}_{ab} + u_a u_b \right) \quad \text{\((-++++)\) but faster!} \]

\( c > 1 \)

Excitations:

A-fields: hyperbolic in \( g^{(A)}_{ab} \)

B-fields: hyperbolic in \( g^{(B)}_{ab} \)

Local interactions
2. Covariant Lorentz violation: Einstein-æther

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- $u^a$ with $u_a u^a = -1$ (æther)
  \[ \Rightarrow \text{Distinguished timelike direction at each point} \]

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\[ c > 1 \]

**Excitations:**

- particles: geodesic
- A-fields: \underline{hyperbolic in} $g_{ab}^{(A)}$
- particles: geodesic
- B-fields: \underline{hyperbolic in} $g_{ab}^{(B)}$

**Local interactions**

→ **Collisions** conserving 4-momentum
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Dynamical fields:

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- $u^a$ with $u_a u^a = −1$ (æther)
  $⇒$ Distinguished timelike direction at each point

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Excitations:

- particles: geodesic
- A-fields: hyperbolic in $g^{(A)}_{ab}$
- particles: geodesic
- B-fields: hyperbolic in $g^{(B)}_{ab}$

Local interactions

$⇒$ **Collisions** conserving 4-momentum (1-form)
3. Lorentz-violating black hole

$g^{(A)}_{ab}$:
- static, spherically symmetric, asymptotically flat
- $\chi^a$ Killing, asymptotically Minkowski $\partial_t$ at infinity
- future $A$-horizon: $\chi_a \chi^a$ changes sign
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\( u^a \):
- stationary, spherically symmetric, asymptotically \( \partial_t \) at infinity
- regular on \( A \)-horizon

\( \Rightarrow A \)-horizon not an event horizon in \( g^{(B)}_{ab} \)
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$\Rightarrow$ $A$-horizon not an event horizon in $g^{(B)}_{ab}$
Penrose process

cf Eling et al 2007

Radial motion (by assumption)

- $\Sigma$ (A or B) dropped from infinity
- $\Sigma \rightarrow A + B$ split in ergoregion
- B-ejectum escapes to infinity

$g_{ab}$

$I^+$

$I^-$

$I^0$
Penrose process

cf Eling et al 2007

Radial motion (by assumption)

• $\Sigma \ (A \text{ or } B)$ dropped from infinity
• $\Sigma \rightarrow A + B$ split in ergoregion
• $B$-ejectum escapes to infinity

Killing energy at infinity?

Iff $-k^A_a \chi^a < 0$, Killing energy at infinity increases
Penrose process

cf Eling et al 2007

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$\Rightarrow$ End point of energy extraction?
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⇒ End point of energy extraction?
⇒ Perpetual motion?
⇒ Violation of 2nd law of BH thermodynamics?!?

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⇒ End point of energy extraction?
⇒ Perpetual motion?
⇒ Violation of 2nd law of BH thermodynamics?!?


For which $(g_{ab}^{(A)}, u^a)$ does the process exist?
4. Results

1. Energy extraction admission theorem

For any \( g^{(A)}_{ab} \), the process exists for some \( u^a \).
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1. Energy extraction admission theorem

For any $g^{(A)}_{ab}$, the process exists for some $u^a$

Construction:

- $\Sigma$: massive $A$
- $B$-ejectum massless

- At splitting event, make $u^a$ point to the left of $v^a$ by sufficiently large relative $A$-velocity ($> c^{-1}$) 
  
  cf Eling et al. 2007
4. Results

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Does this happen for ‘reasonable’ field equations?
4. Results (cont’d)

2. Energy extraction no-go theorem

If

\[-g_{ab}^{(B)} \chi^a \chi^b < 1\] (1)

in exterior \(\cup\) ergosurface \(\cup\) ergoregion, the process does not exist.
4. Results (cont’d)

2. Energy extraction no-go theorem

If

\[- g_{ab} B^a B^b < 1 \quad (1)\]

in exterior \( \cup \) ergosurface \( \cup \) ergoregion, the process does not exist.

Comments

• Physics of (1): \(- g^{(B)}_{00} < 1 \Rightarrow B\)-gravity attractive

• (1) implies \(- g^{(A)}_{ab} \chi^a \chi^b < 1 \Rightarrow A\)-gravity attractive too

• (1) holds in all known Einstein-æther and Hořava solutions, analytic and numerical

• Might (1) necessarily follow from (reasonable) field equations?
4. Results (cont’d)

2. Energy extraction no-go theorem

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• (1) holds in all known Einstein-æther and Hořava solutions, analytic and numerical
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Proof: conceptually straightforward
5. Upshots

No-go theorem for Penrose splitting processes in spherically symmetric black holes without local Lorentz symmetry

- **Strong** despite the limitations (e.g. radial motion)
  - no perpetual motion
  - no violation of 2nd law of thermodynamics

**Nonradial motion generalisation:**

- Exists under additional assumptions about the area-radius
  
  Paul Ezra, JL and William Smith (in preparation)

**Conjecture:**

- If field equations allow $-g^{(B)}_{ab} \chi^a \chi^b < 1$ to be violated and energy extraction to occur, there must be new charges at infinity
Happy birthday Jurek!