Weighing the spacetime along the line of sight

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Warsaw

Jurekfest - Conference on the occasion

of Jerzy Lewandowski's 60th birthday

Faculty of Physics, University of Warsaw

16th September 2019

Idea

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- Determining mass distribution in the spacetime = one of the fundamental problems of astronomy

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- **Possible application:** dark and ordinary matter mapping, cosmological isotropy tests, …

Hand-waving explanation

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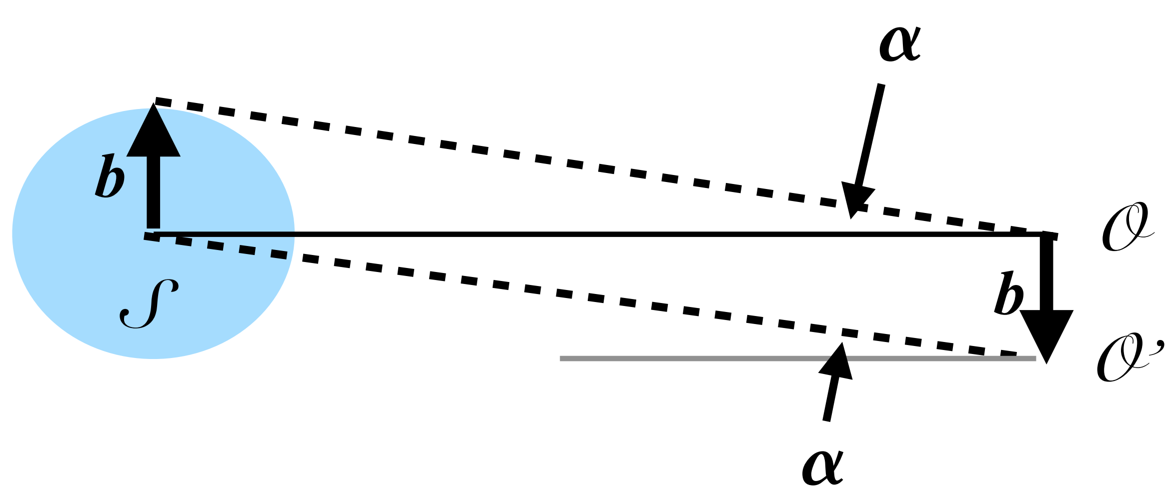
- Flat spacetime, no matter: measurements of distance to an object by parallax (D_{par}) and by angular size (D_{ang}) must give the same result



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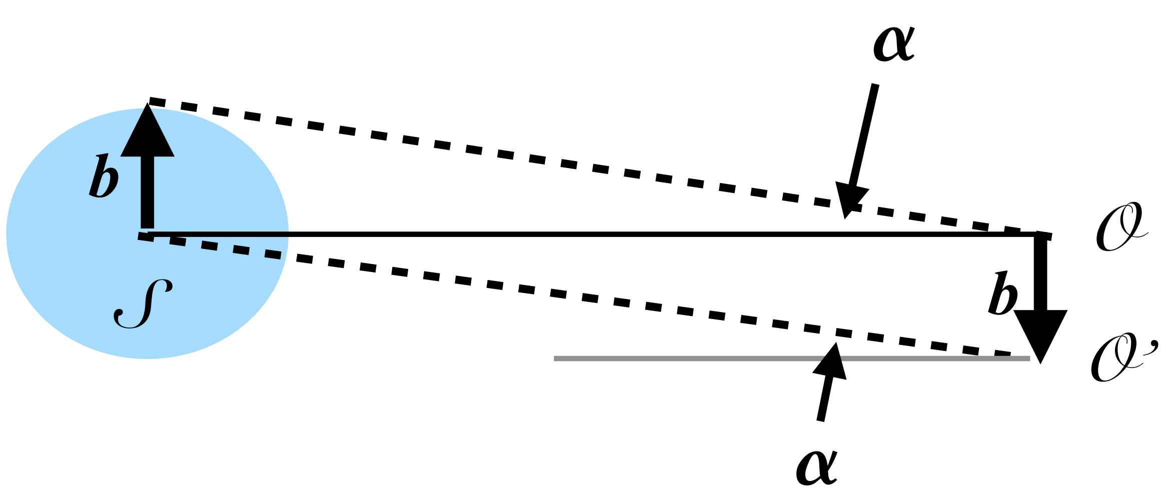

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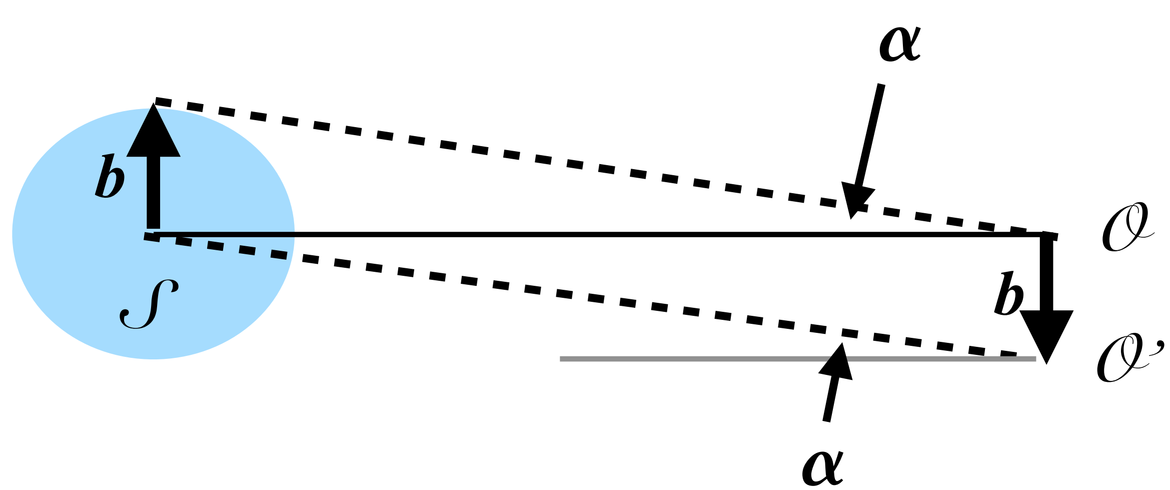

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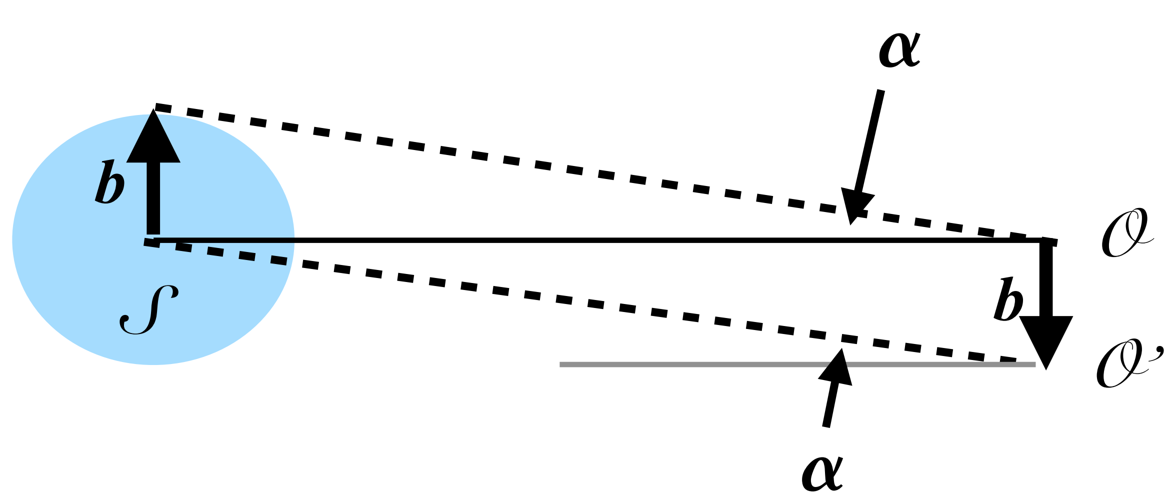

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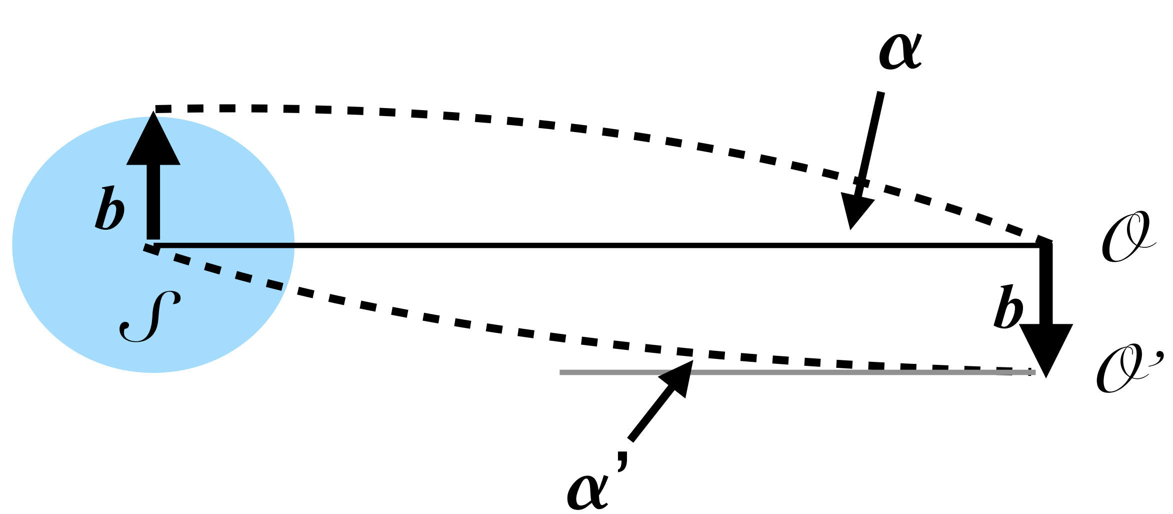

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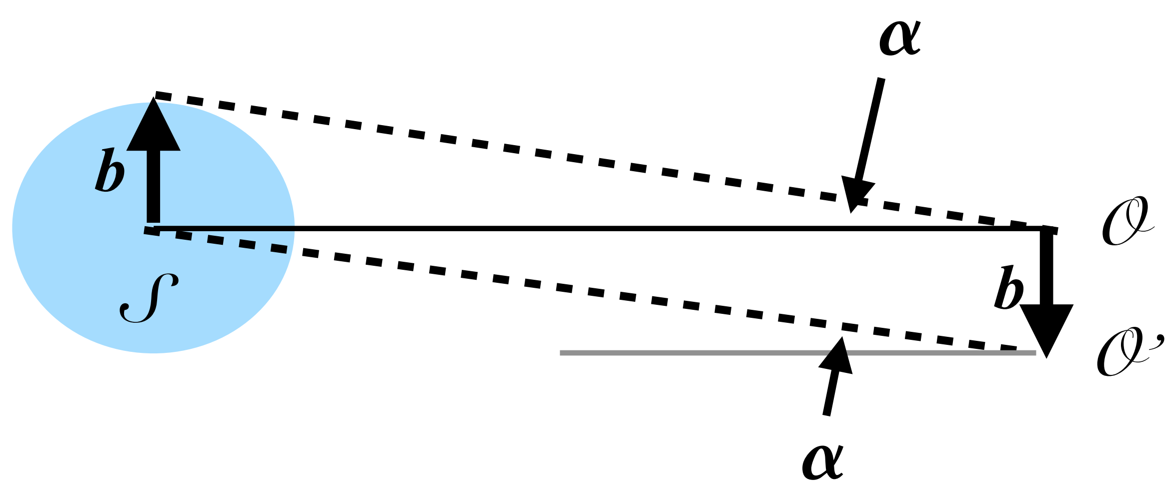

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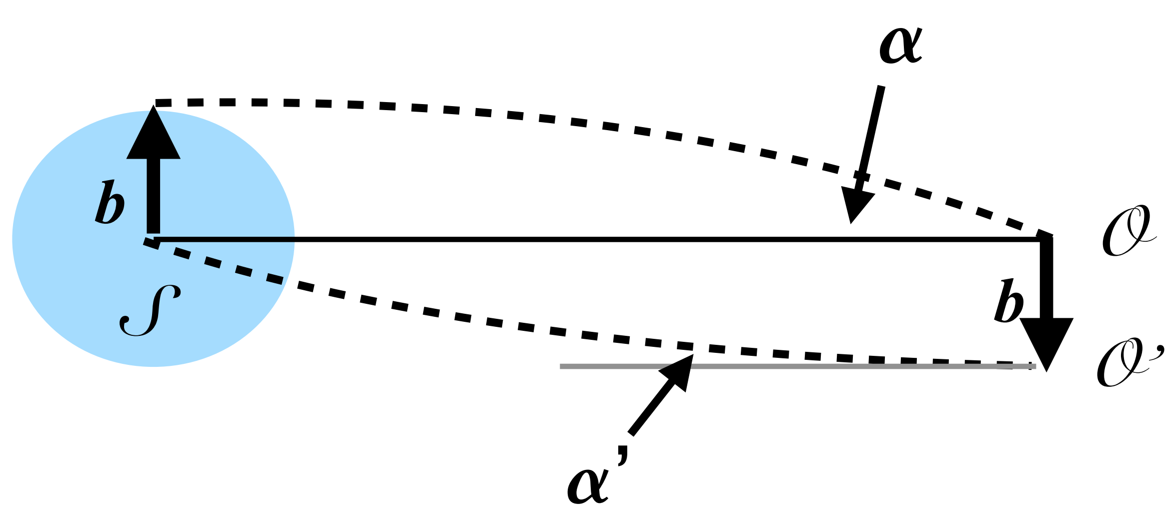

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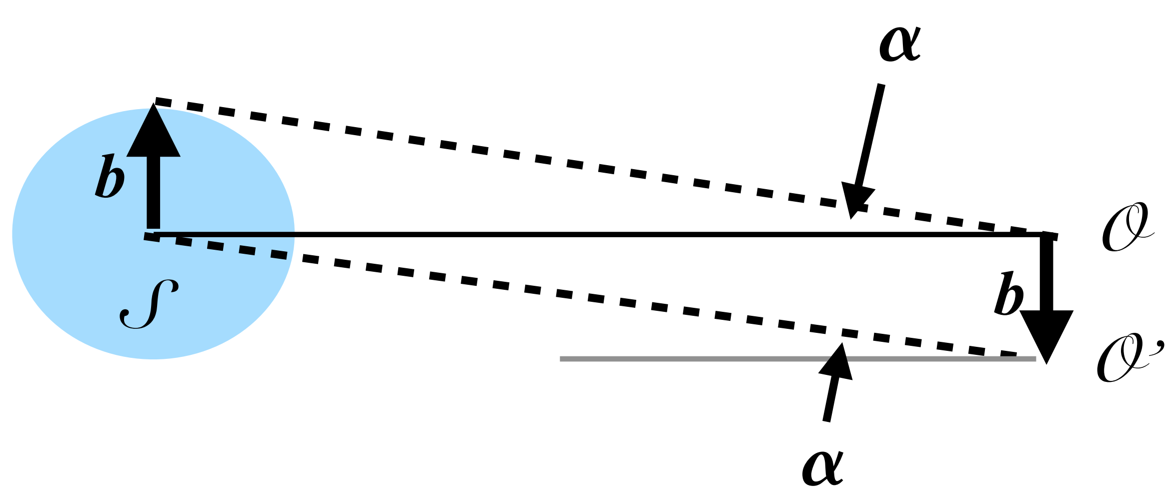

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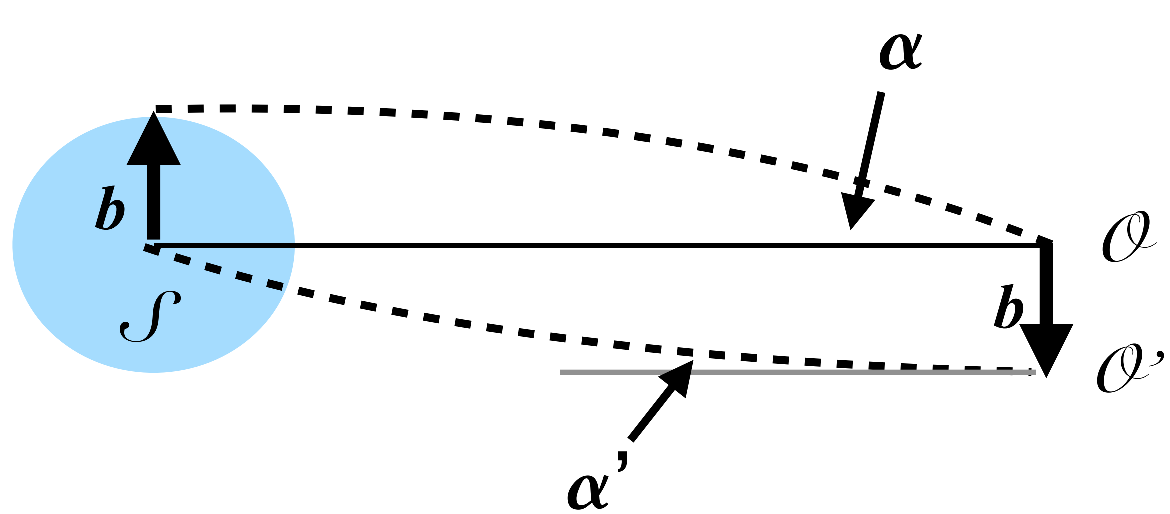

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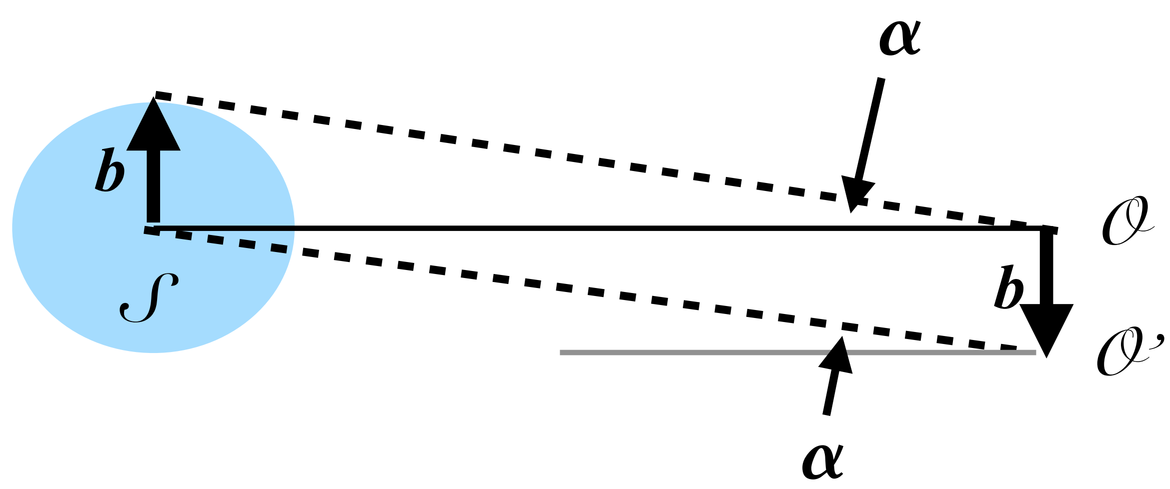

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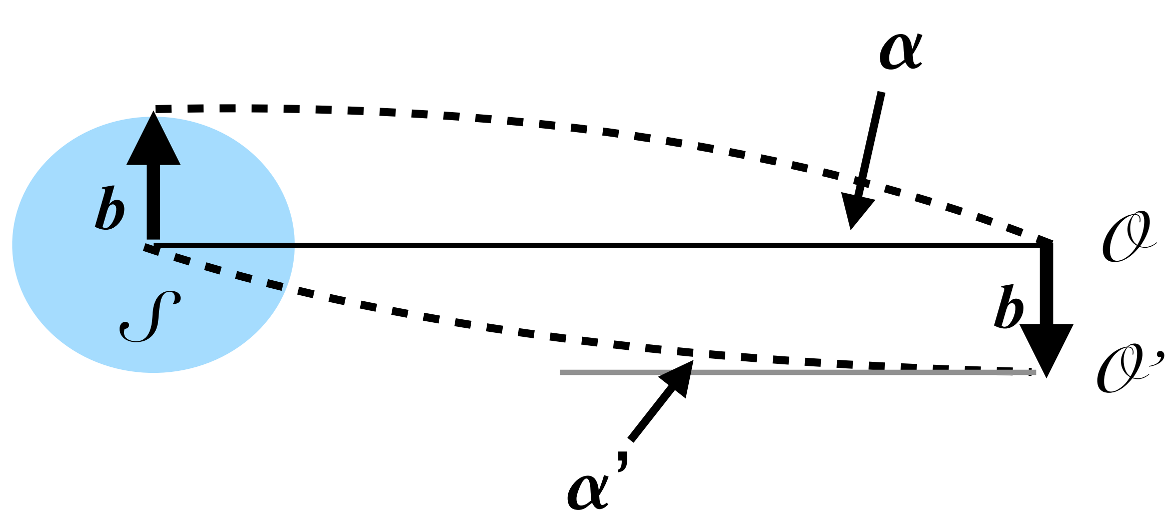

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- **Claim:** the difference measures the amount of matter along the LOS between $\mathcal{S}$ and $\mathcal{O}$

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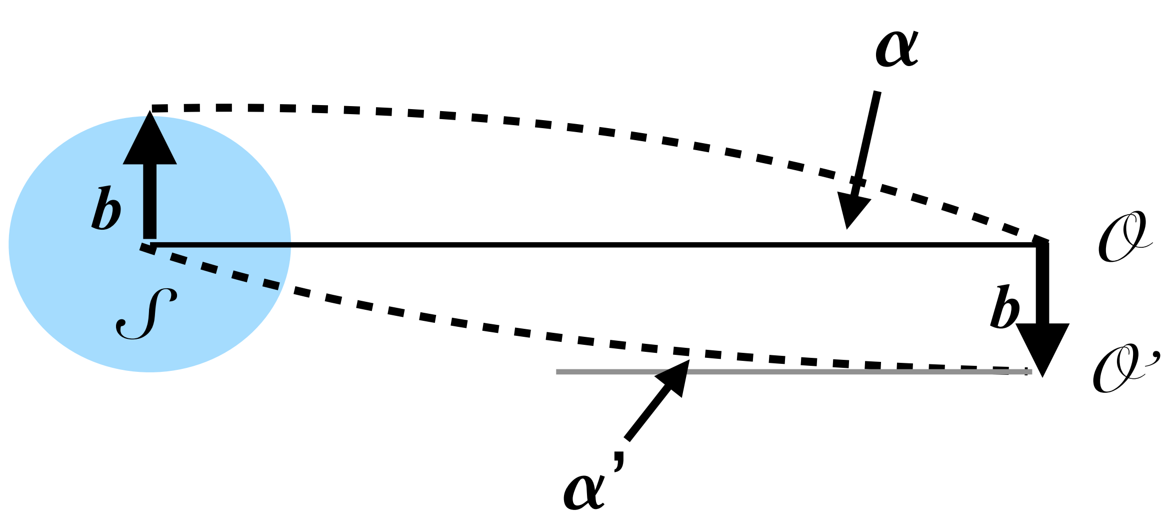

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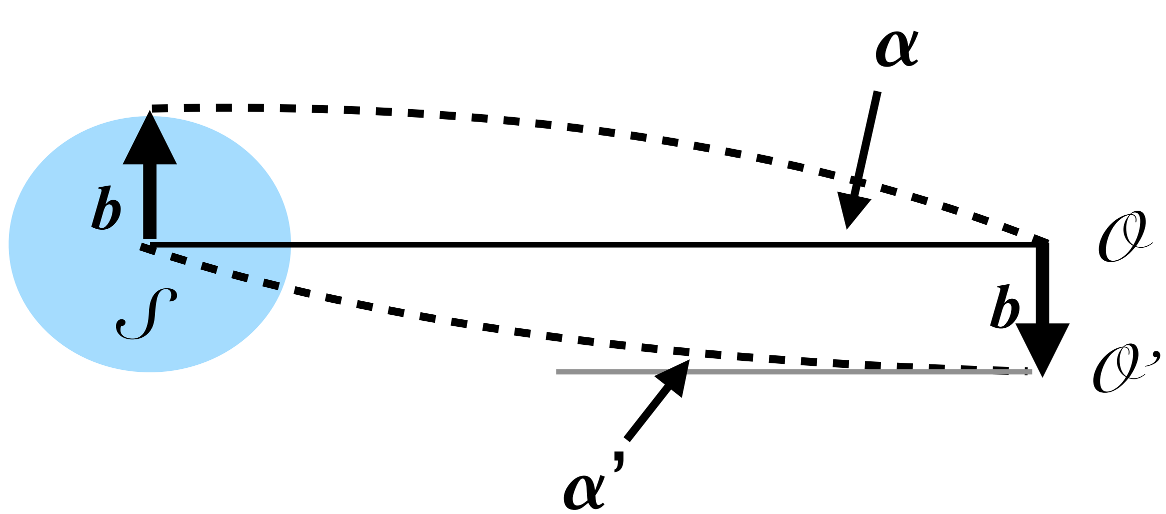

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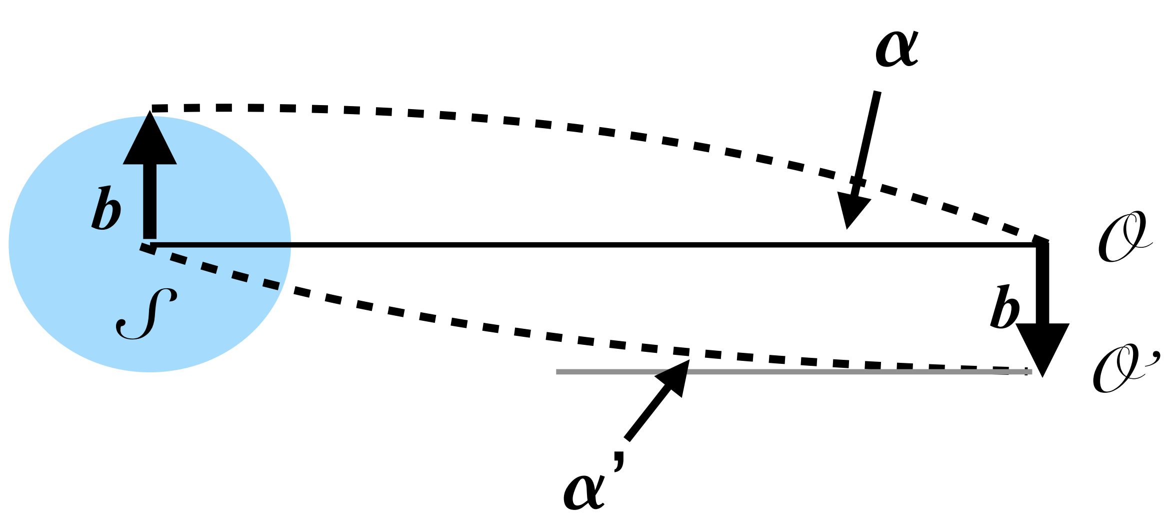

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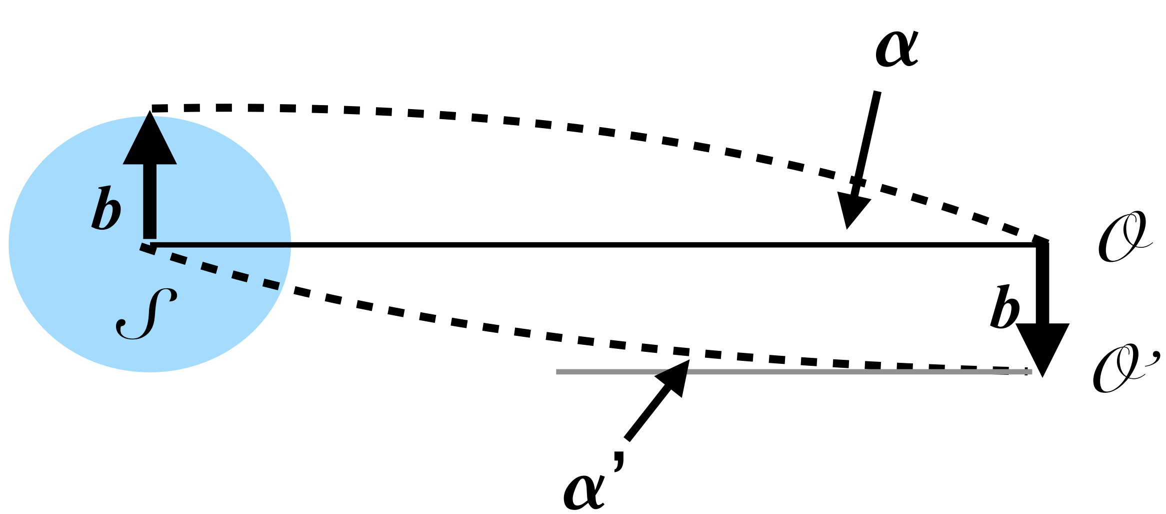

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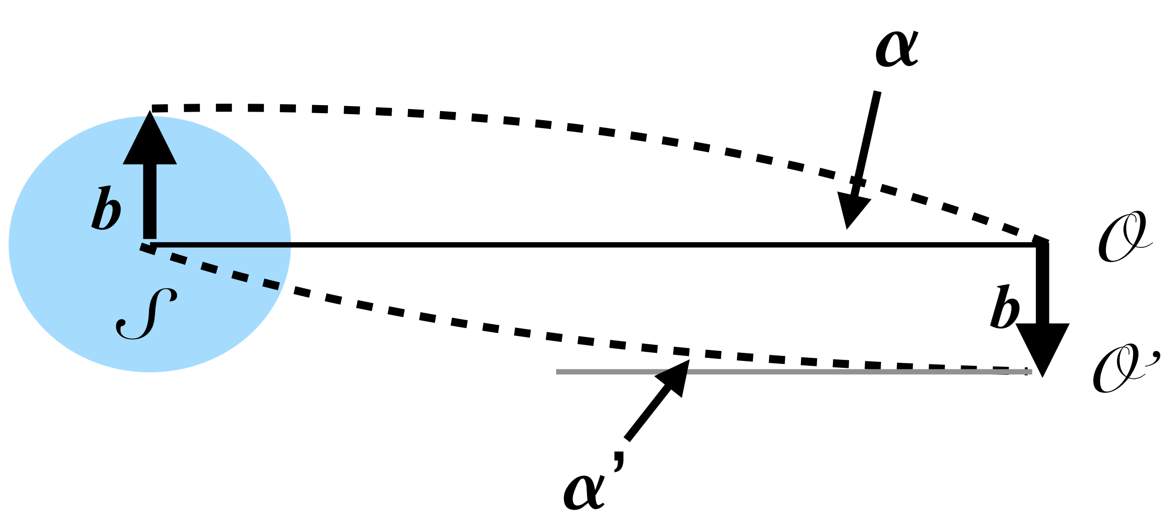

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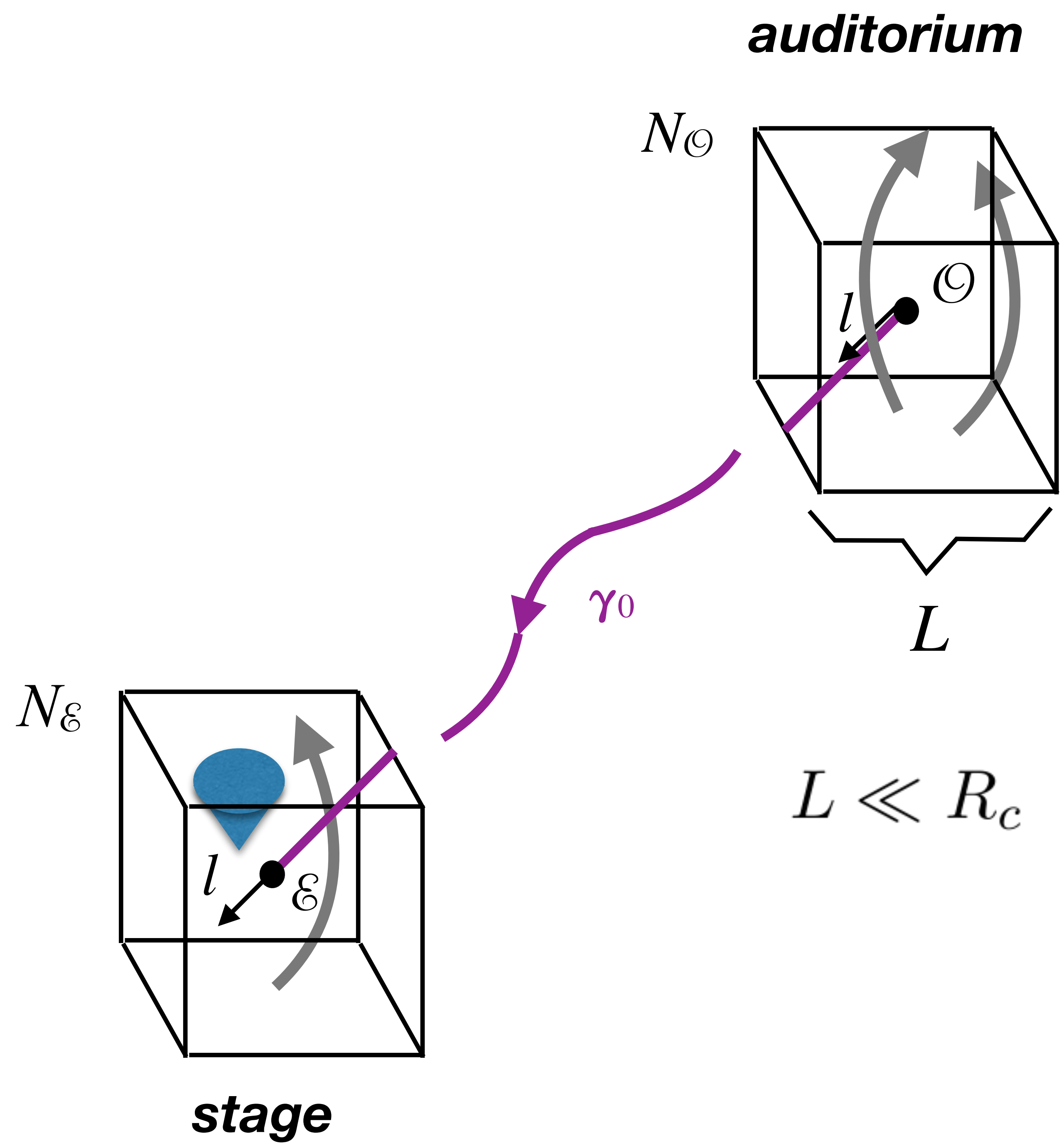
- ***Need of a fully relativistic approach! (all GR and SR effects)***

Fully relativistic approach



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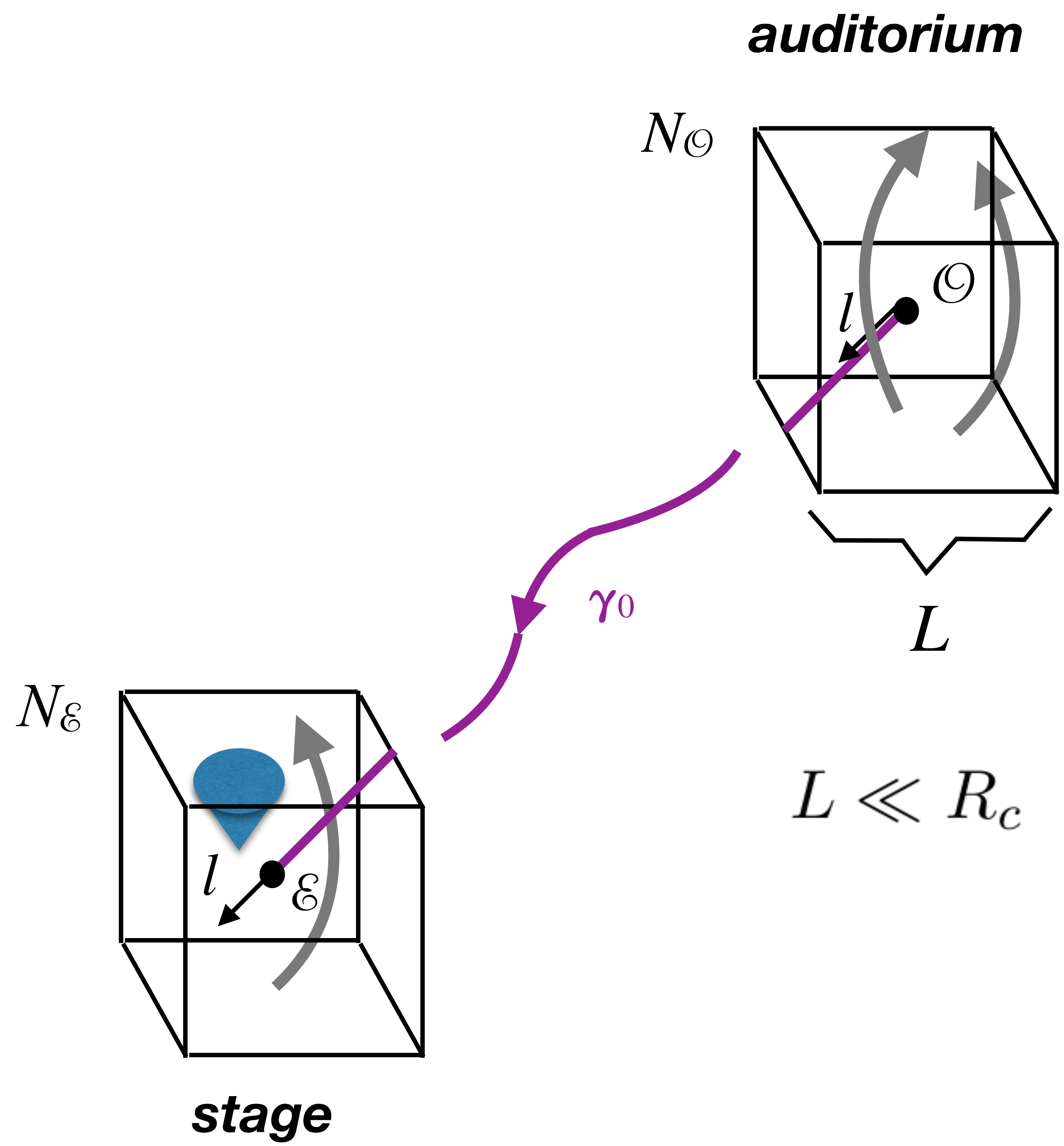
M. Grasso, MK, J. Serbenta, *Geometric optics in general relativity using bilocal operators*, Phys. Rev. **D 99**, 064038 (2019) (Editors' suggestion)



$L \ll R_c$

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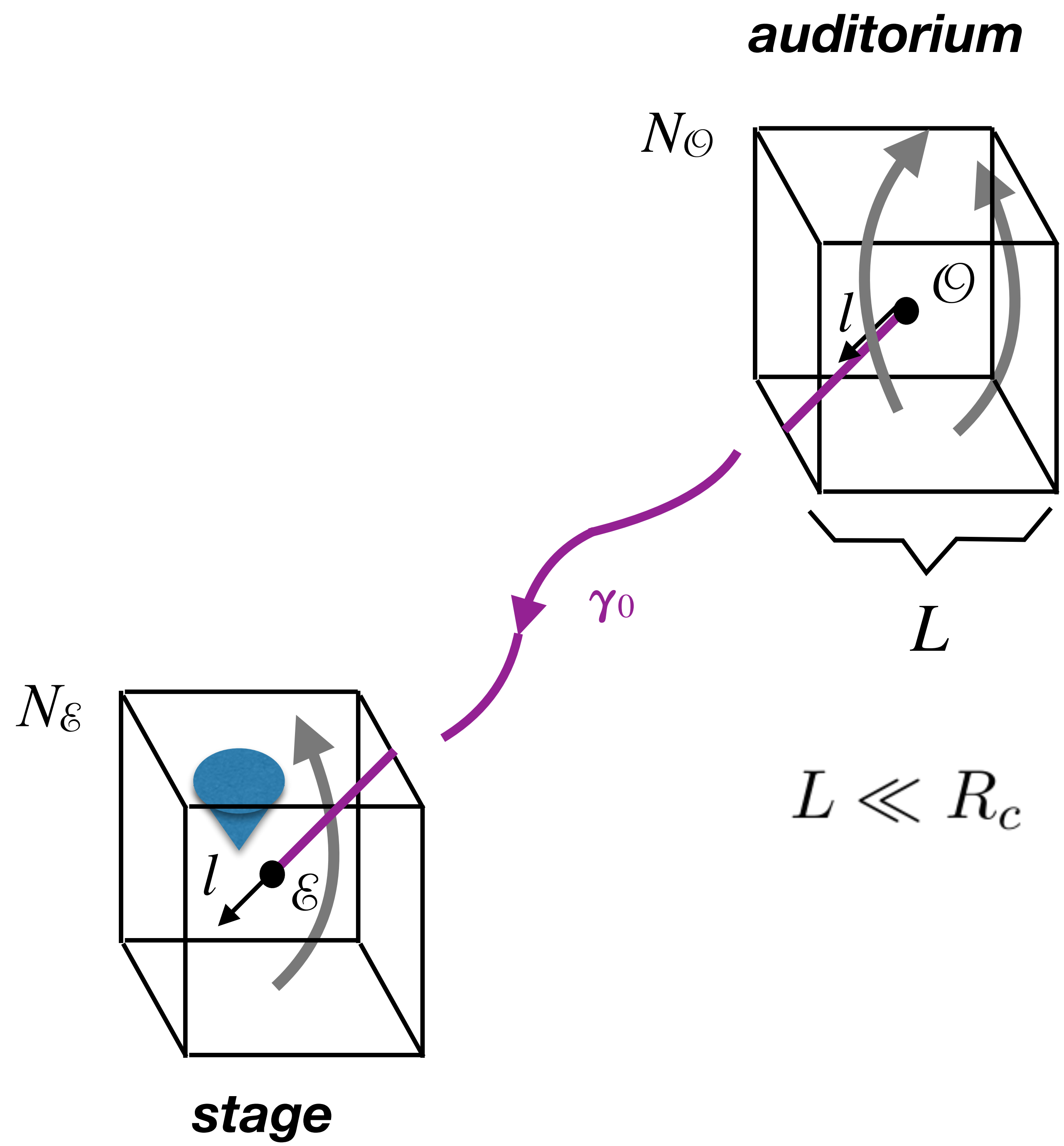

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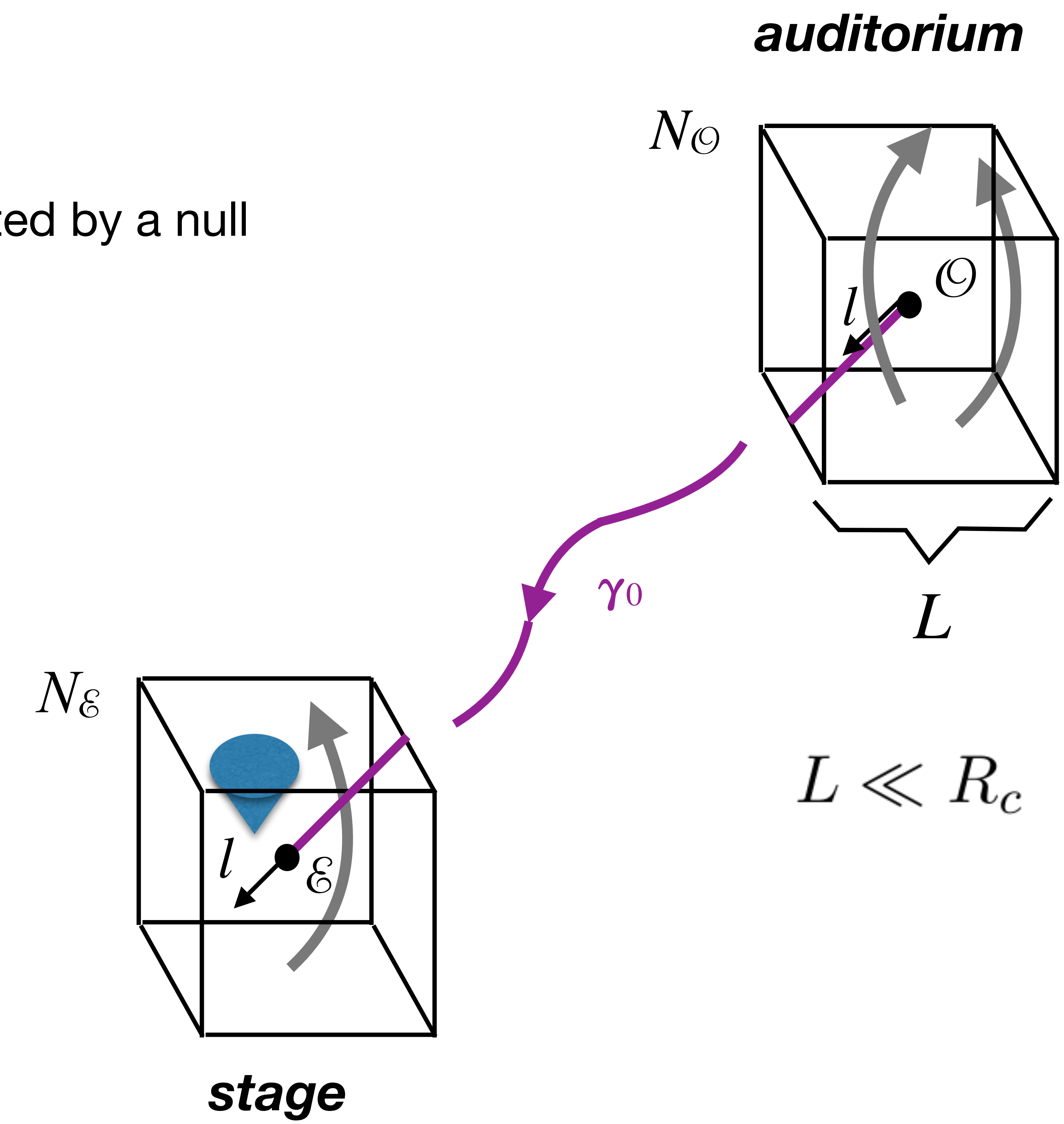

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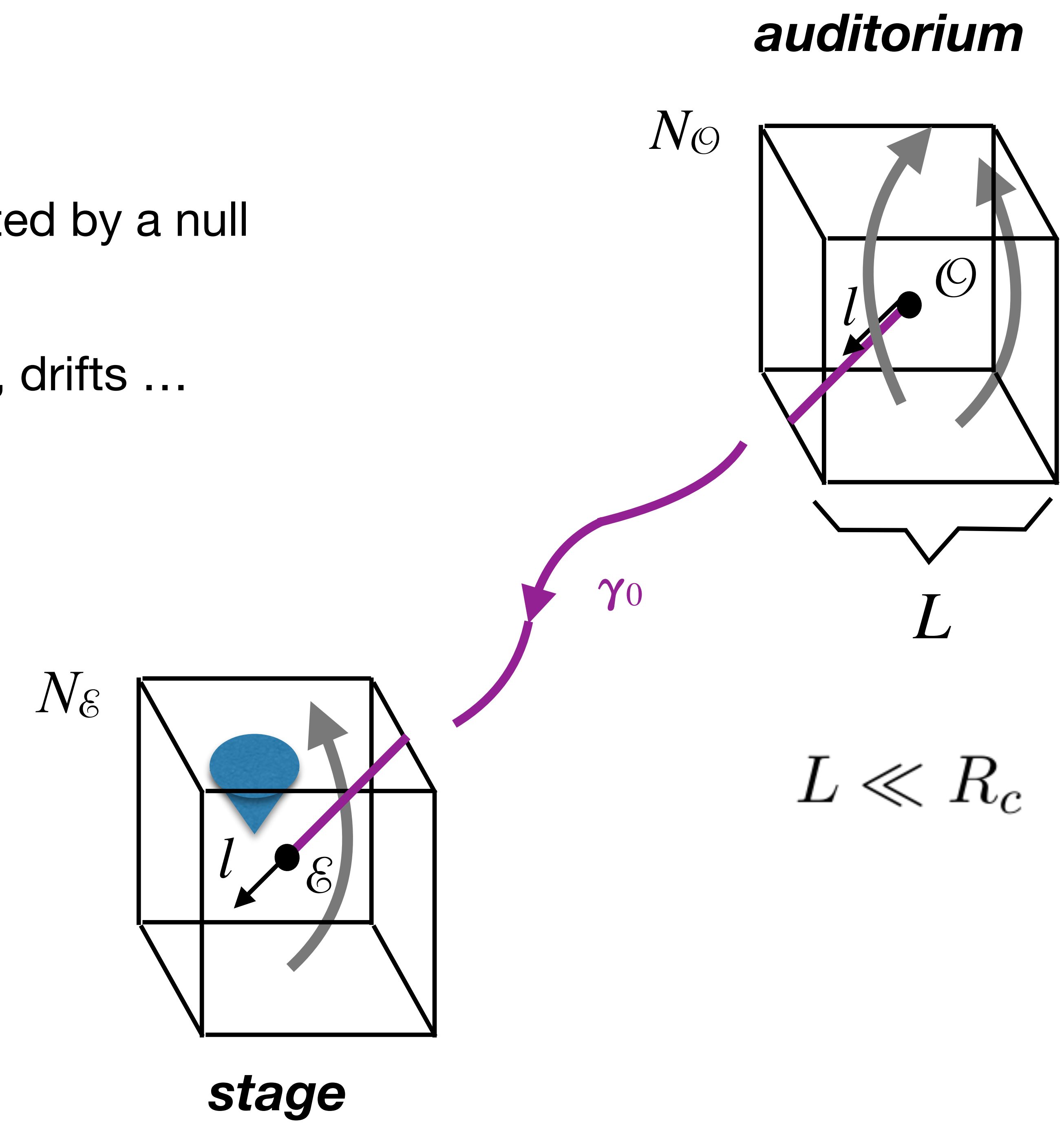

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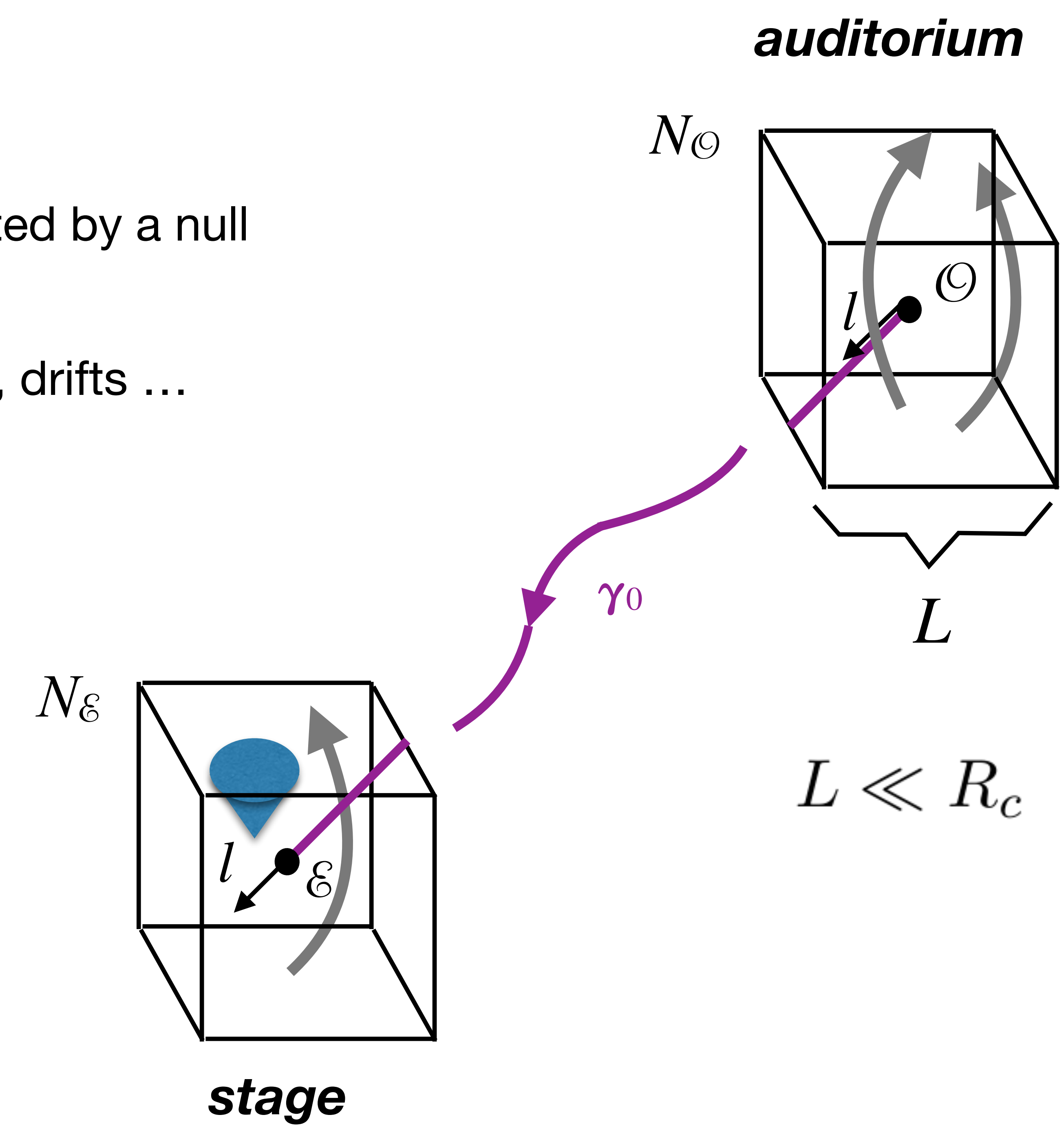

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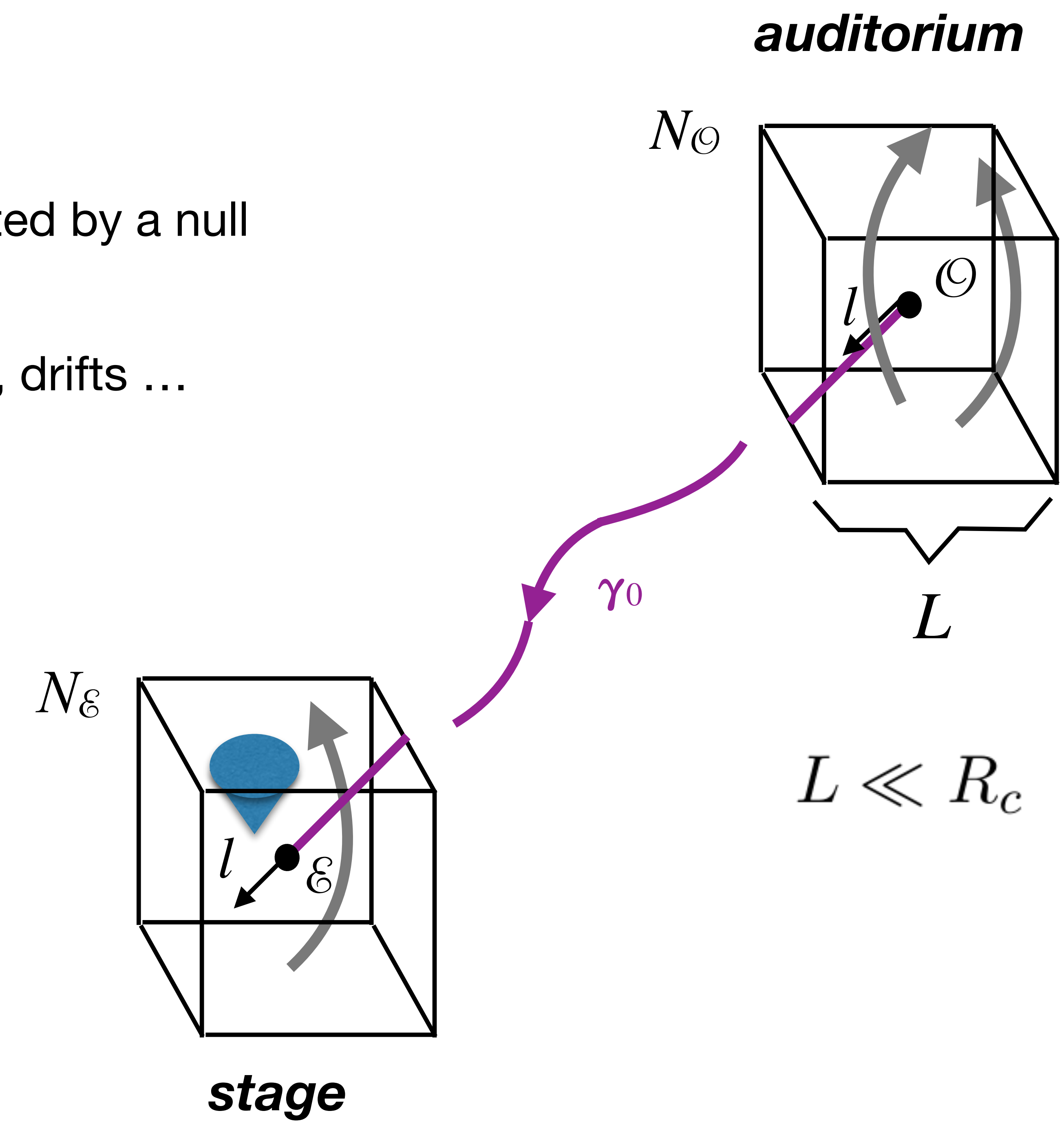

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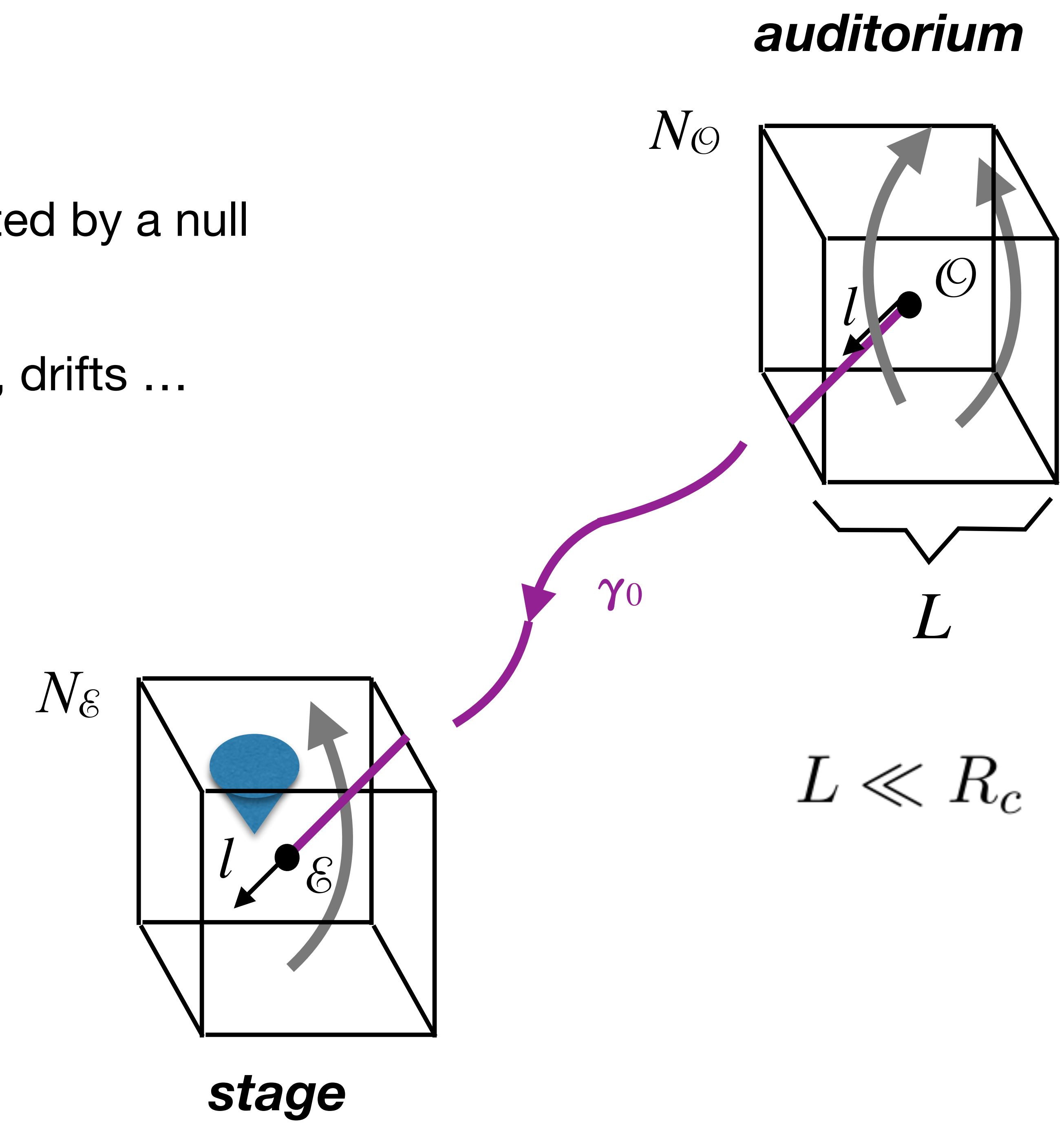

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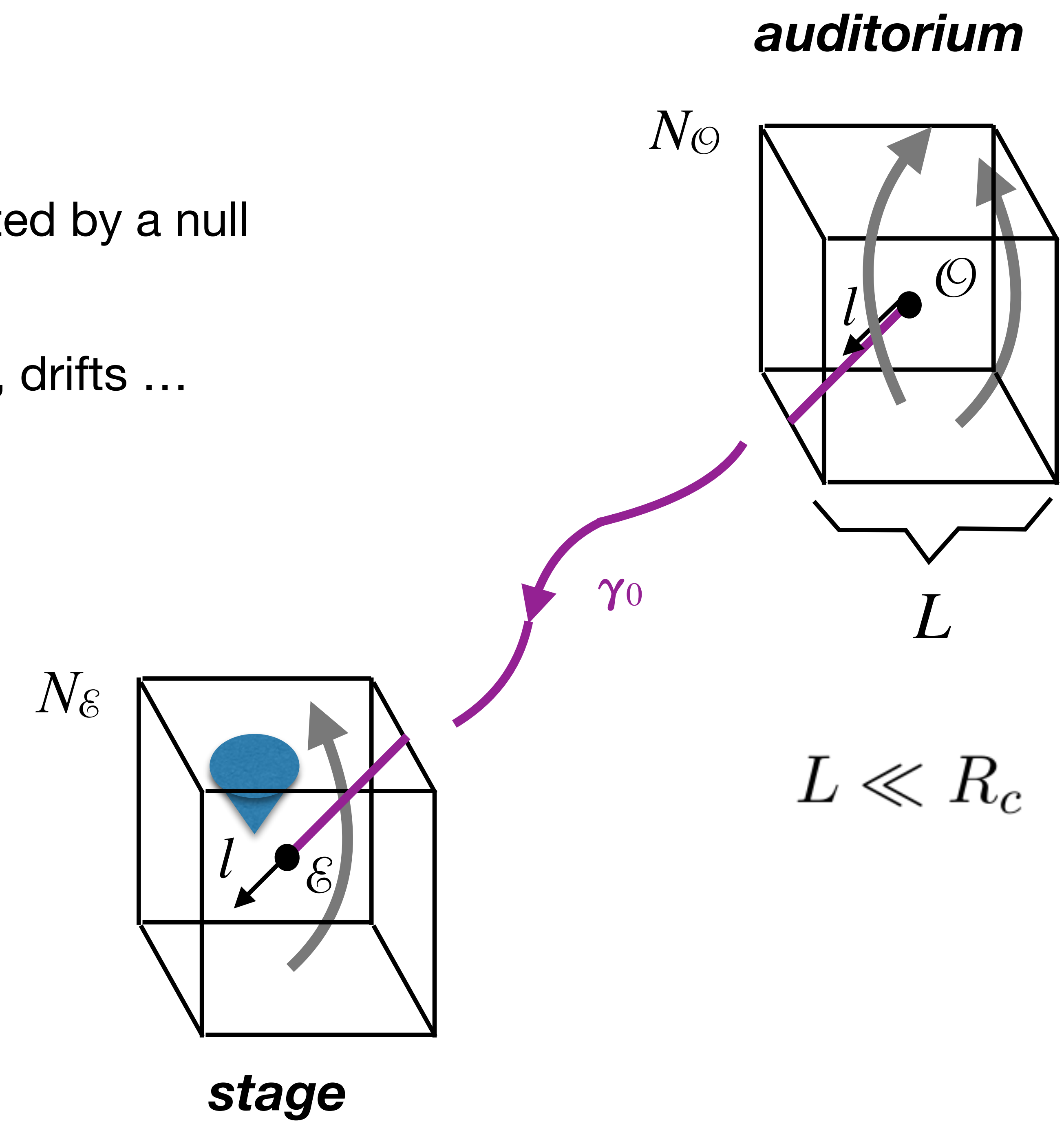
Separate the dependence in the expressions



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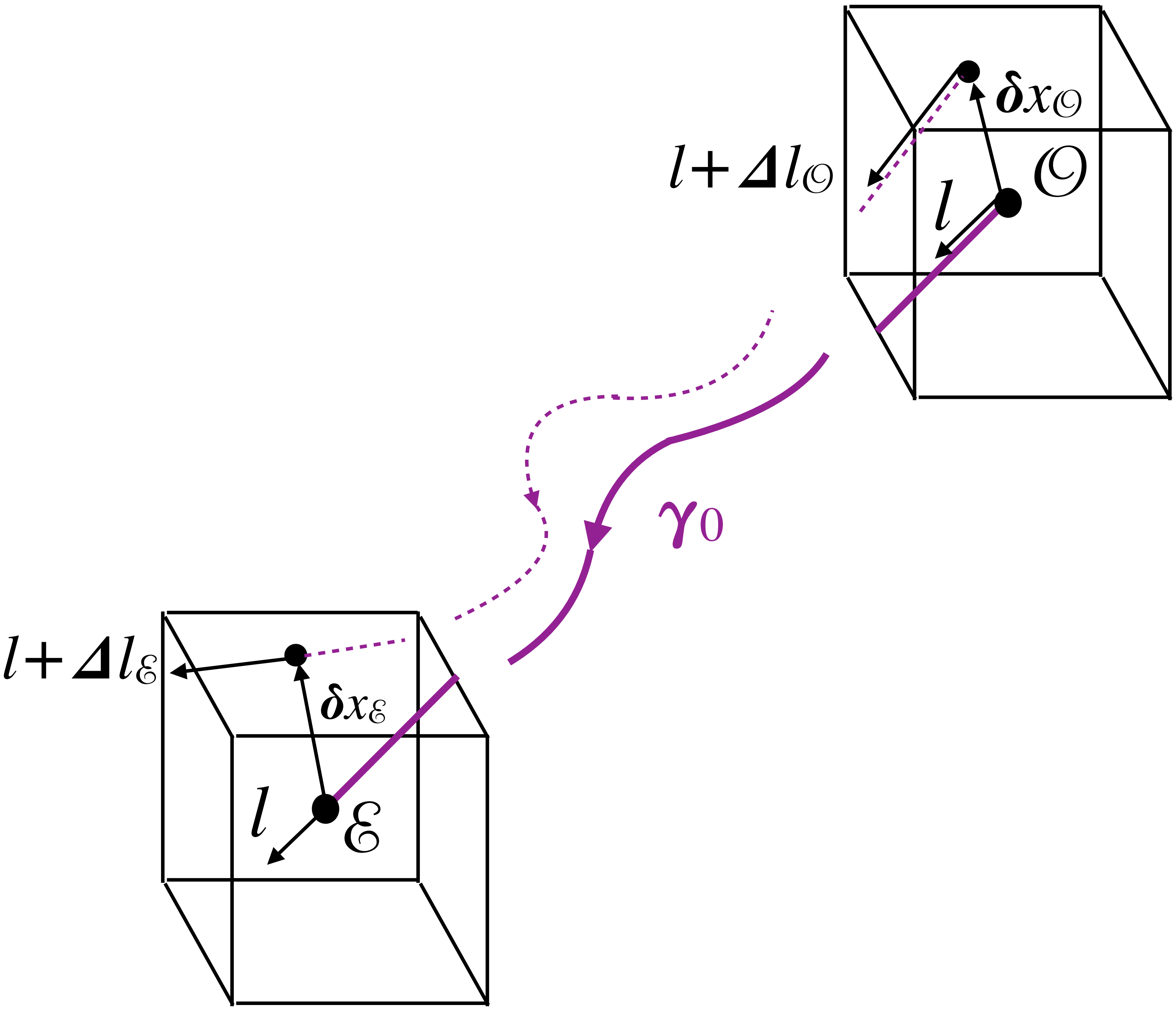
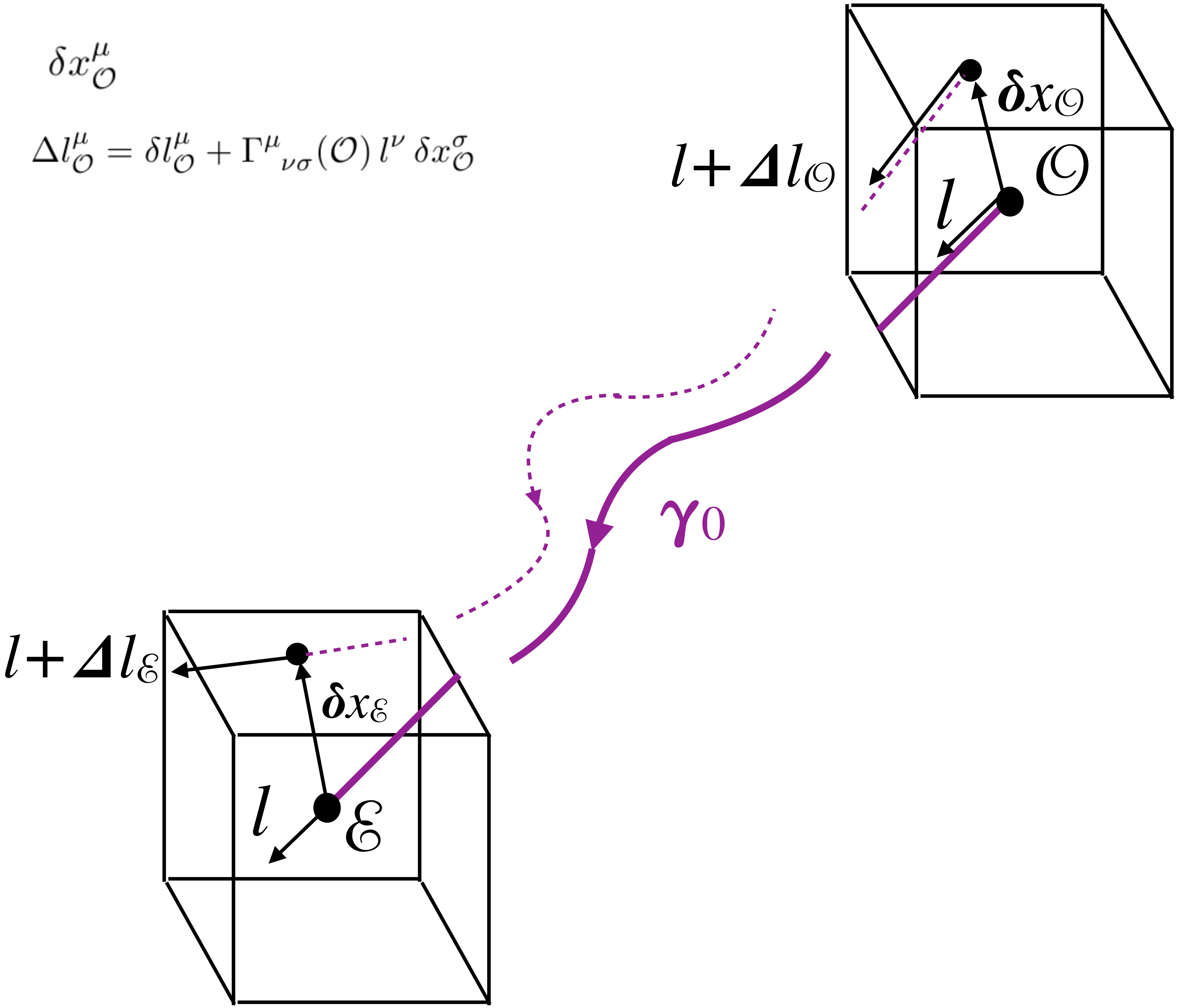
Light propagation effects



Light propagation effects

$\delta x^{\mu}_{\mathcal{O}}$

$\Delta l^{\mu}_{\mathcal{O}} = \delta l^{\mu}_{\mathcal{O}} + \Gamma^{\mu}{}_{\nu\sigma}(\mathcal{O})\, l^{\nu}\, \delta x^{\sigma}_{\mathcal{O}}$



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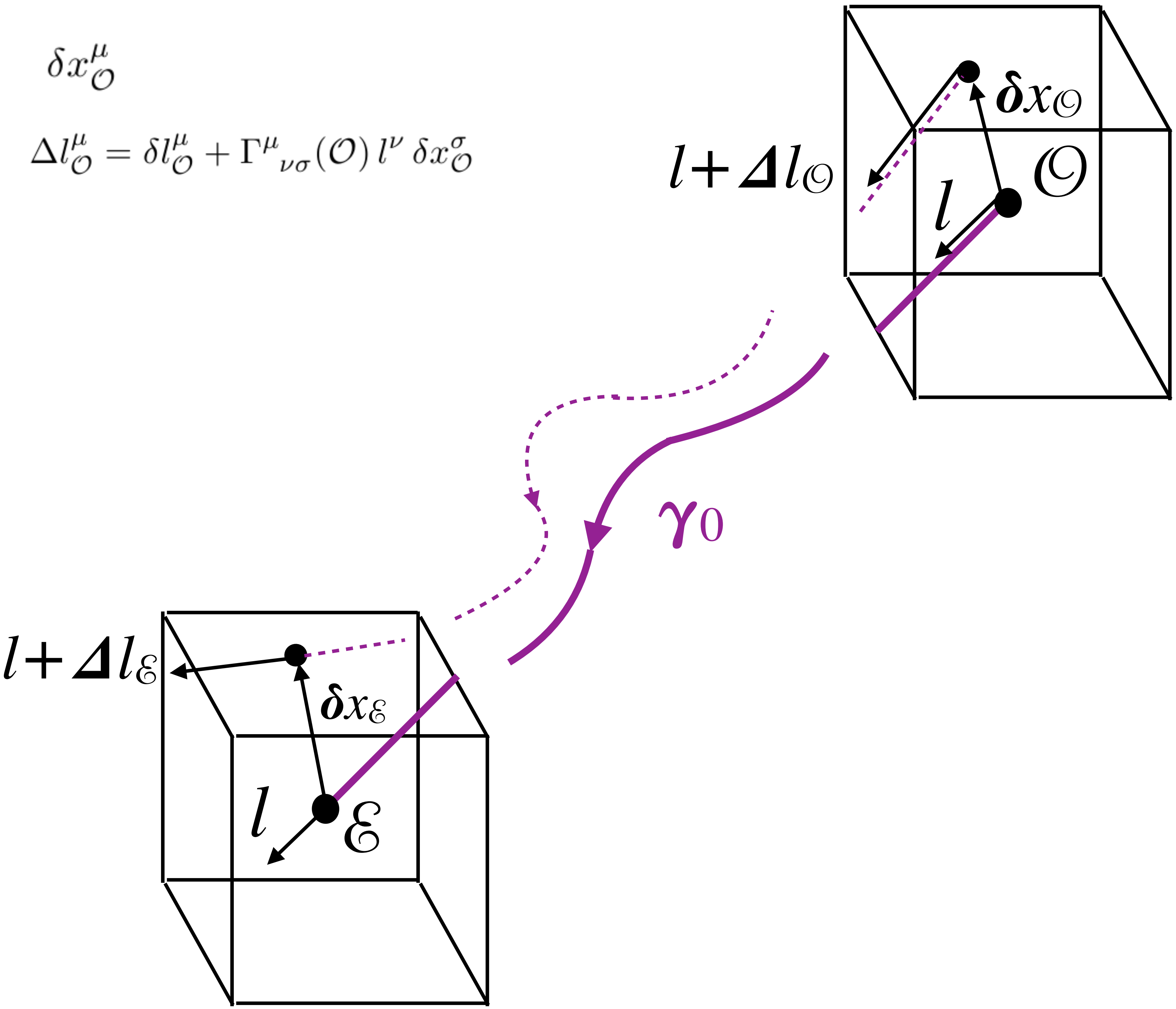

$\nabla_l \nabla_l \xi^\mu - R^\mu{}_{\alpha\beta\nu}\, l^\alpha\, l^\beta\, \xi^\nu = 0$

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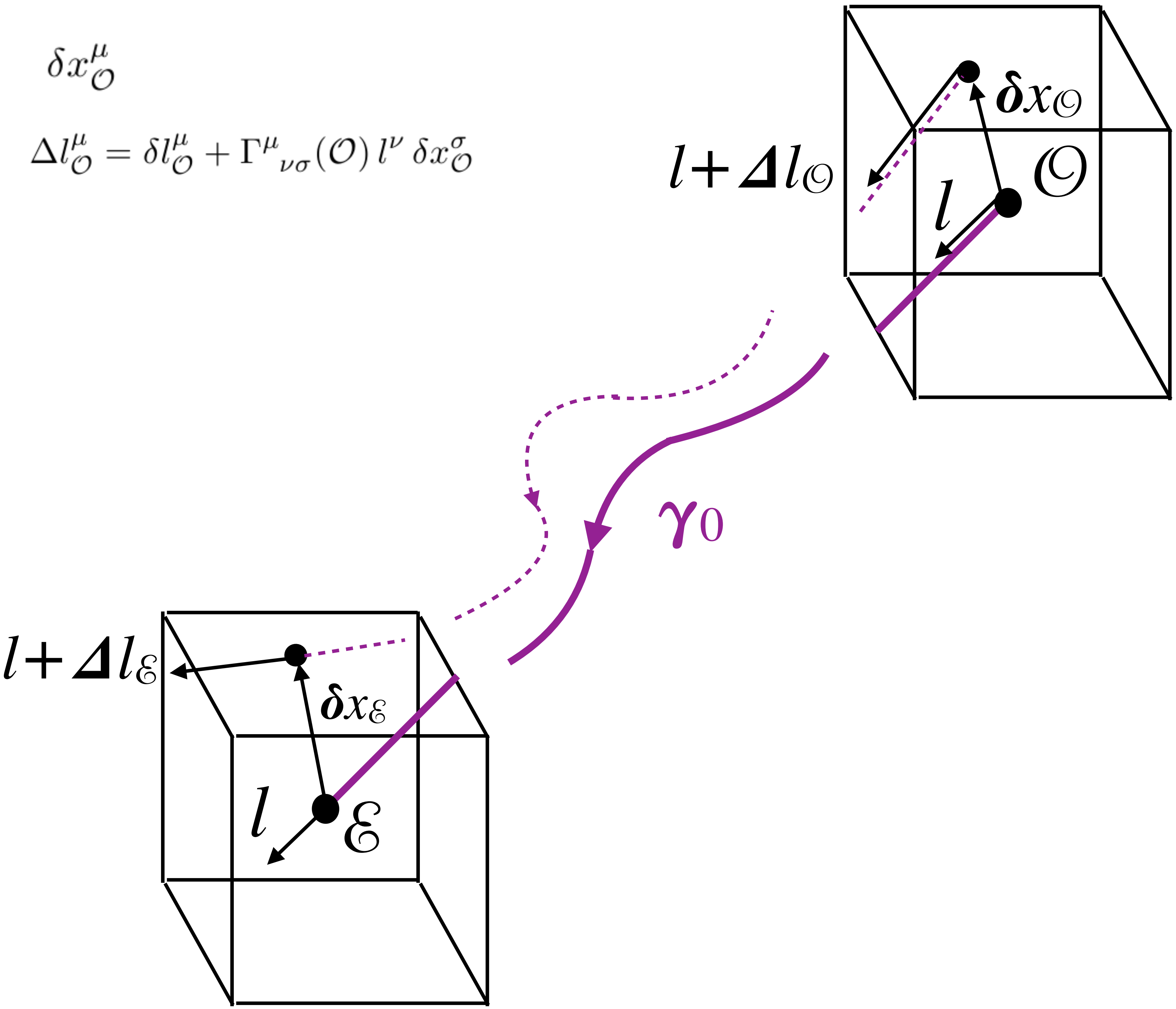

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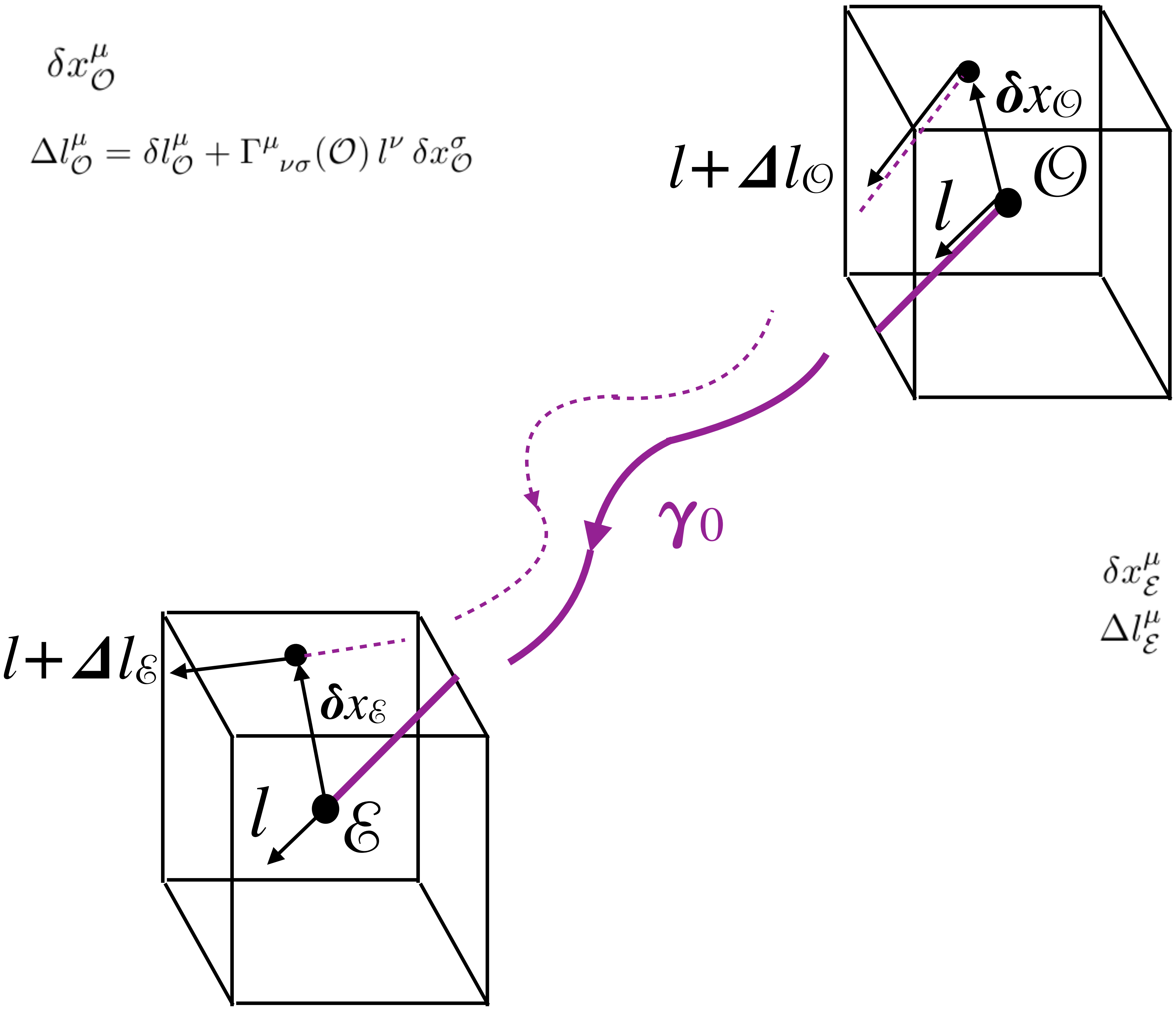

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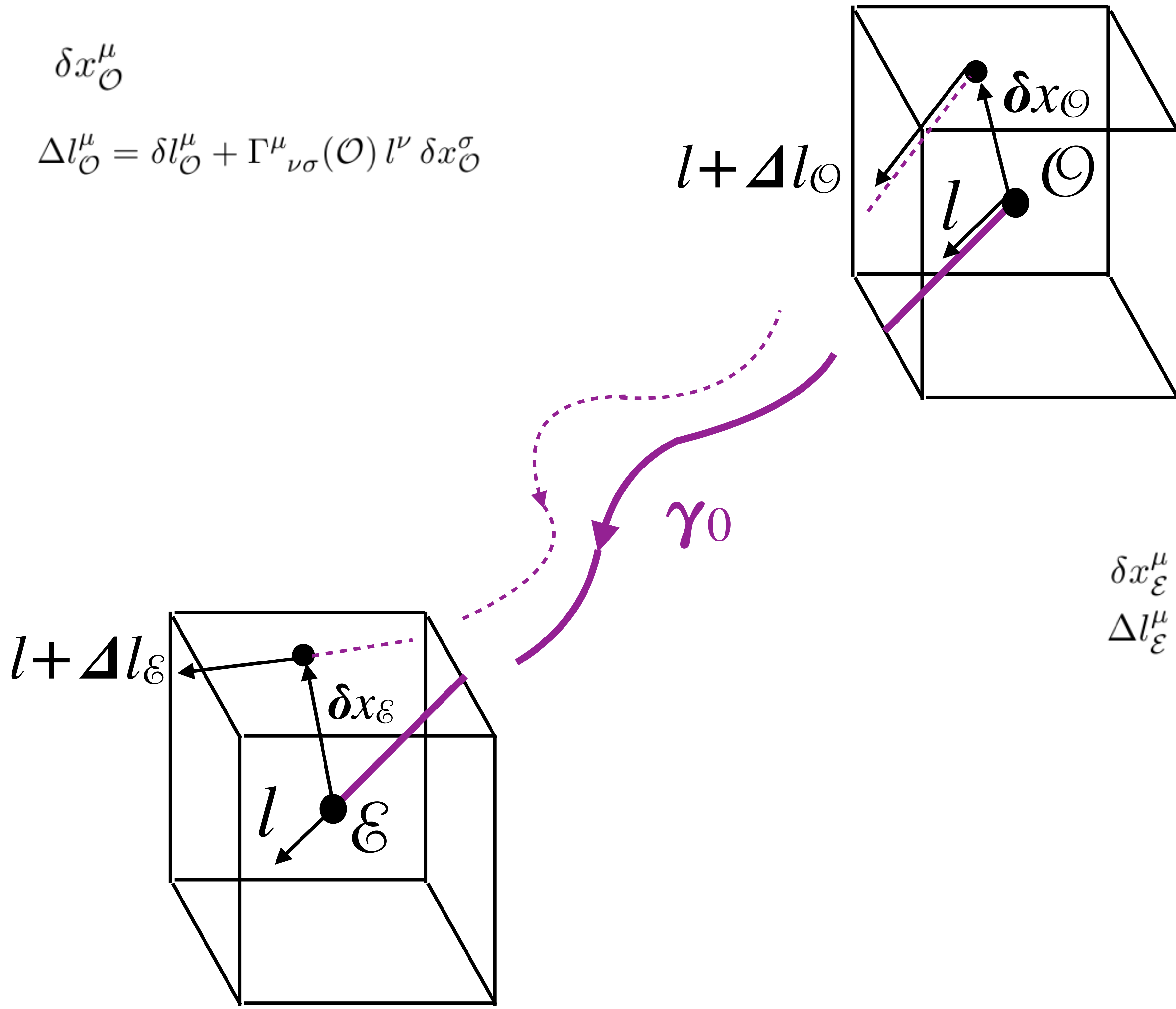

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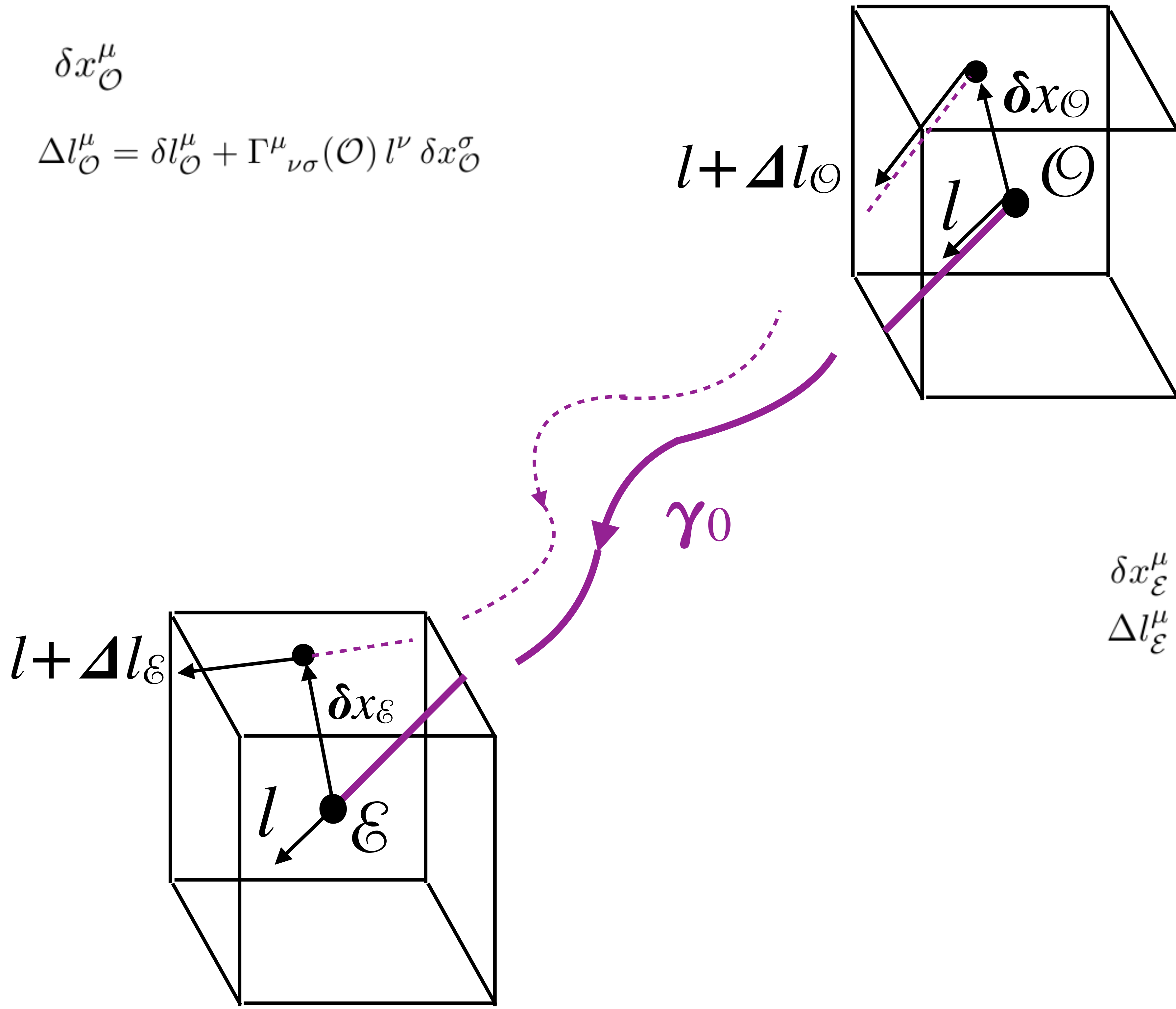

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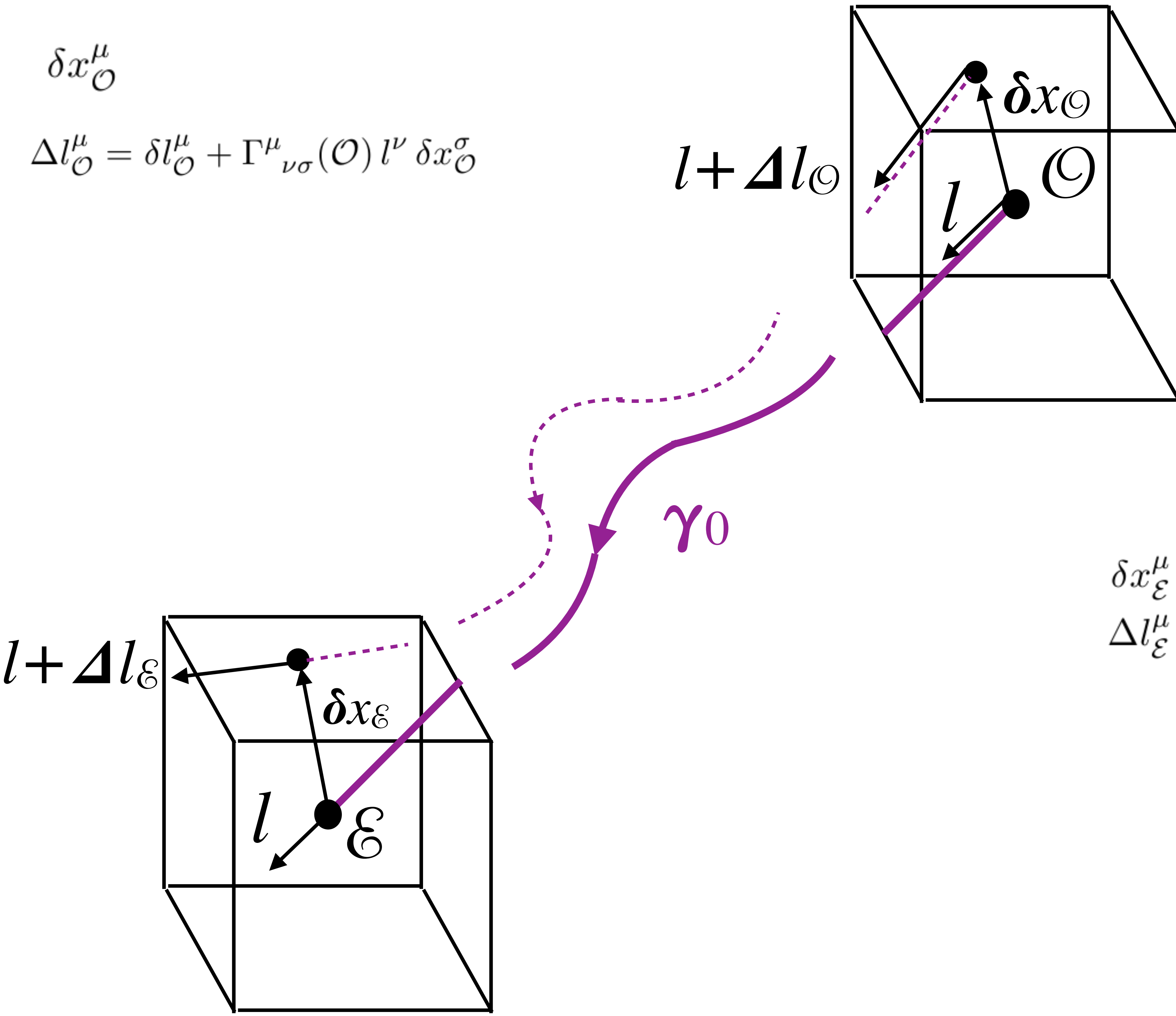

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$$\dot{B}^\mu{}_\nu(\lambda_{\mathcal{O}}) = \delta^\mu{}_\nu$$

$$W_{lx}{}^\mu{}_\nu = B^\mu{}_\nu(\lambda_{\mathcal{E}})$$
$$W_{ll}{}^\mu{}_\nu = \dot{B}^\mu{}_\nu(\lambda_{\mathcal{E}})$$

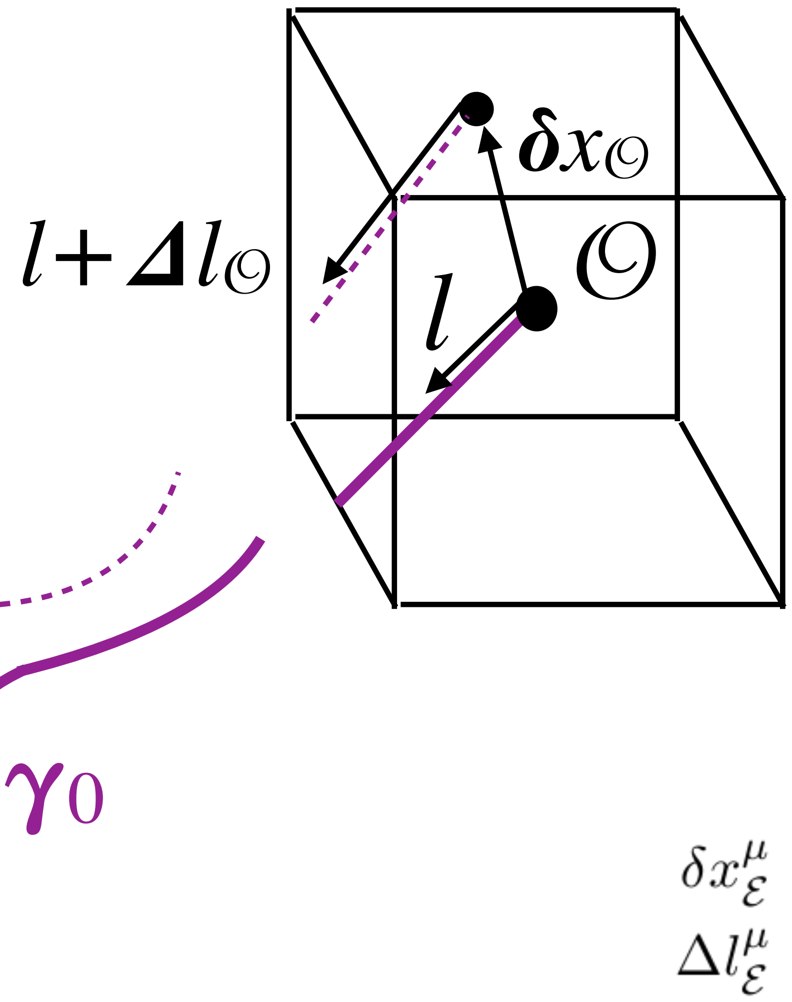
$$\begin{aligned} \delta x^\mu_{\mathcal{E}} &= W_{xx}{}^\mu{}_\nu\, \delta x^\nu_{\mathcal{O}} + W_{xl}{}^\mu{}_\nu\, \Delta l^\nu_{\mathcal{O}} \\ \Delta l^\mu_{\mathcal{E}} &= W_{lx}{}^\mu{}_\nu\, \delta x^\nu_{\mathcal{O}} + W_{ll}{}^\mu{}_\nu\, \Delta l^\nu_{\mathcal{O}} \end{aligned}$$

$$W_{**} : T_{\mathcal{O}}M \mapsto T_{\mathcal{E}}M$$

Light propagation effects

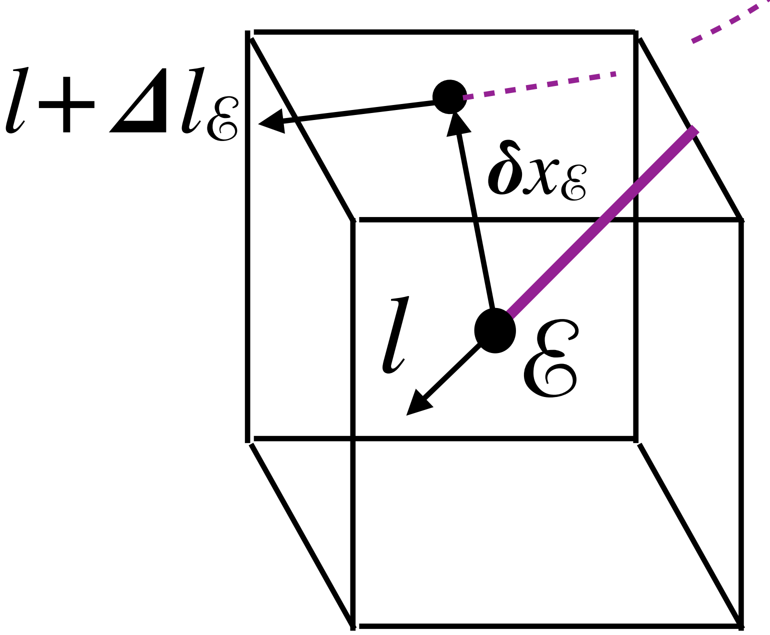
$$\delta x^\mu_{\mathcal{O}}$$

$$\Delta l^\mu_{\mathcal{O}} = \delta l^\mu_{\mathcal{O}} + \Gamma^\mu{}_{\nu\sigma}(\mathcal{O})\, l^\nu\, \delta x^\sigma_{\mathcal{O}}$$



$$\ddot{B}^\mu{}_\nu - R^\mu{}_{\alpha\beta\sigma}\, l^\alpha\, l^\beta\, B^\sigma{}_\nu = 0$$
$$B^\mu{}_\nu(\lambda_{\mathcal{O}}) = 0$$
$$\dot{B}^\mu{}_\nu(\lambda_{\mathcal{O}}) = \delta^\mu{}_\nu$$

$$W_{lx}{}^\mu{}_\nu = B^\mu{}_\nu(\lambda_{\mathcal{E}})$$
$$W_{ll}{}^\mu{}_\nu = \dot{B}^\mu{}_\nu(\lambda_{\mathcal{E}})$$



$$\begin{aligned} \delta x^\mu_{\mathcal{E}} &= W_{xx}{}^\mu{}_\nu\, \delta x^\nu_{\mathcal{O}} + W_{xl}{}^\mu{}_\nu\, \Delta l^\nu_{\mathcal{O}} \\ \Delta l^\mu_{\mathcal{E}} &= W_{lx}{}^\mu{}_\nu\, \delta x^\nu_{\mathcal{O}} + W_{ll}{}^\mu{}_\nu\, \Delta l^\nu_{\mathcal{O}} \end{aligned}$$

$$W_{**} : T_{\mathcal{O}}M \mapsto T_{\mathcal{E}}M$$

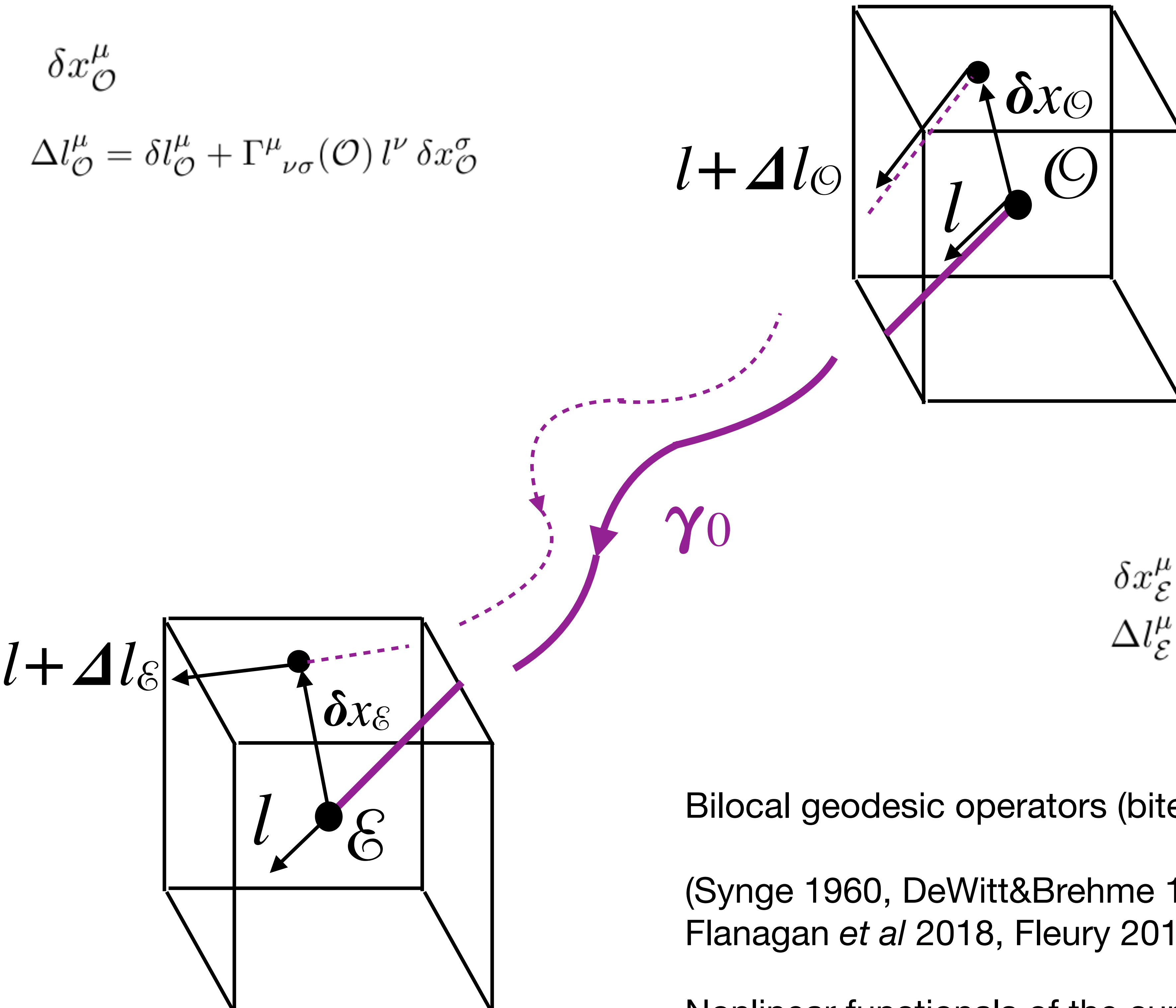
Bilocal geodesic operators (bitensors)

(Synge 1960, DeWitt&Brehme 1960, Dixon 1970, Vines 2015, Flanagan *et al* 2018, Fleury 2014, Uzun 2018…)

Light propagation effects

$\delta x^\mu_{\mathcal{O}}$

$\Delta l^\mu_{\mathcal{O}} = \delta l^\mu_{\mathcal{O}} + \Gamma^\mu{}_{\nu\sigma}(\mathcal{O})\, l^\nu\, \delta x^\sigma_{\mathcal{O}}$



$$\ddot{B}^\mu{}_\nu - R^\mu{}_{\alpha\beta\sigma}\, l^\alpha\, l^\beta\, B^\sigma{}_\nu = 0$$
$$B^\mu{}_\nu(\lambda_{\mathcal{O}}) = 0$$
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$$W_{lx}{}^\mu{}_\nu = B^\mu{}_\nu(\lambda_{\mathcal{E}})$$
$$W_{ll}{}^\mu{}_\nu = \dot{B}^\mu{}_\nu(\lambda_{\mathcal{E}})$$

$$\begin{aligned} \delta x^\mu_{\mathcal{E}} &= W_{xx}{}^\mu{}_\nu\, \delta x^\nu_{\mathcal{O}} + W_{xl}{}^\mu{}_\nu\, \Delta l^\nu_{\mathcal{O}} \\ \Delta l^\mu_{\mathcal{E}} &= W_{lx}{}^\mu{}_\nu\, \delta x^\nu_{\mathcal{O}} + W_{ll}{}^\mu{}_\nu\, \Delta l^\nu_{\mathcal{O}} \end{aligned}$$

$$W_{**} : T_{\mathcal{O}}M \mapsto T_{\mathcal{E}}M$$

Bilocal geodesic operators (bitensors)

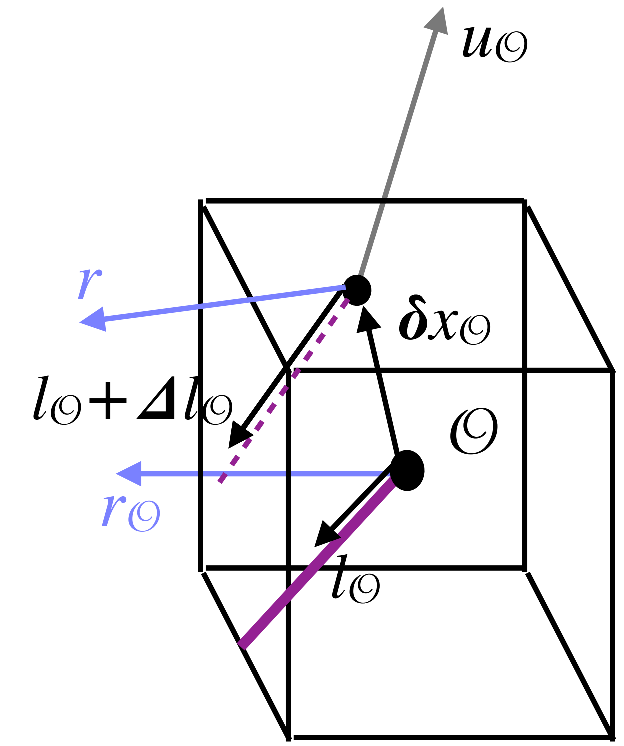
(Synge 1960, DeWitt&Brehme 1960, Dixon 1970, Vines 2015, Flanagan *et al* 2018, Fleury 2014, Uzun 2018…)

Nonlinear functionals of the curvature tensor

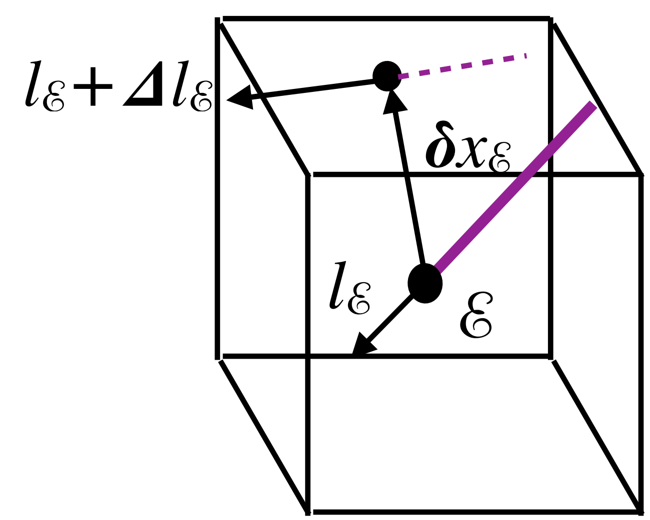
Observables

Apparent position on the observer's sky

γ_0 as the reference null geodesic



auditorium



stage

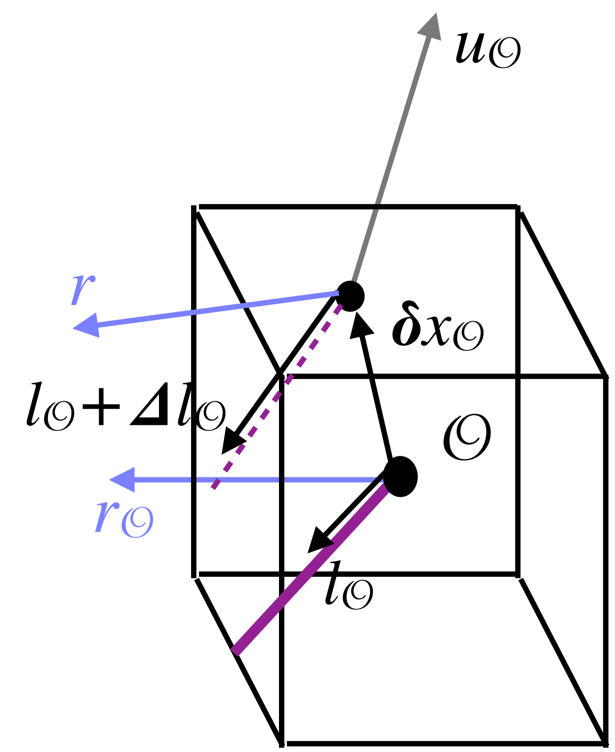
Observables

Apparent position on the observer's sky

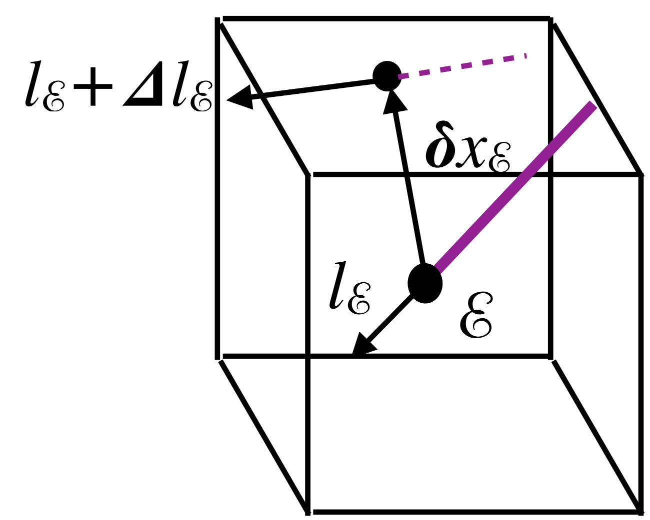
γ_0 as the reference null geodesic

- **position the sky** $\quad r^\mu = \dfrac{l^\mu_{\mathcal{O}} + \Delta l^\mu_{\mathcal{O}}}{(l^\sigma_{\mathcal{O}} + \Delta l^\sigma_{\mathcal{O}})\, u_{\mathcal{O}_\sigma}} + u^\mu_{\mathcal{O}}$

$$\delta r^A = \frac{\Delta l^A_{\mathcal{O}}}{(l^\sigma_{\mathcal{O}} + \Delta l^\sigma_{\mathcal{O}})\, u_{\mathcal{O}_\sigma}} = \frac{\Delta l^A_{\mathcal{O}}}{l^\sigma_{\mathcal{O}}\, u_{\mathcal{O}_\sigma}} + O(\Delta l^2_{\mathcal{O}})$$



auditorium



stage

Observables

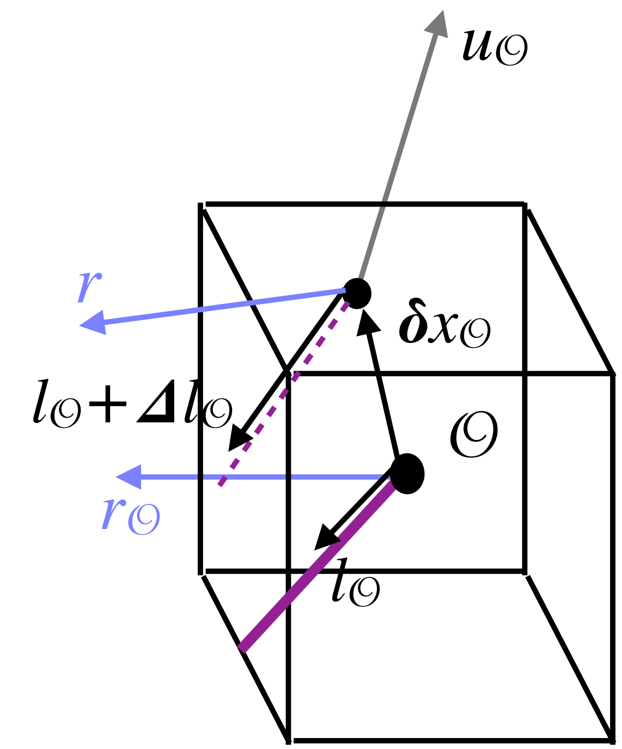
Apparent position on the observer's sky

γ_0 as the reference null geodesic

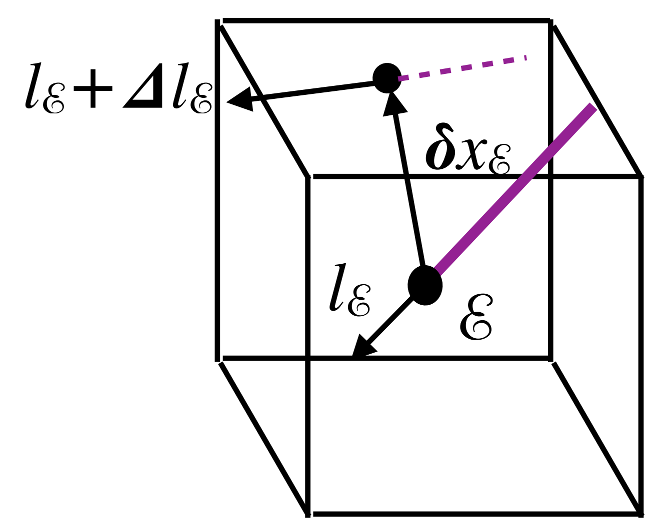
- **position the sky** $$r^\mu = \frac{l^\mu_{\mathcal{O}} + \Delta l^\mu_{\mathcal{O}}}{(l^\sigma_{\mathcal{O}} + \Delta l^\sigma_{\mathcal{O}})\, u_{\mathcal{O}_\sigma}} + u^\mu_{\mathcal{O}}$$

$$\delta r^A = \frac{\Delta l^A_{\mathcal{O}}}{(l^\sigma_{\mathcal{O}} + \Delta l^\sigma_{\mathcal{O}})\, u_{\mathcal{O}_\sigma}} = \frac{\Delta l^A_{\mathcal{O}}}{l^\sigma_{\mathcal{O}}\, u_{\mathcal{O}_\sigma}} + \cancel{O(\Delta l^2_{\mathcal{O}})}$$

Parallel rays approximation (PRA)



auditorium



stage

Observables

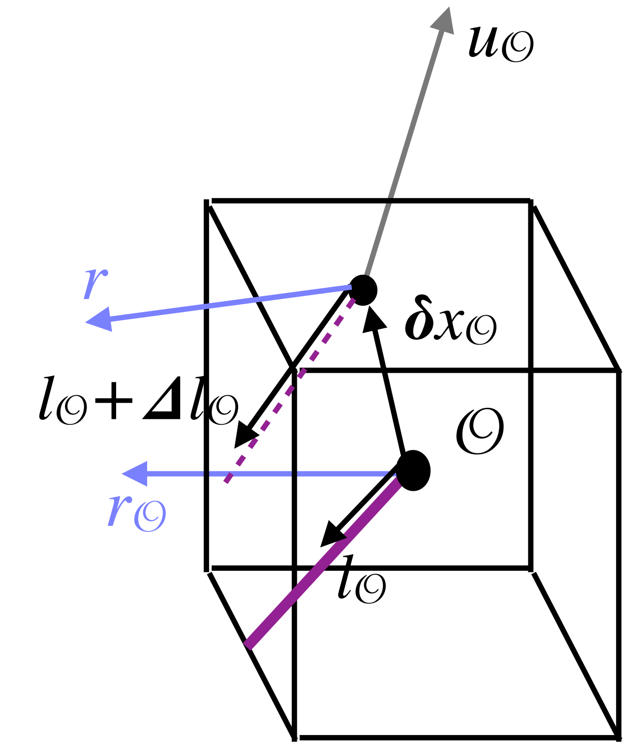
Apparent position on the observer's sky

γ_0 as the reference null geodesic

- **position the sky** $$r^\mu = \frac{l^\mu_{\mathscr{O}} + \Delta l^\mu_{\mathscr{O}}}{(l^\sigma_{\mathscr{O}} + \Delta l^\sigma_{\mathscr{O}})\, u_{\mathscr{O}_\sigma}} + u^\mu_{\mathscr{O}}$$

$$\delta r^A = \frac{\Delta l^A_{\mathscr{O}}}{(l^\sigma_{\mathscr{O}} + \Delta l^\sigma_{\mathscr{O}})\, u_{\mathscr{O}_\sigma}} = \frac{\Delta l^A_{\mathscr{O}}}{l^\sigma_{\mathscr{O}}\, u_{\mathscr{O}_\sigma}} + \cancel{O(\Delta l^2_{\mathscr{O}})}$$

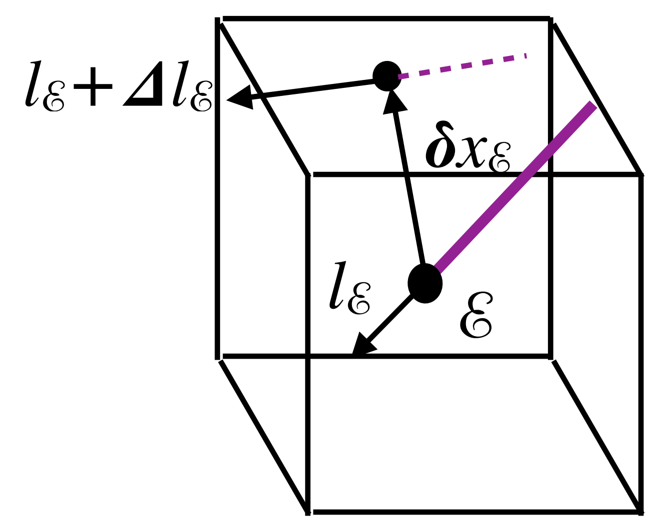
Parallel rays approximation (PRA)



auditorium

- **time of arrival** $$g_{\mu\nu}\,(l^\mu_{\mathscr{O}} + \Delta l^\mu_{\mathscr{O}})\,(l^\nu_{\mathscr{O}} + \Delta l^\nu_{\mathscr{O}}) = 0$$

$$g_{\mu\nu}\, l^\mu_{\mathscr{O}}\, \Delta l^\mu_{\mathscr{O}} + O(\Delta l^2_{\mathscr{O}}) = 0$$



stage

Observables

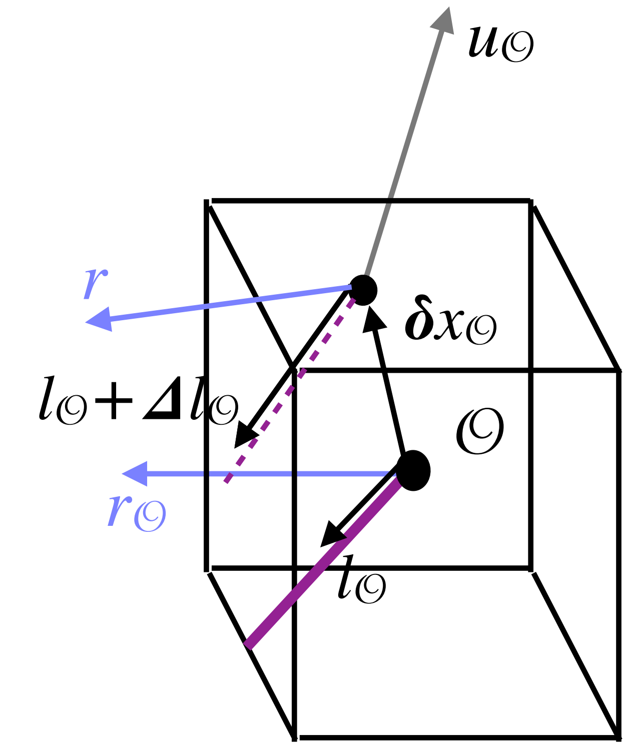
Apparent position on the observer's sky

γ_0 as the reference null geodesic

- **position the sky** $$r^\mu = \frac{l^\mu_{\mathcal{O}} + \Delta l^\mu_{\mathcal{O}}}{(l^\sigma_{\mathcal{O}} + \Delta l^\sigma_{\mathcal{O}})\, u_{\mathcal{O}_\sigma}} + u^\mu_{\mathcal{O}}$$

$$\delta r^A = \frac{\Delta l^A_{\mathcal{O}}}{(l^\sigma_{\mathcal{O}} + \Delta l^\sigma_{\mathcal{O}})\, u_{\mathcal{O}_\sigma}} = \frac{\Delta l^A_{\mathcal{O}}}{l^\sigma_{\mathcal{O}}\, u_{\mathcal{O}_\sigma}} \cancel{+ O(\Delta l^2_{\mathcal{O}})}$$

Parallel rays approximation (PRA)

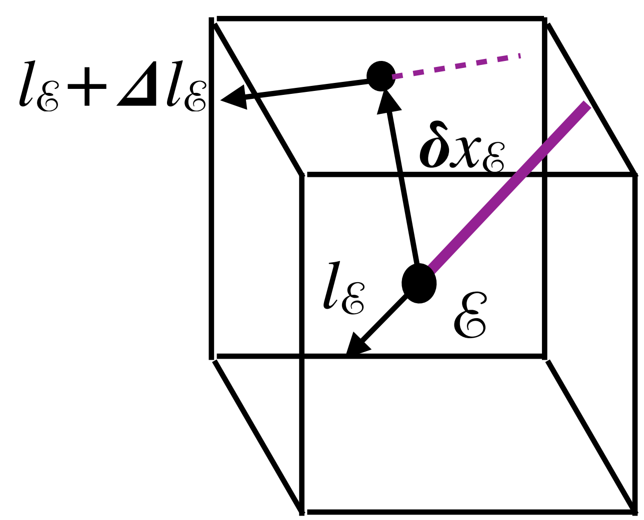


auditorium

- **time of arrival** $$g_{\mu\nu}\,(l^\mu_{\mathcal{O}} + \Delta l^\mu_{\mathcal{O}})\,(l^\nu_{\mathcal{O}} + \Delta l^\nu_{\mathcal{O}}) = 0$$

$$g_{\mu\nu}\, l^\mu_{\mathcal{O}}\, \Delta l^\mu_{\mathcal{O}} \cancel{+ O(\Delta l^2_{\mathcal{O}})} = 0$$

Flat light cones approximation (FLA)



stage

Observables

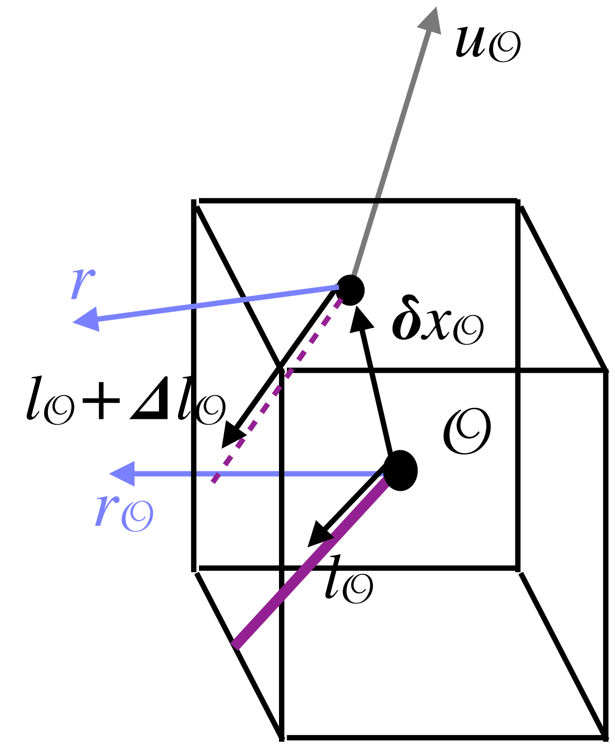
Apparent position on the observer's sky

γ_0 as the reference null geodesic

- **position the sky** $$r^\mu = \frac{l^\mu_{\mathcal{O}} + \Delta l^\mu_{\mathcal{O}}}{(l^\sigma_{\mathcal{O}} + \Delta l^\sigma_{\mathcal{O}})\, u_{\mathcal{O}_\sigma}} + u^\mu_{\mathcal{O}}$$

$$\delta r^A = \frac{\Delta l^A_{\mathcal{O}}}{(l^\sigma_{\mathcal{O}} + \Delta l^\sigma_{\mathcal{O}})\, u_{\mathcal{O}_\sigma}} = \frac{\Delta l^A_{\mathcal{O}}}{l^\sigma_{\mathcal{O}}\, u_{\mathcal{O}_\sigma}} \xcancel{+ O(\Delta l^2_{\mathcal{O}})}$$

Parallel rays approximation (PRA)



auditorium

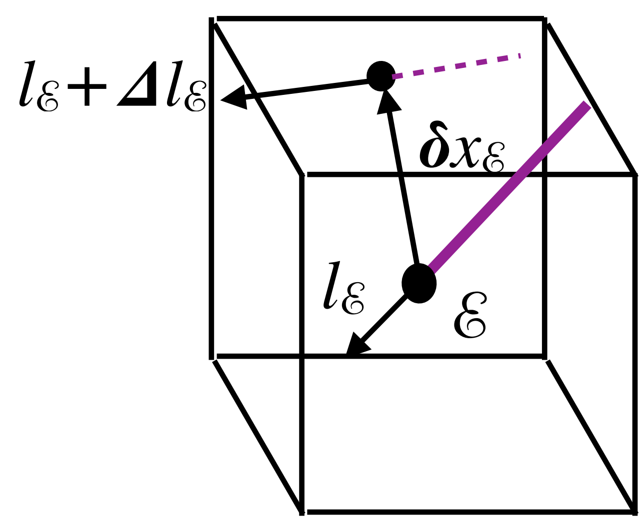
- **time of arrival** $$g_{\mu\nu}\,(l^\mu_{\mathcal{O}} + \Delta l^\mu_{\mathcal{O}})\,(l^\nu_{\mathcal{O}} + \Delta l^\nu_{\mathcal{O}}) = 0$$

$$g_{\mu\nu}\, l^\mu_{\mathcal{O}}\, \Delta l^\mu_{\mathcal{O}} \xcancel{+ O(\Delta l^2_{\mathcal{O}})} = 0$$

Flat light cones approximation (FLA)

$\Updownarrow$ geodesic deviation equation

$$\delta x^\mu_{\mathcal{O}}\, l_{\mathcal{O}\,\mu} = \delta x^\mu_{\mathcal{E}}\, l_{\mathcal{E}\,\mu}$$



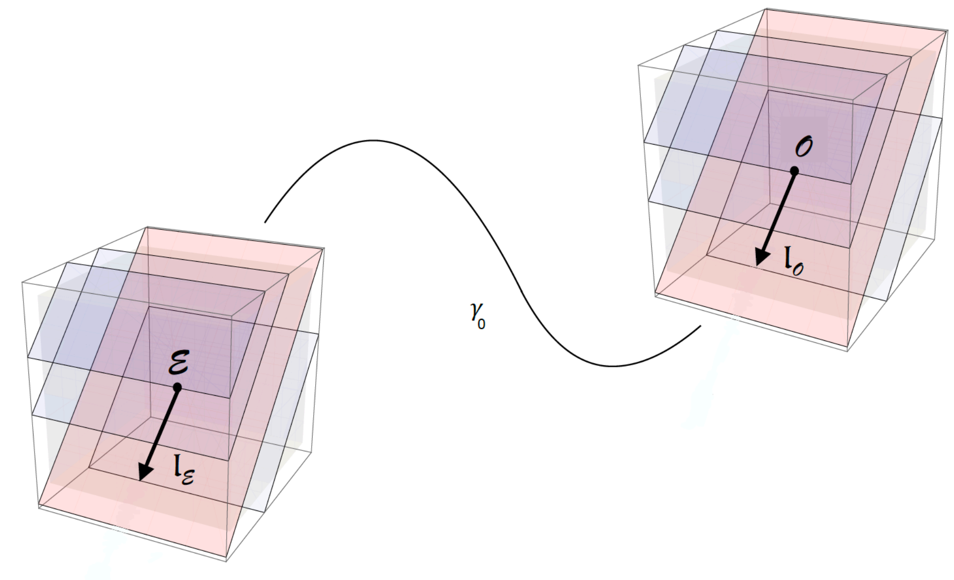
stage

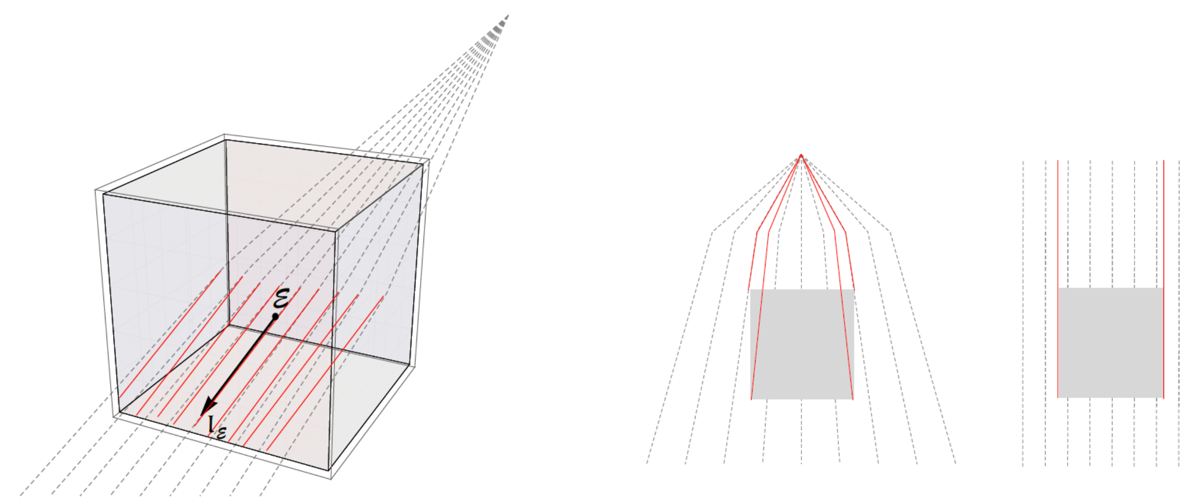
Approximations

Flat light cones approximation

$$\delta x^{\mu}_{\mathscr{O}}\, l_{\mathscr{O}\,\mu} = \delta x^{\mu}_{\mathscr{E}}\, l_{\mathscr{E}\,\mu}$$

No transverse Rømer delays



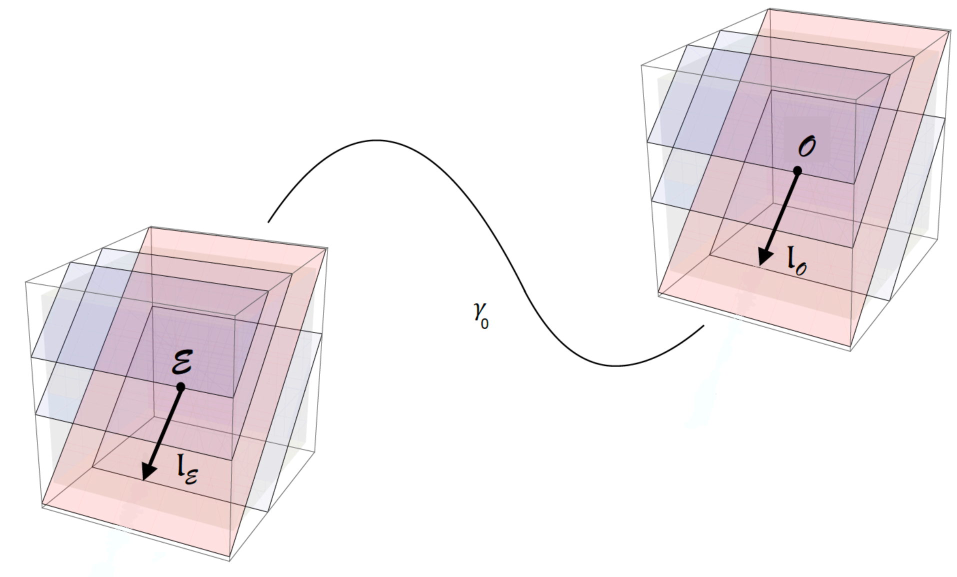


Approximations

Flat light cones approximation

$$\delta x^\mu_{\mathscr{O}}\, l_{\mathscr{O}\,\mu} = \delta x^\mu_{\mathscr{E}}\, l_{\mathscr{E}\,\mu}$$

No transverse Rømer delays

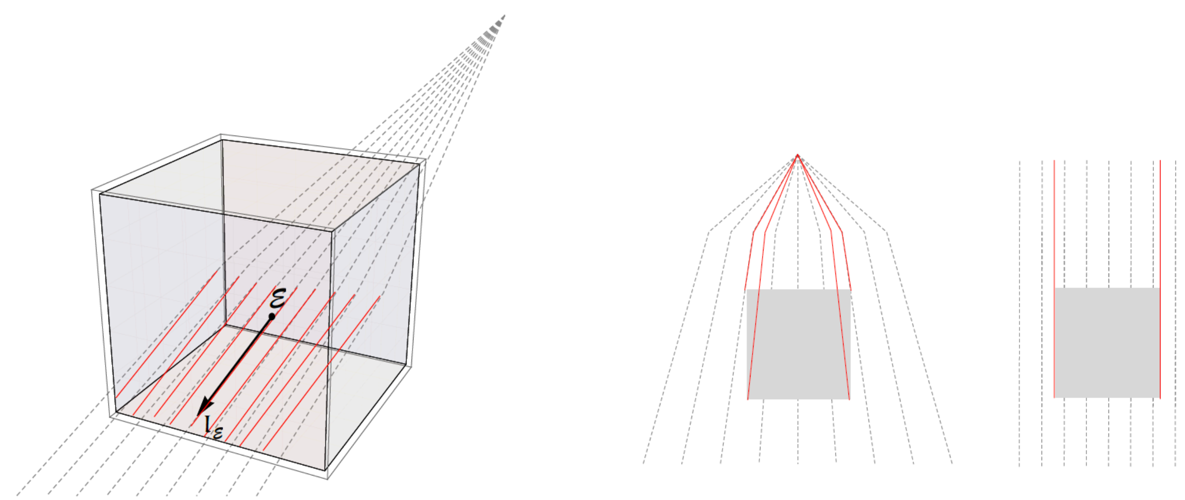


Parallel rays approximation

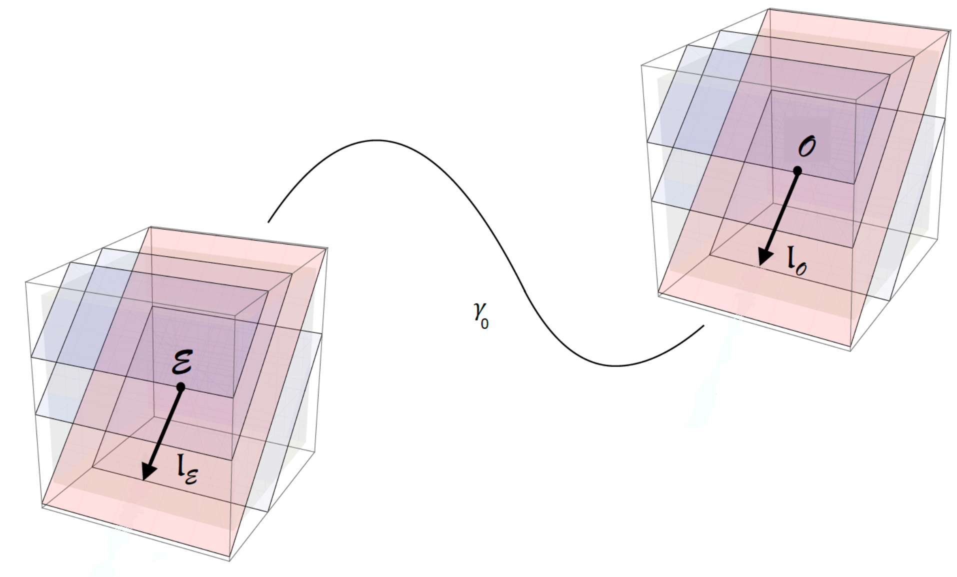
$$\delta r^A = \frac{\Delta l^A_{\mathscr{O}}}{l^\sigma_{\mathscr{O}}\, u_{\mathscr{O}_\sigma}}$$

No perspective distortions

l components drop out,
only the timelike and transverse remain



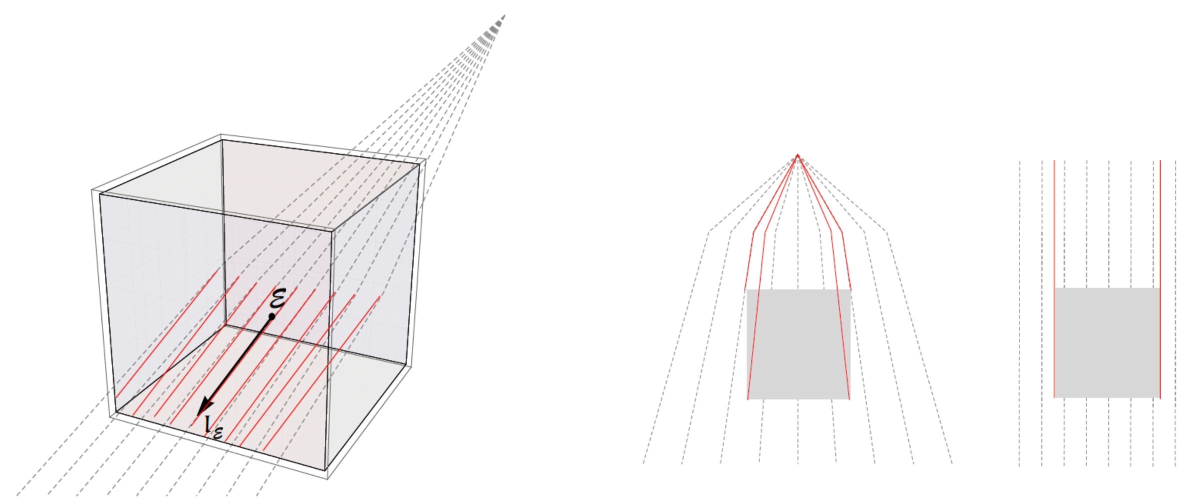
Approximations



Flat light cones approximation

$$\delta x^{\mu}_{\mathcal{O}}\, l_{\mathcal{O}\,\mu} = \delta x^{\mu}_{\mathcal{E}}\, l_{\mathcal{E}\,\mu}$$

No transverse Rømer delays



Parallel rays approximation

$$\delta r^{A} = \frac{\Delta l^{A}_{\mathcal{O}}}{l^{\sigma}_{\mathcal{O}}\, u_{\mathcal{O}_{\sigma}}}$$

No perspective distortions

l components drop out,
only the timelike and transverse remain

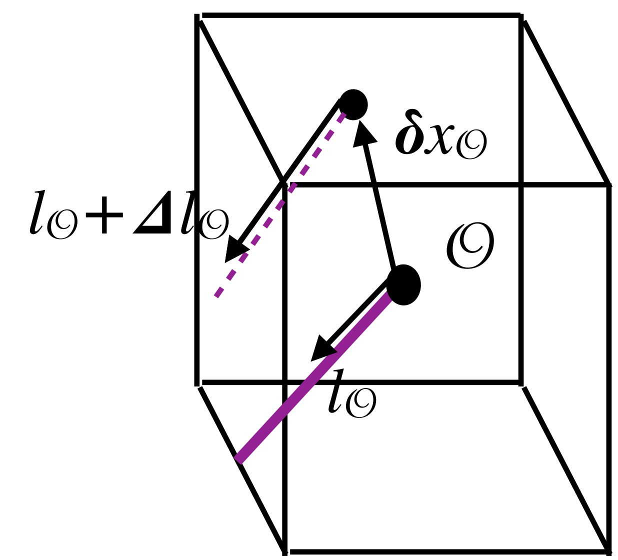
Applicability

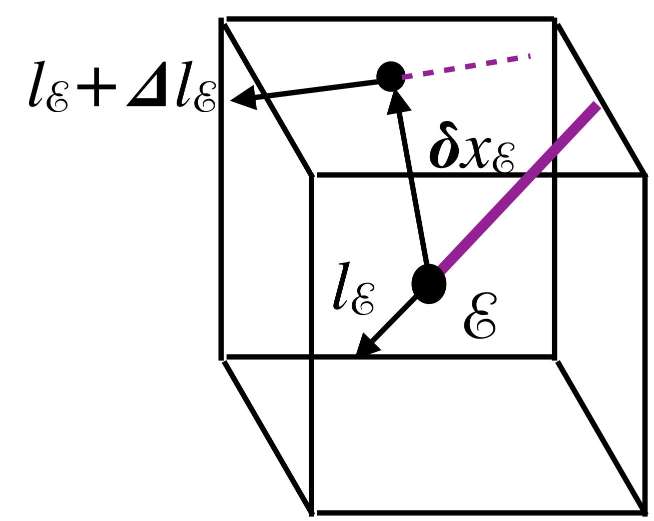
almost flat: keep $\dfrac{L}{R}$, disregard $\left(\dfrac{L}{R}\right)^2$

Displacement formulas

$$\delta x^{\mu}_{\mathscr{E}} = W_{XX}{}^{\mu}{}_{\nu}\, \delta x^{\nu}_{\mathscr{O}} + W_{XL}{}^{\mu}{}_{\nu}\, \Delta l^{\nu}_{\mathscr{O}}$$

$$\delta x^{\mu}_{\mathscr{O}}\, l_{\mathscr{O}\,\mu} = \delta x^{\mu}_{\mathscr{E}}\, l_{\mathscr{E}\,\mu}$$

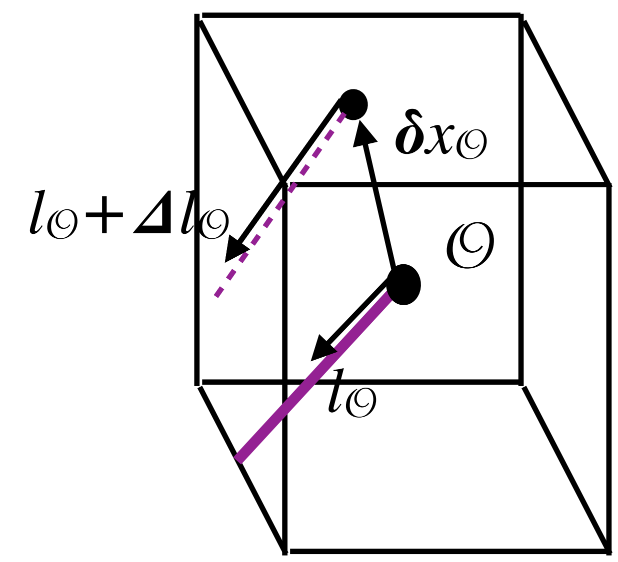


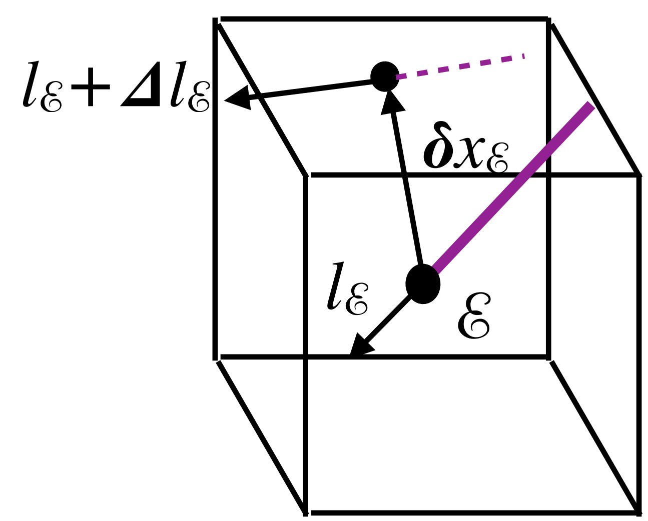


Displacement formulas

$$\mathscr{D}^A{}_B\, \Delta l^B_{\mathscr{O}} = \delta x^A_{\mathscr{E}} - \delta \hat{x}^A_{\mathscr{O}} - m^A{}_\mu\, \delta x^\mu_{\mathscr{O}}$$

$$\delta x^\mu_{\mathscr{O}}\, l_{\mathscr{O}\mu} = \delta x^\mu_{\mathscr{E}}\, l_{\mathscr{E}\mu}$$





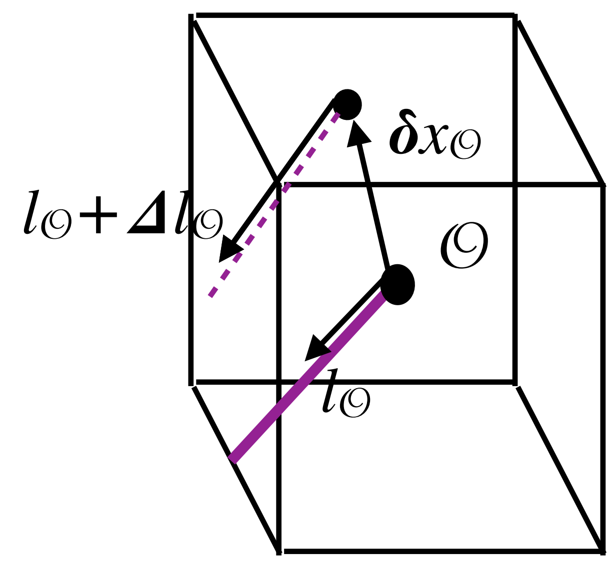
Displacement formulas

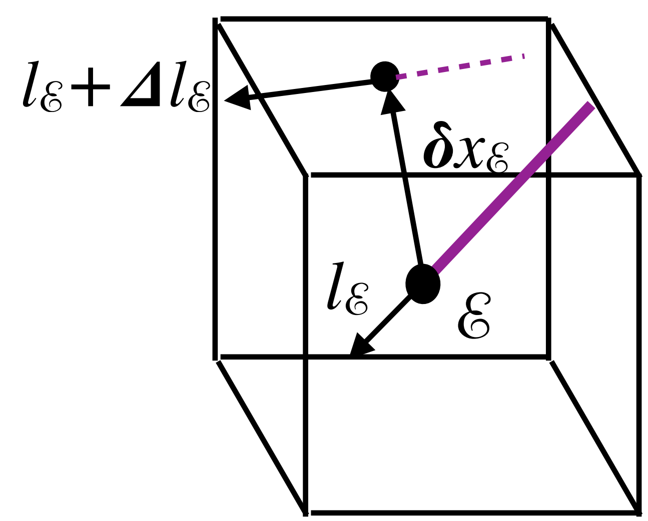
$$\mathscr{D}^A{}_B\,\Delta l^B_{\mathscr{O}} = \delta x^A_{\mathscr{E}} - \delta\hat{x}^A_{\mathscr{O}} - m^A{}_\mu\,\delta x^\mu_{\mathscr{O}}$$

$$\delta x^\mu_{\mathscr{O}}\,l_{\mathscr{O}\mu} = \delta x^\mu_{\mathscr{E}}\,l_{\mathscr{E}\mu}$$

where

^ = parallel transport

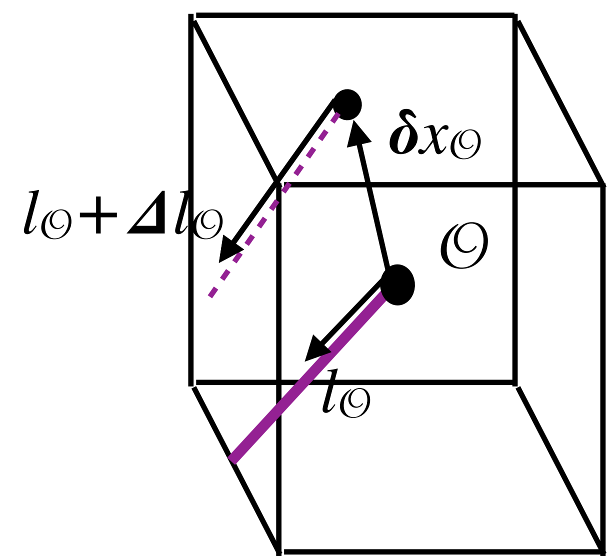




Displacement formulas

$$\mathscr{D}^A{}_B\,\Delta l^B_{\mathscr{O}} = \delta x^A_{\mathscr{E}} - \delta\hat{x}^A_{\mathscr{O}} - m^A{}_\mu\,\delta x^\mu_{\mathscr{O}}$$

$$\delta x^\mu_{\mathscr{O}}\, l_{\mathscr{O}\mu} = \delta x^\mu_{\mathscr{E}}\, l_{\mathscr{E}\mu}$$

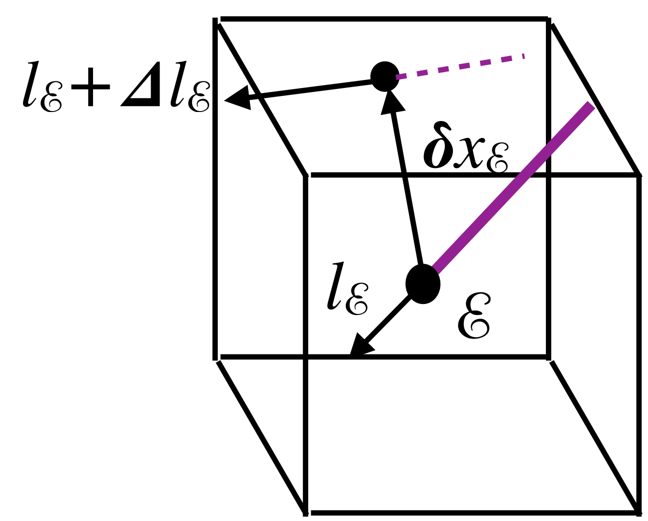


where

^ = parallel transport

$\mathscr{D} : \mathscr{P}_{\mathscr{O}} \to \mathscr{P}_{\mathscr{E}}$ Jacobi operator

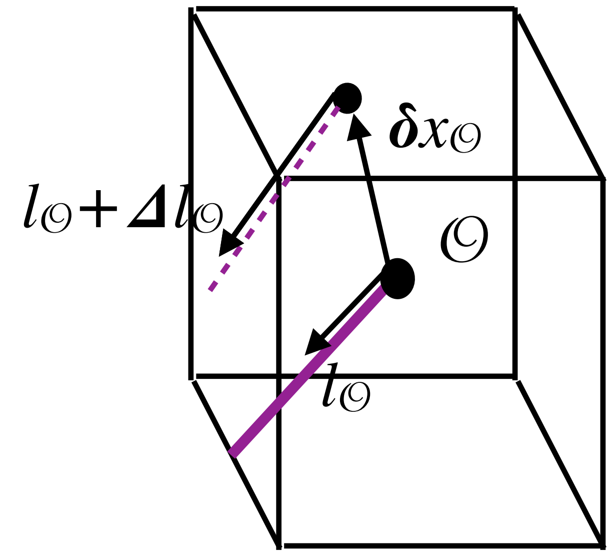
$$\ddot{\mathcal{D}}^A{}_B - R^A{}_{\mu\nu C}\, l^\mu\, l^\nu\, \mathcal{D}^C{}_B = 0$$
$$\mathcal{D}^A{}_B(\lambda_{\mathcal{O}}) = 0$$
$$\dot{\mathcal{D}}^A{}_B(\lambda_{\mathcal{O}}) = \delta^A{}_B$$



Displacement formulas

$$\mathscr{D}^A{}_B\,\Delta l^B_{\mathscr{O}} = \delta x^A_{\mathscr{E}} - \delta\hat{x}^A_{\mathscr{O}} - m^A{}_\mu\,\delta x^\mu_{\mathscr{O}}$$

$$\delta x^\mu_{\mathscr{O}}\,l_{\mathscr{O}\mu} = \delta x^\mu_{\mathscr{E}}\,l_{\mathscr{E}\mu}$$



where

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$\mathscr{D} : \mathscr{P}_{\mathscr{O}} \to \mathscr{P}_{\mathscr{E}}$ Jacobi operator

$$\ddot{\mathcal{D}}^A{}_B - R^A{}_{\mu\nu C}\,l^\mu\,l^\nu\,\mathcal{D}^C{}_B = 0$$
$$\mathcal{D}^A{}_B(\lambda_{\mathcal{O}}) = 0$$
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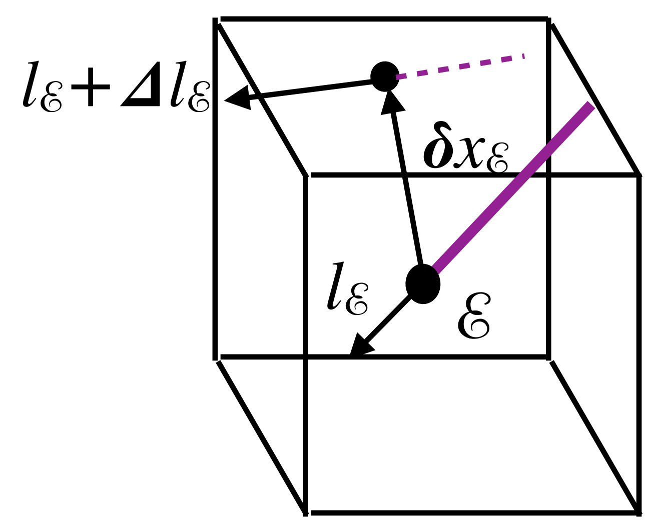


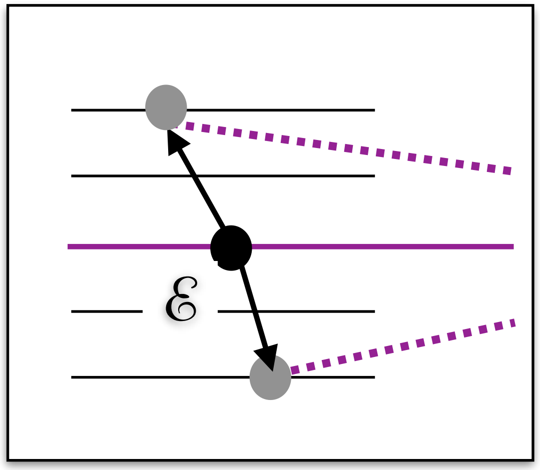
$m : \mathscr{Q}_{\mathscr{O}} \to \mathscr{P}_{\mathscr{E}}$ E/O asymmetry operator

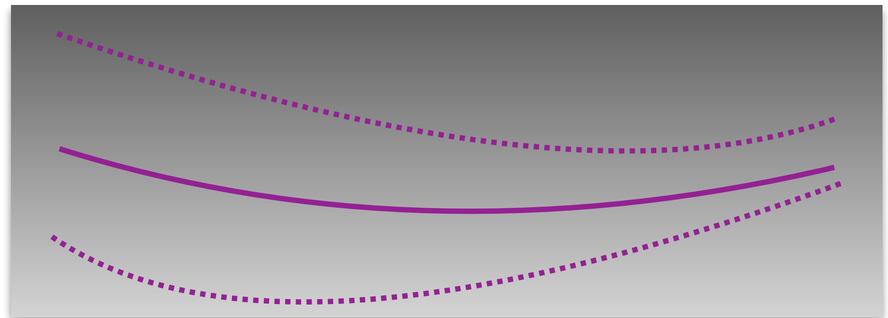
$$\ddot{m}^A{}_\sigma - R^A{}_{\mu\nu C}\,l^\mu\,l^\nu\,m^C{}_\sigma = R^A{}_{\mu\nu\sigma}\,l^\mu\,l^\nu$$
$$m^A{}_\mu(\lambda_{\mathcal{O}}) = 0$$
$$\dot{m}^A{}_\mu(\lambda_{\mathcal{O}}) = 0$$

vanishes in a flat space!

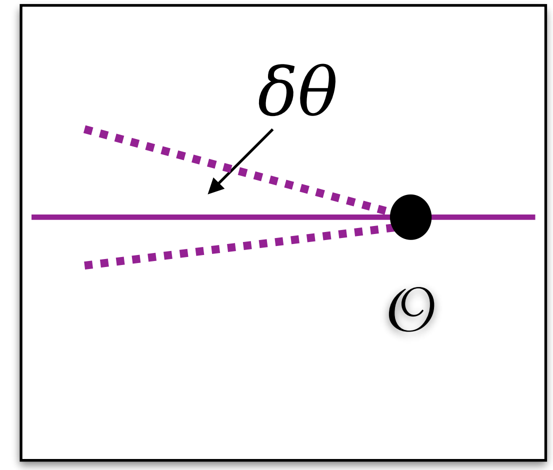
$$m|_{\mathscr{P}_{\mathscr{O}}} \equiv m_\perp : \mathscr{P}_{\mathscr{O}} \to \mathscr{P}_{\mathscr{E}}$$

Linear image distortion

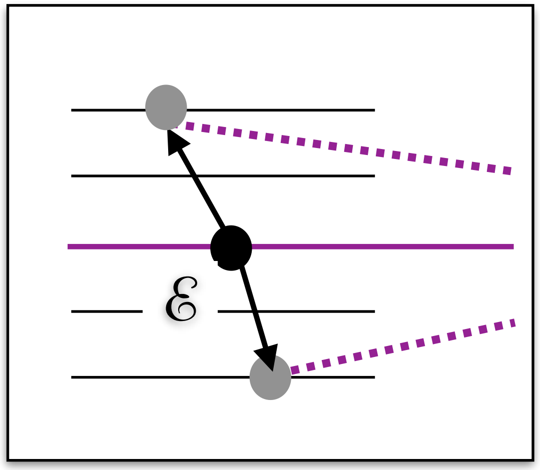


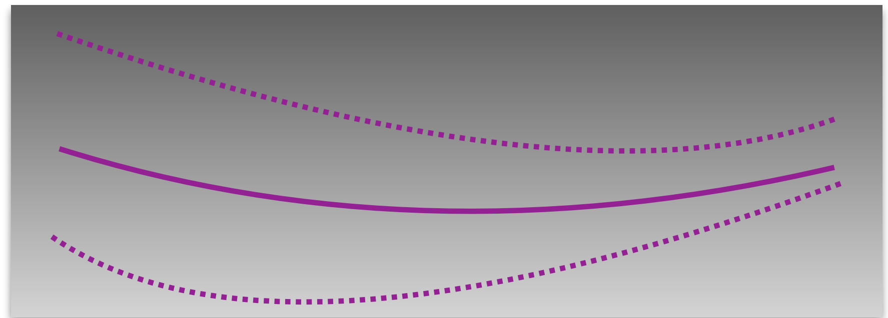


$$\mathcal{D}^A{}_B \, \Delta l^B_{\mathcal{O}} = \delta x^A_{\mathcal{E}}$$

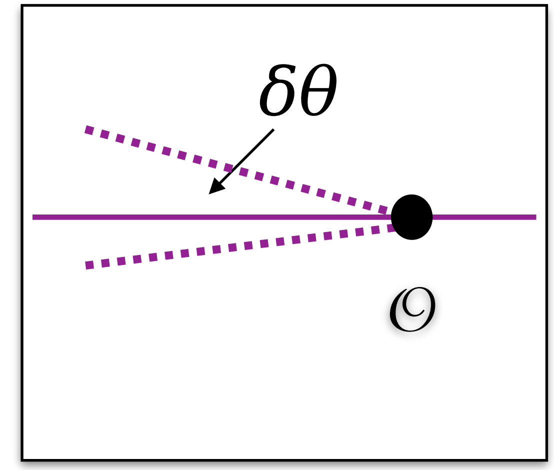


Linear image distortion



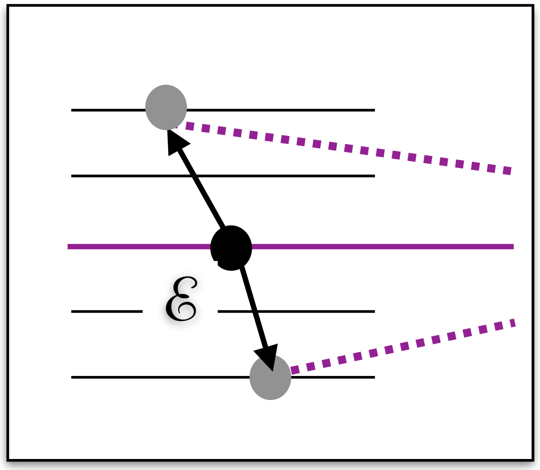


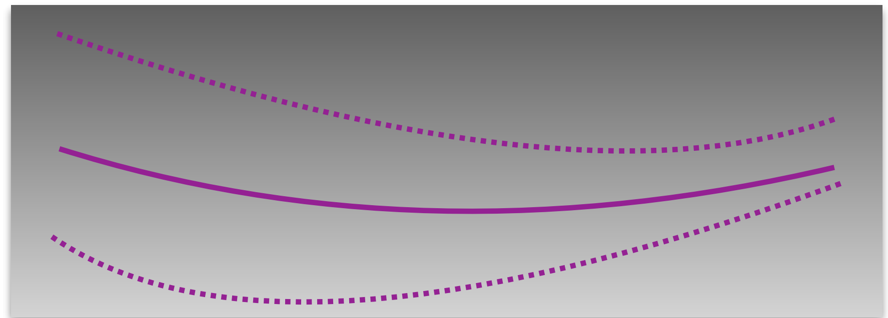
$\mathcal{D}^A{}_B\,\Delta l^B_{\mathcal{O}} = \delta x^A_{\mathcal{E}}$

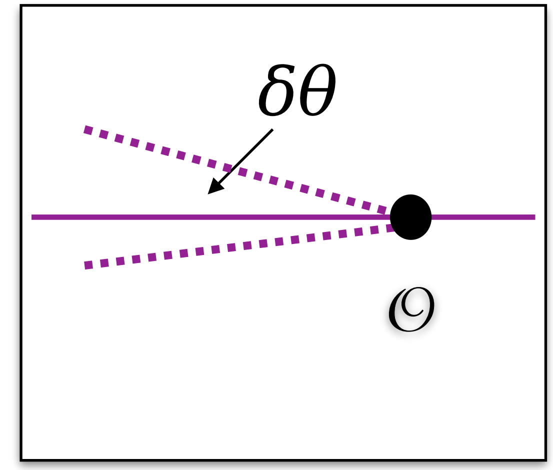


$$\delta\theta^A \approx \delta r^A = \frac{1}{u^\sigma_{\mathscr{O}}\, l_{\mathscr{O}\,\sigma}}\, \mathscr{D}^{-1\,A}{}_B\,\delta x^B_{\mathscr{E}}$$

Linear image distortion





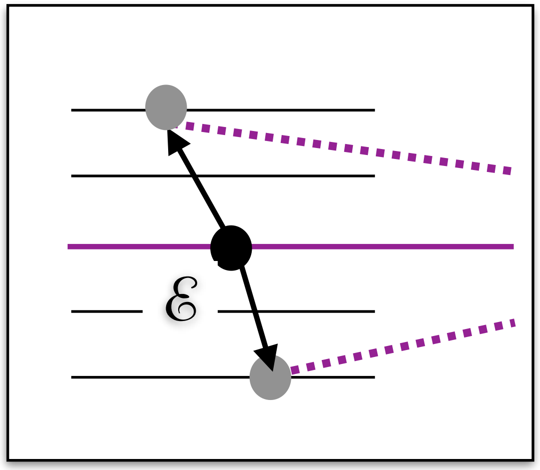


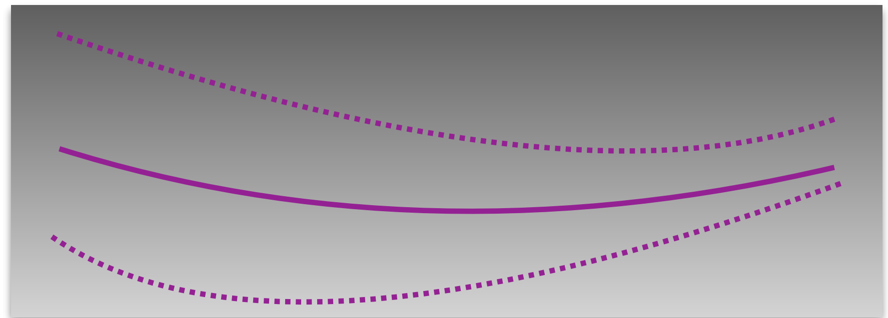
$$\mathcal{D}^A{}_B\,\Delta l^B_{\mathcal{O}} = \delta x^A_{\mathcal{E}}$$

$$\delta\theta^A \approx \delta r^A = \frac{1}{u^\sigma_{\mathcal{O}}\, l_{\mathcal{O}\,\sigma}}\,\mathscr{D}^{-1\,A}{}_B\,\delta x^B_{\mathscr{E}}$$

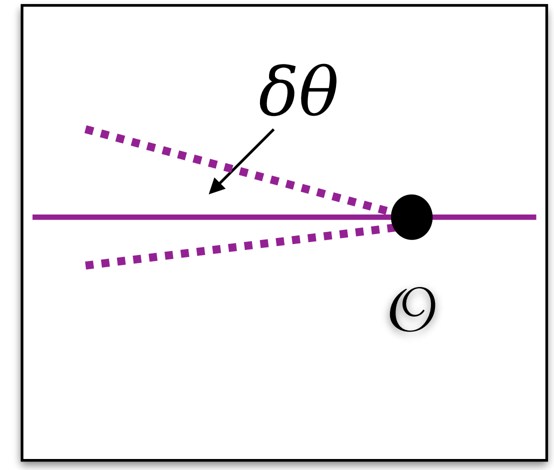
magnification matrix $M^A{}_B$

Linear image distortion



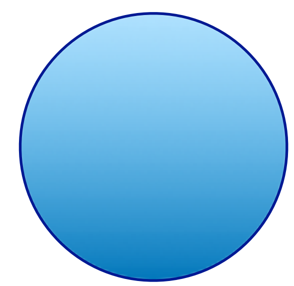


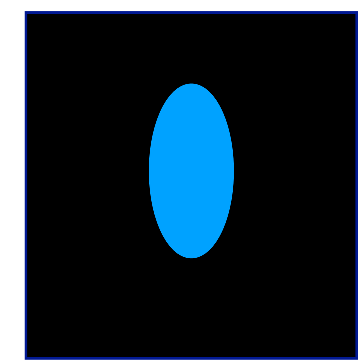
$$\mathcal{D}^A{}_B \, \Delta l^B_{\mathcal{O}} = \delta x^A_{\mathcal{E}}$$



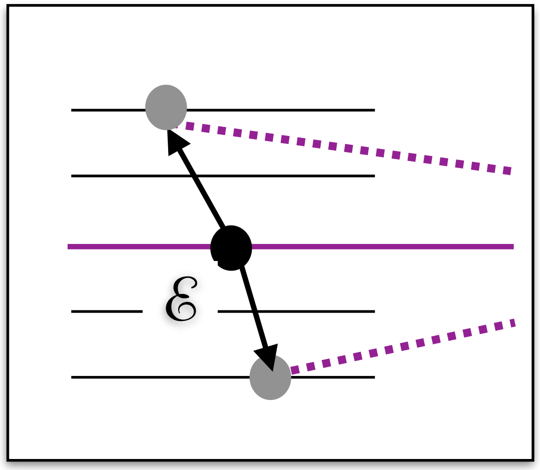
$$\delta\theta^A \approx \delta r^A = \frac{1}{u^\sigma_{\mathcal{O}}\, l_{\mathcal{O}\,\sigma}} \mathcal{D}^{-1A}{}_B \, \delta x^B_{\mathcal{E}}$$

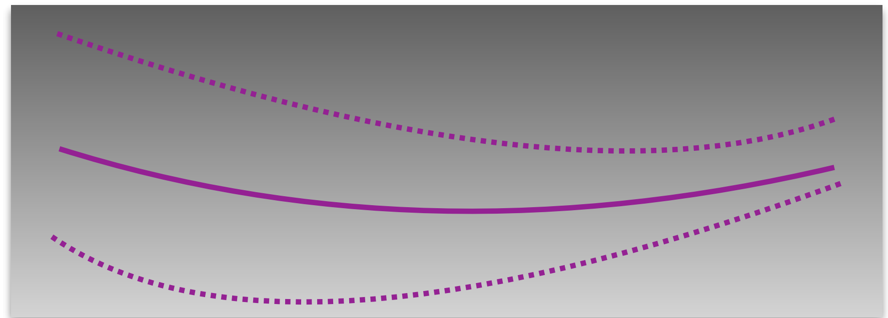
magnification matrix $M^A{}_B$



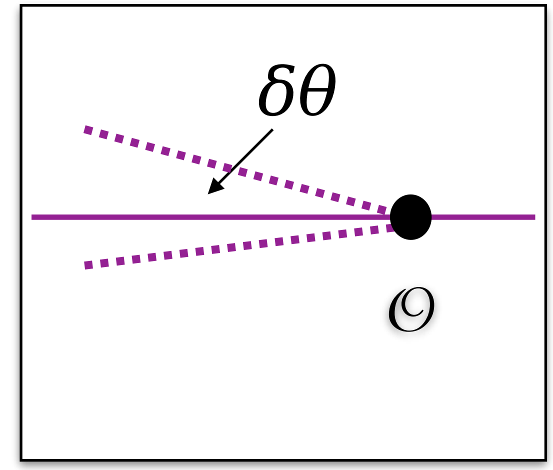


Linear image distortion



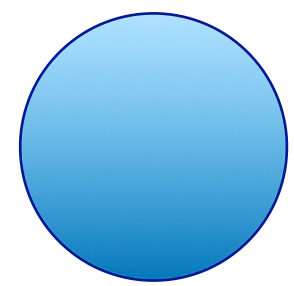


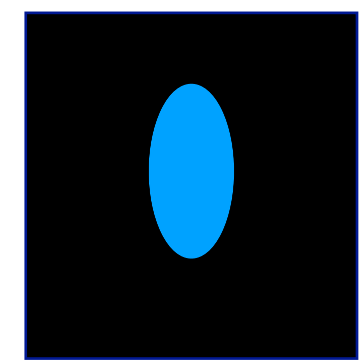
$$\mathcal{D}^A{}_B \, \Delta l^B_{\mathcal{O}} = \delta x^A_{\mathcal{E}}$$



$$\delta\theta^A \approx \delta r^A = \boxed{\frac{1}{u^\sigma_{\mathscr{O}} \, l_{\mathscr{O}\,\sigma}} \mathscr{D}^{-1\,A}{}_B} \, \delta x^B_{\mathscr{E}}$$

magnification matrix $M^A{}_B$

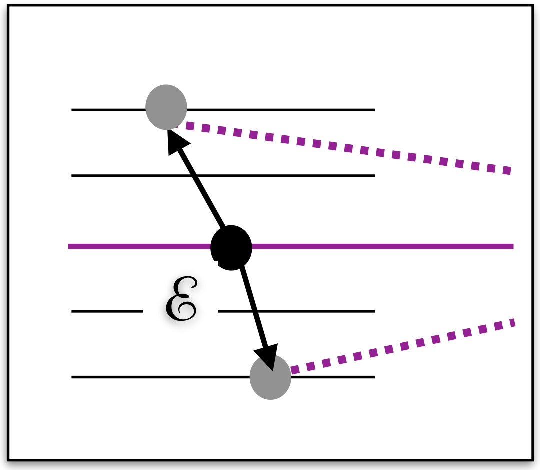


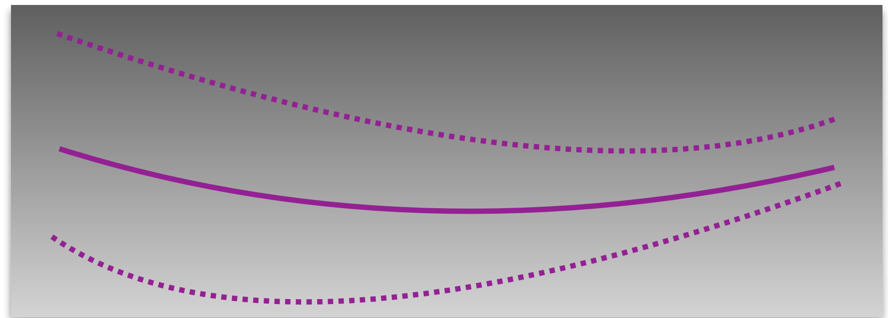


angular diameter distance

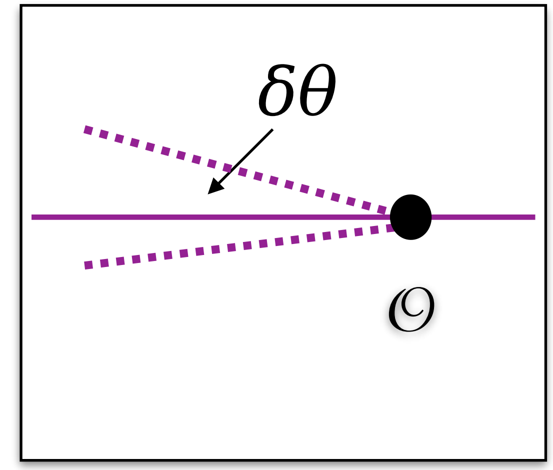
$$D_{ang} = u^\sigma_{\mathscr{O}} \, l_{\mathscr{O}\,\sigma} \left| \det \mathscr{D}^A{}_B \right|^{1/2}$$

Linear image distortion



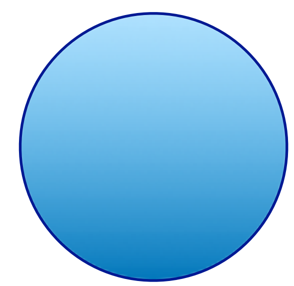


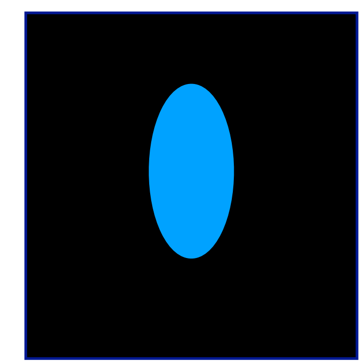
$$\mathcal{D}^A{}_B \, \Delta l^B_{\mathcal{O}} = \delta x^A_{\mathcal{E}}$$



$$\delta\theta^A \approx \delta r^A = \frac{1}{u^\sigma_{\mathscr{O}} \, l_{\mathscr{O}\,\sigma}} \mathscr{D}^{-1\,A}{}_B \, \delta x^B_{\mathscr{E}}$$

magnification matrix $M^A{}_B$





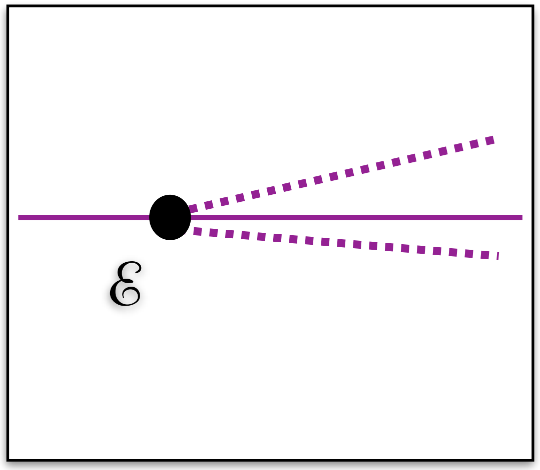
angular diameter distance

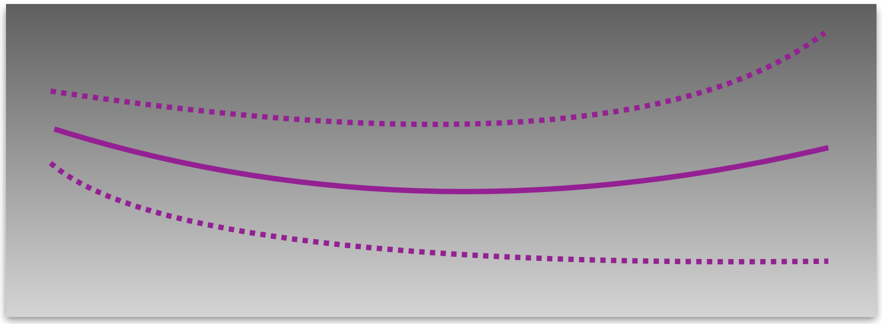
$$D_{ang} = u^\sigma_{\mathscr{O}} \, l_{\mathscr{O}\,\sigma} \left| \det \mathscr{D}^A{}_B \right|^{1/2}$$

$$M^A{}_B \equiv M^A{}_B \left(R^\mu{}_{\nu\alpha\beta}\Big|_{\gamma_0}, u^\mu_{\mathscr{O}} \right)$$

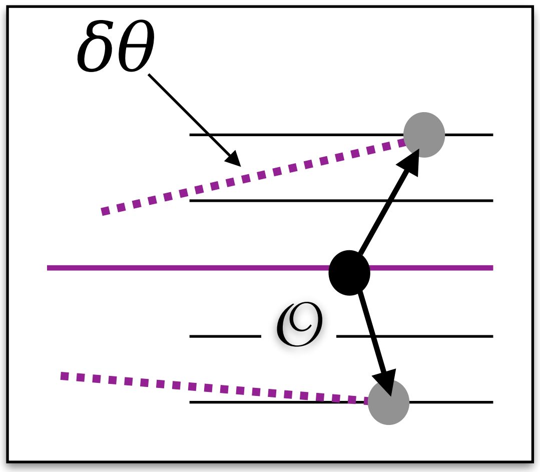
$$D_{ang} \equiv D_{ang} \left(R^\mu{}_{\nu\alpha\beta}\Big|_{\gamma_0}, u^\mu_{\mathscr{O}} \right)$$

Parallax

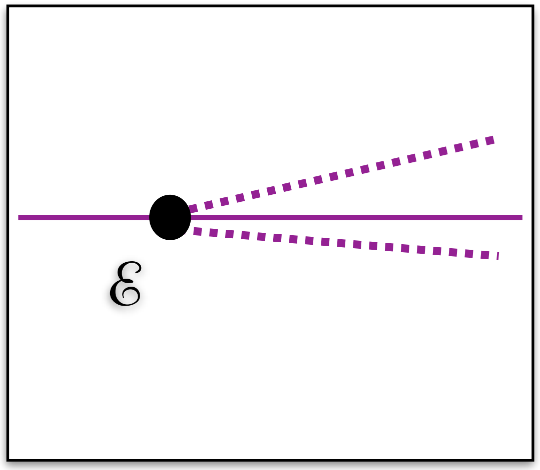


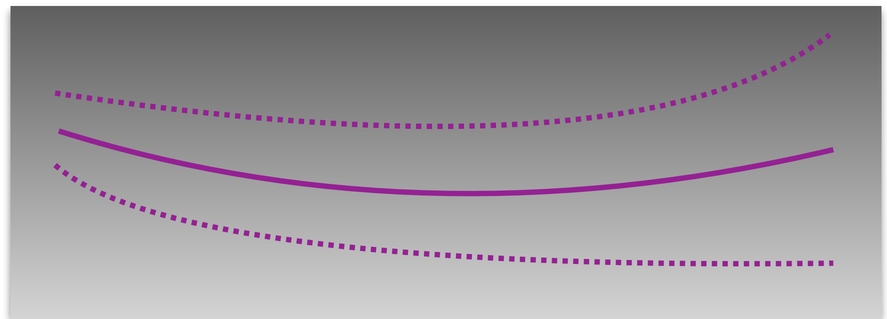


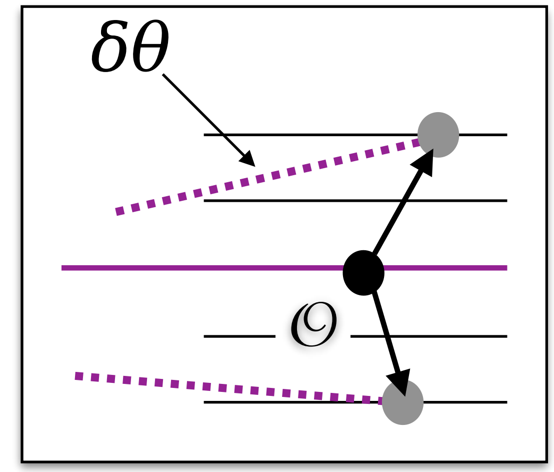
$$\mathcal{D}^A{}_B \, \Delta l^B = -\,\delta\hat{x}^A_{\mathcal{O}} - m_\perp{}^A{}_B \, \delta x^B_{\mathcal{O}}$$



Parallax



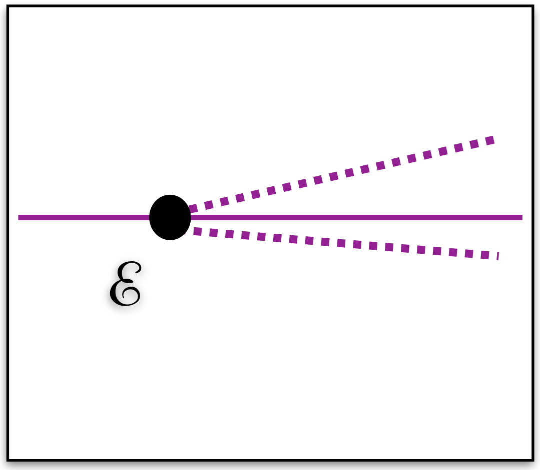


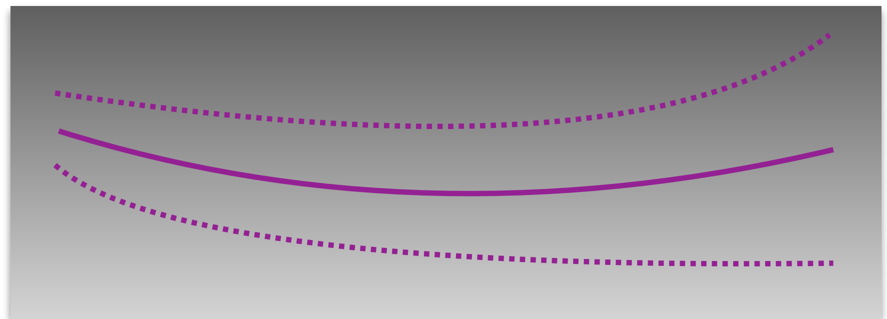


$$\mathscr{D}^A{}_B \, \Delta l^B = -\, \delta\hat{x}^A_{\mathcal{O}} - m_\perp{}^A{}_B \, \delta x^B_{\mathcal{O}}$$

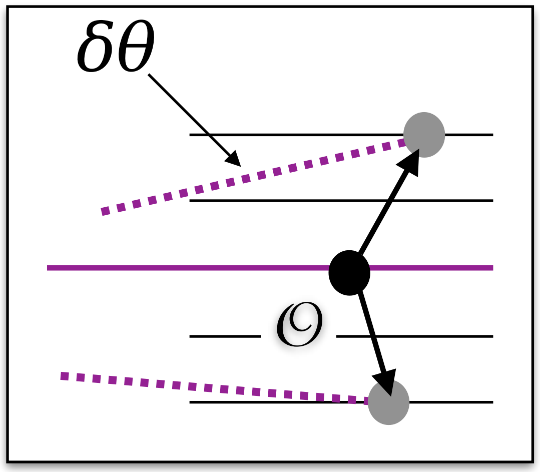
$$\delta\theta^A \approx \delta r^A = -\frac{1}{u^\sigma_{\mathcal{O}} \, l_{\mathcal{O}\,\sigma}} \, \mathscr{D}^{-1A}{}_C \left(\delta^C{}_B + m_\perp{}^C{}_B\right) \delta x^B_{\mathcal{O}}$$

Parallax





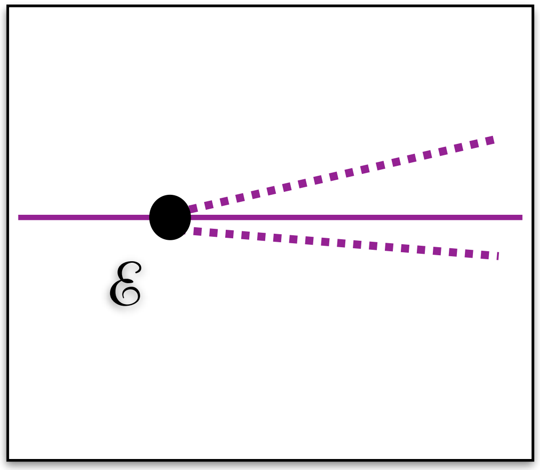
$$\mathscr{D}^A{}_B \, \Delta l^B = -\, \delta \hat{x}^A_{\mathscr{O}} - m_\perp{}^A{}_B \, \delta x^B_{\mathscr{O}}$$

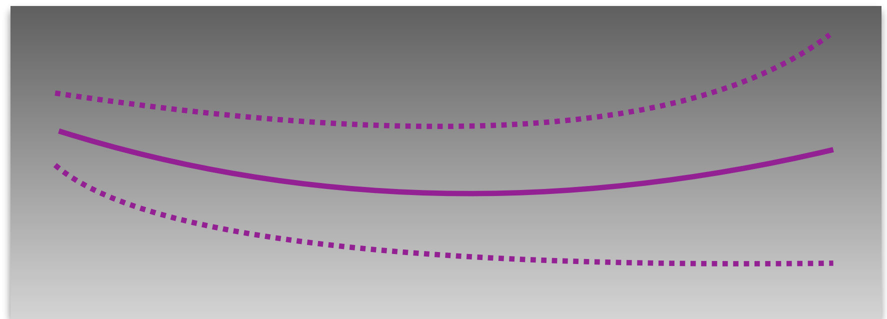


$$\delta\theta^A \approx \delta r^A = -\underbrace{\frac{1}{u^\sigma_{\mathscr{O}}\, l_{\mathscr{O}\,\sigma}} \mathscr{D}^{-1\,A}{}_C \left(\delta^C{}_B + m_\perp{}^C{}_B \right)}_{\Pi^A{}_B} \delta x^B_{\mathscr{O}}$$

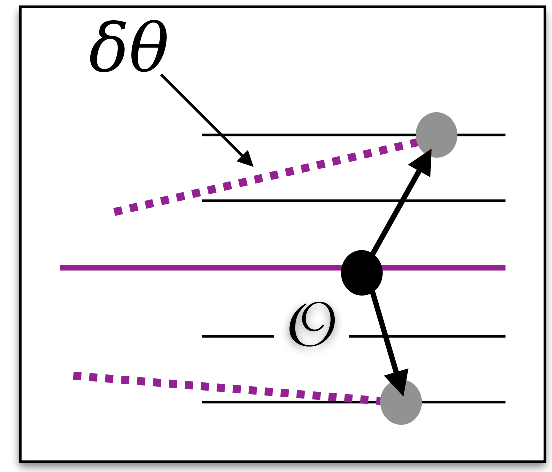
parallax matrix $\Pi^A{}_B$

Parallax



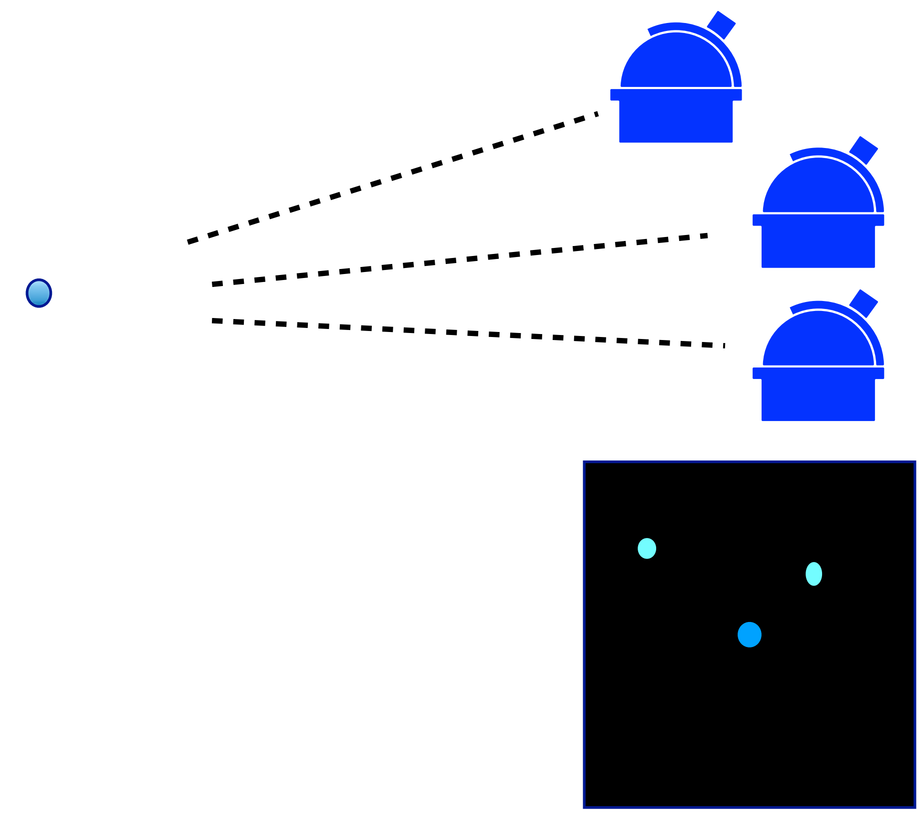


$$\mathscr{D}^A{}_B\,\Delta l^B = -\,\delta\hat{x}^A_{\mathscr{O}} - m_\perp{}^A{}_B\,\delta x^B_{\mathscr{O}}$$

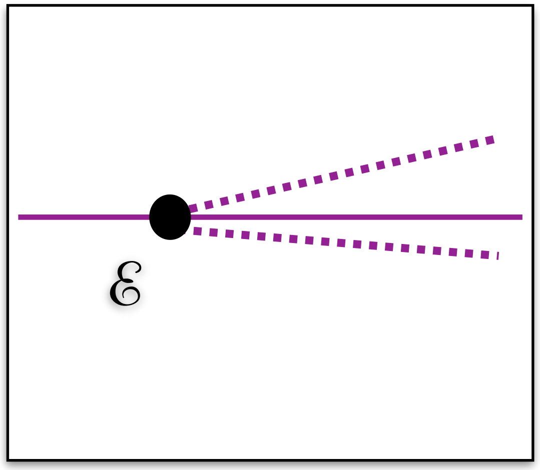


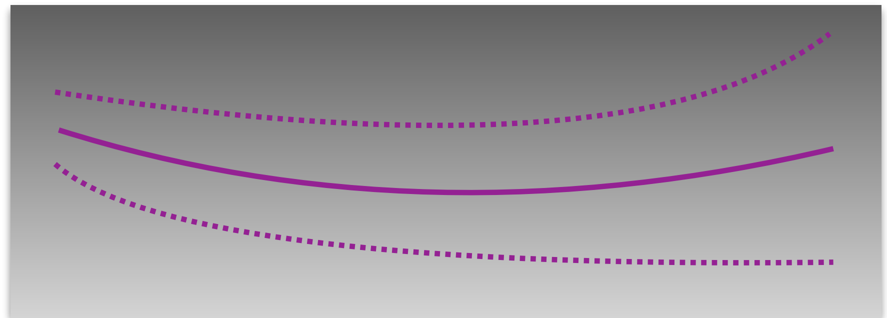
$$\delta\theta^A \approx \delta r^A = -\underbrace{\frac{1}{u^\sigma_{\mathscr{O}}\, l_{\mathscr{O}\,\sigma}}\,\mathscr{D}^{-1A}{}_C\left(\delta^C{}_B + m_\perp{}^C{}_B\right)}_{\Pi^A{}_B}\delta x^B_{\mathscr{O}}$$

parallax matrix $\Pi^A{}_B$

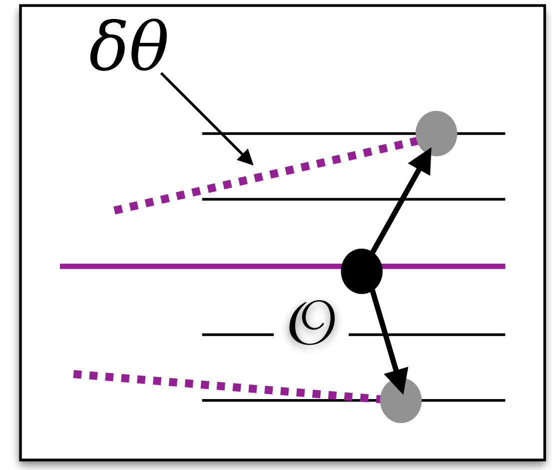


Parallax





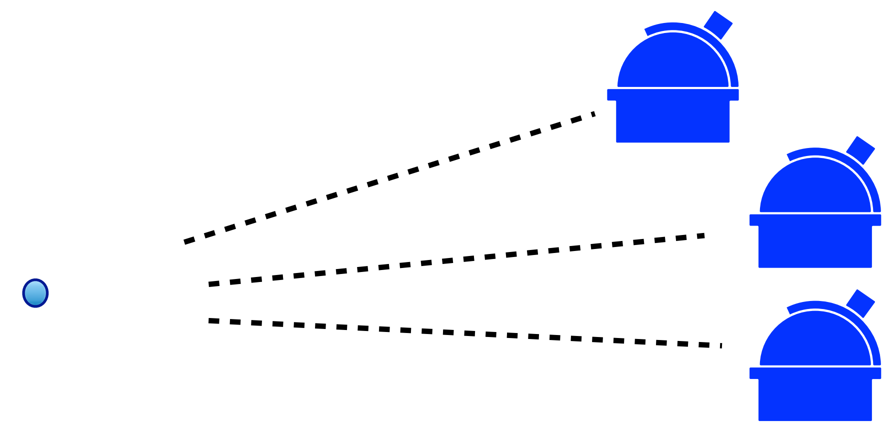
$$\mathscr{D}^A{}_B \, \Delta l^B = -\,\delta\hat{x}^A_{\mathscr{O}} - m_\perp{}^A{}_B \, \delta x^B_{\mathscr{O}}$$

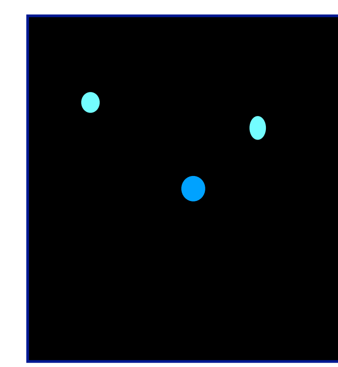


$$\delta\theta^A \approx \delta r^A = -\frac{1}{u^\sigma_{\mathscr{O}} \, l_{\mathscr{O}\,\sigma}} \mathscr{D}^{-1\,A}{}_C \left(\delta^C{}_B + m_\perp{}^C{}_B\right) \delta x^B_{\mathscr{O}}$$

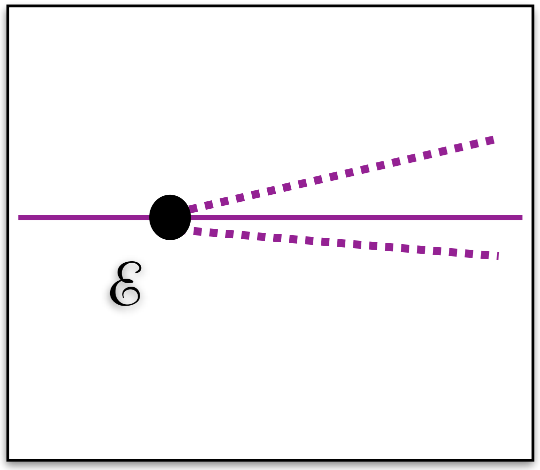
parallax matrix $\Pi^A{}_B$

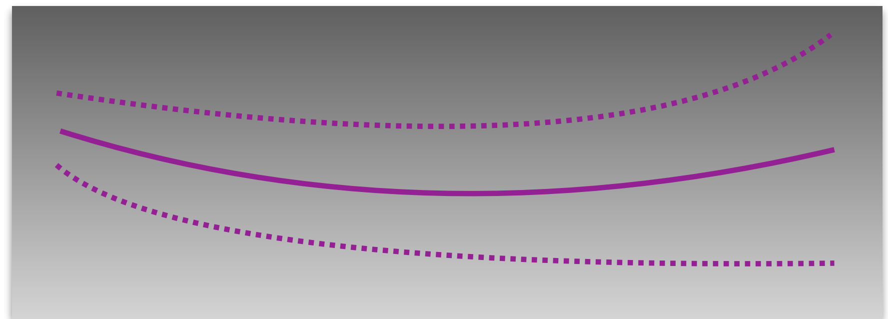
$$\Pi_{AB} = \Pi_{BA}$$

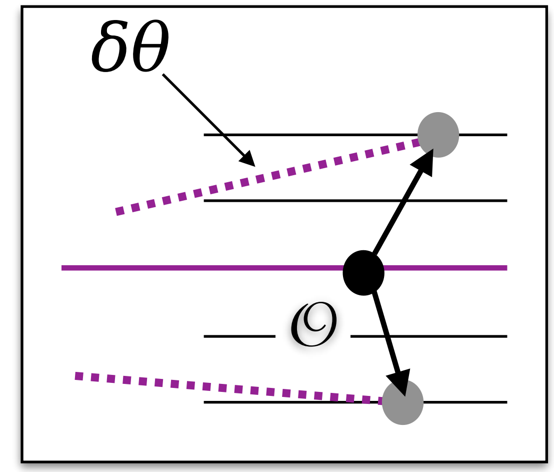




Parallax





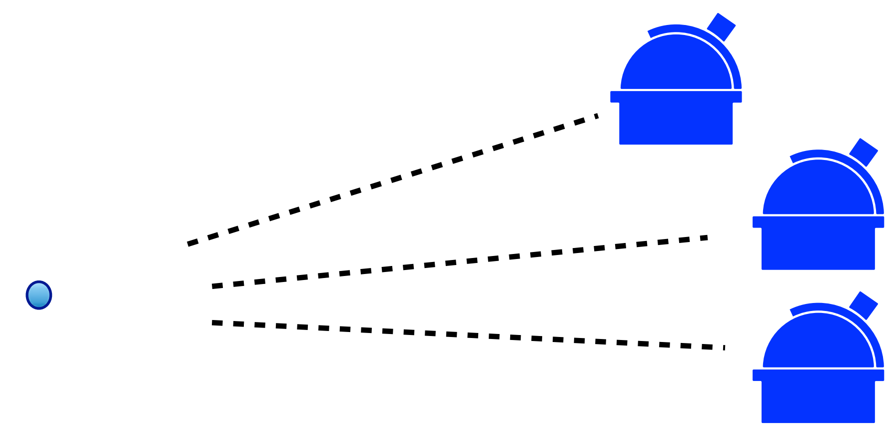


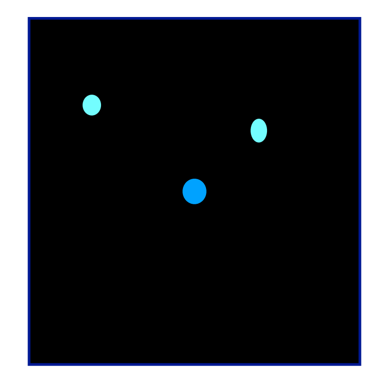
$$\mathscr{D}^A{}_B\,\Delta l^B = -\,\delta\hat{x}^A_{\mathscr{O}} - m_\perp{}^A{}_B\,\delta x^B_{\mathscr{O}}$$

$$\delta\theta^A \approx \delta r^A = -\frac{1}{u^\sigma_{\mathscr{O}}\, l_{\mathscr{O}\,\sigma}}\,\mathscr{D}^{-1A}{}_C \left(\delta^C{}_B + m_\perp{}^C{}_B\right) \delta x^B_{\mathscr{O}}$$

parallax matrix $\Pi^A{}_B$

$$\Pi_{AB} = \Pi_{BA}$$

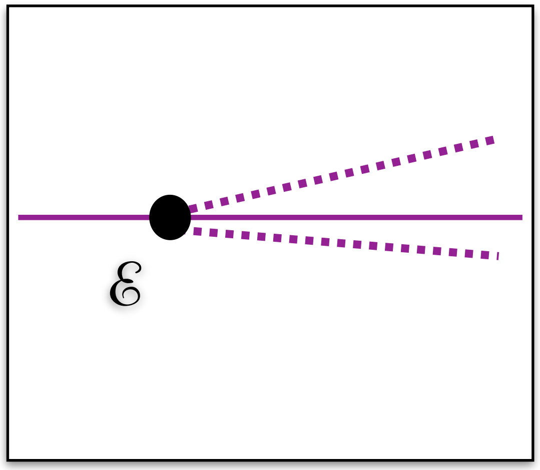


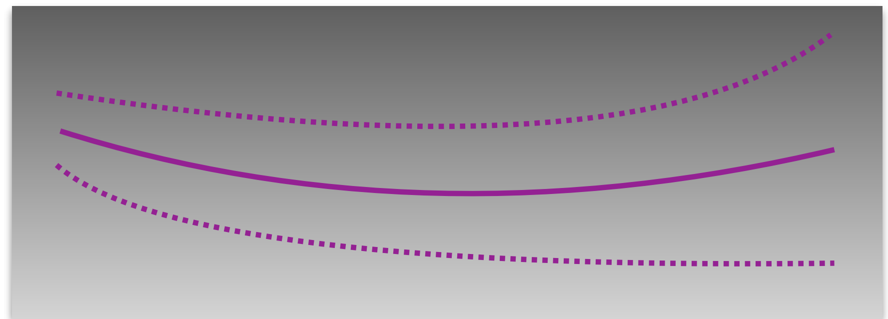


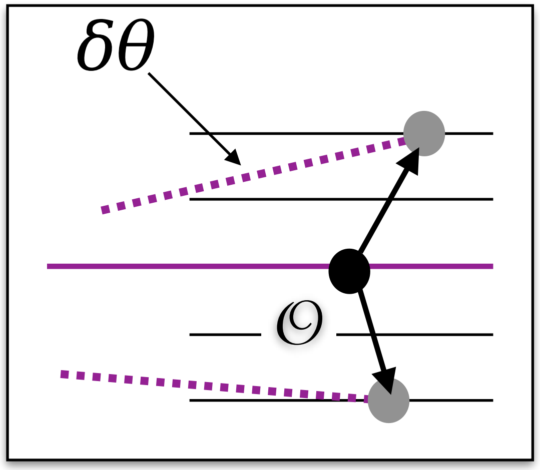
parallax distance

$$D_{par} = u^\sigma_{\mathscr{O}}\, l_{\mathscr{O}\,\sigma}\, \left|\det \mathscr{D}^A{}_B\right|^{1/2} \left|\det\left(\delta^A{}_B + m_\perp{}^A{}_B\right)\right|^{-1/2}$$

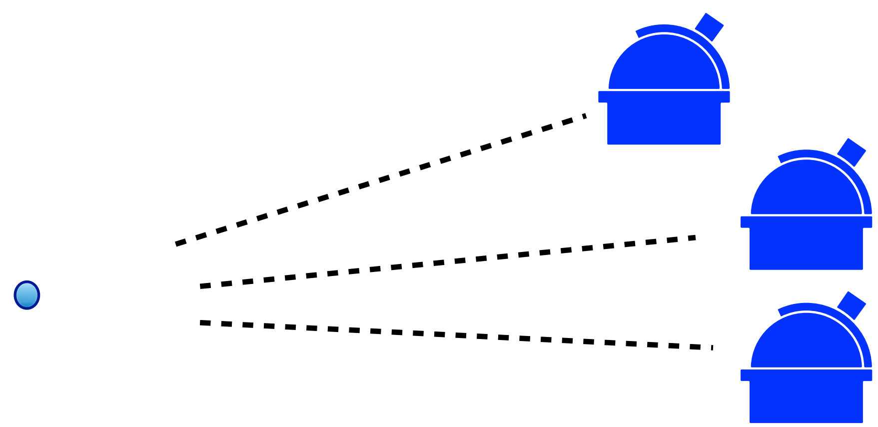
Parallax







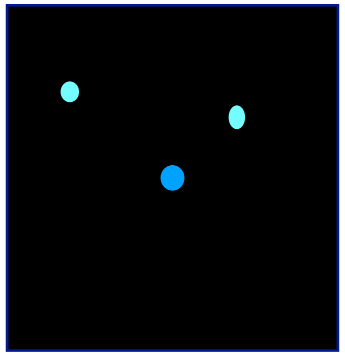
$$\mathcal{D}^A{}_B \, \Delta l^B = -\,\delta \hat{x}^A_{\mathcal{O}} - m_\perp{}^A{}_B \, \delta x^B_{\mathcal{O}}$$



$$\delta\theta^A \approx \delta r^A = -\frac{1}{u^\sigma_{\mathcal{O}} \, l_{\mathcal{O}\,\sigma}} \mathcal{D}^{-1A}{}_C \left(\delta^C{}_B + m_\perp{}^C{}_B\right) \delta x^B_{\mathcal{O}}$$

parallax matrix $\Pi^A{}_B$

$$\Pi_{AB} = \Pi_{BA}$$



$$\Pi^A{}_B \equiv \Pi^A{}_B\left(R^\mu{}_{\nu\alpha\beta}\Big|_{\gamma_0}, u^\mu_{\mathcal{O}}\right)$$

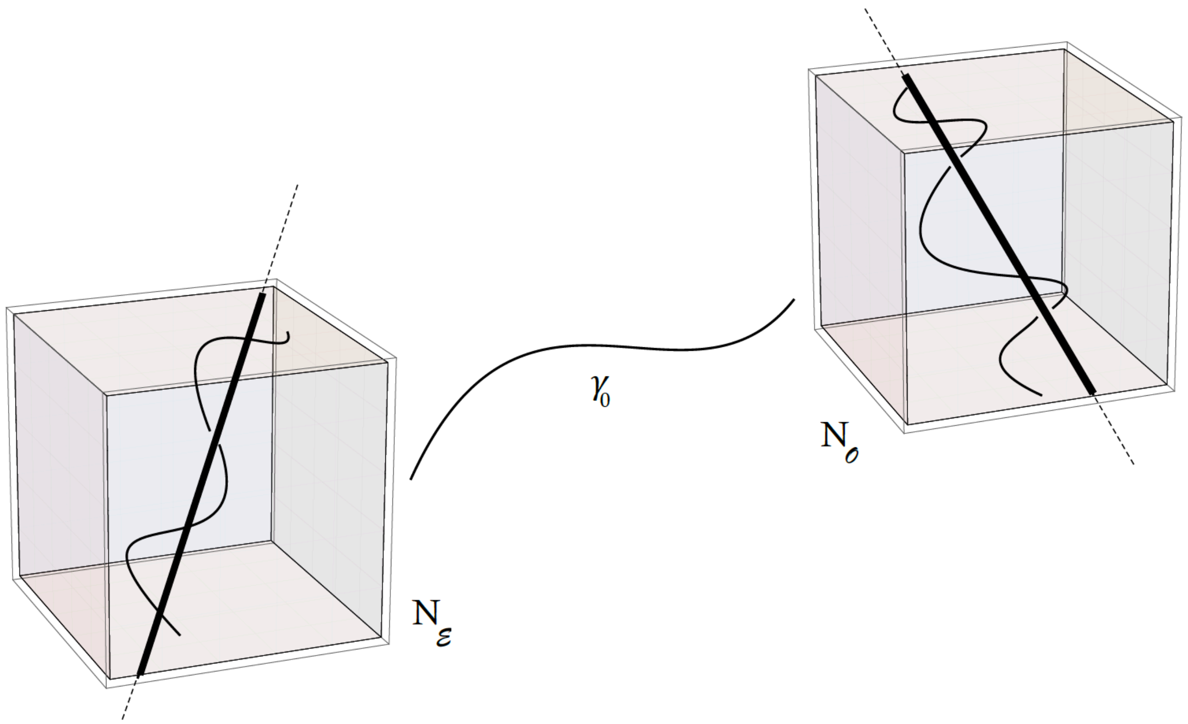
parallax distance

$$D_{par} = u^\sigma_{\mathcal{O}} \, l_{\mathcal{O}\,\sigma} \left|\det \mathcal{D}^A{}_B\right|^{1/2} \left|\det\left(\delta^A{}_B + m_\perp{}^A{}_B\right)\right|^{-1/2}$$

$$D_{par} \equiv D_{par}\left(R^\mu{}_{\nu\alpha\beta}\Big|_{\gamma_0}, u^\mu_{\mathcal{O}}\right)$$

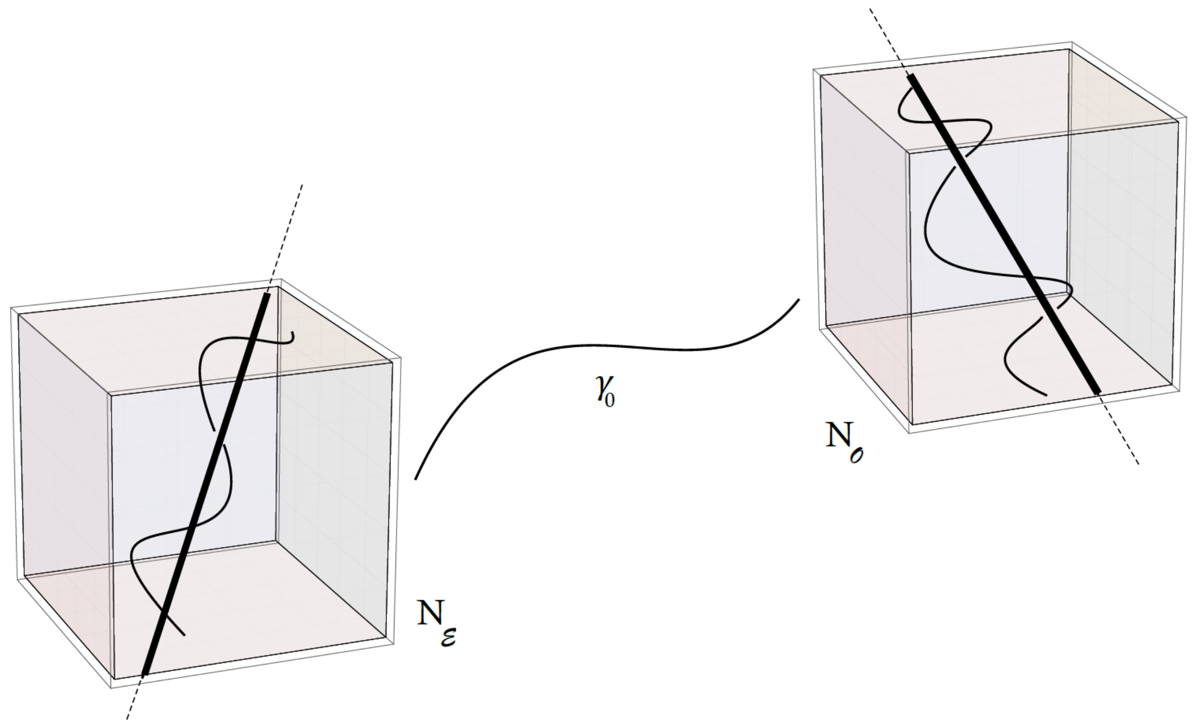
Parallax in a general situation

Parallax in a general situation



Both observer and emitter in bound systems

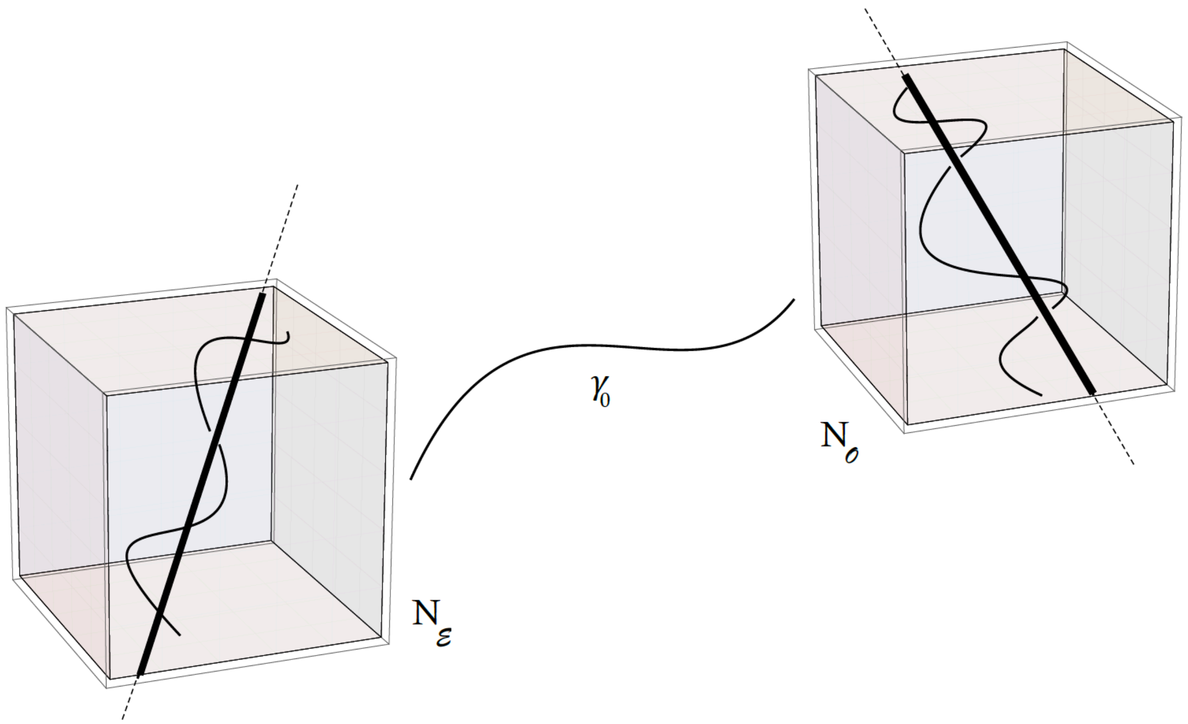
Parallax in a general situation



Both observer and emitter in bound systems

Barycenters in free fall

Parallax in a general situation

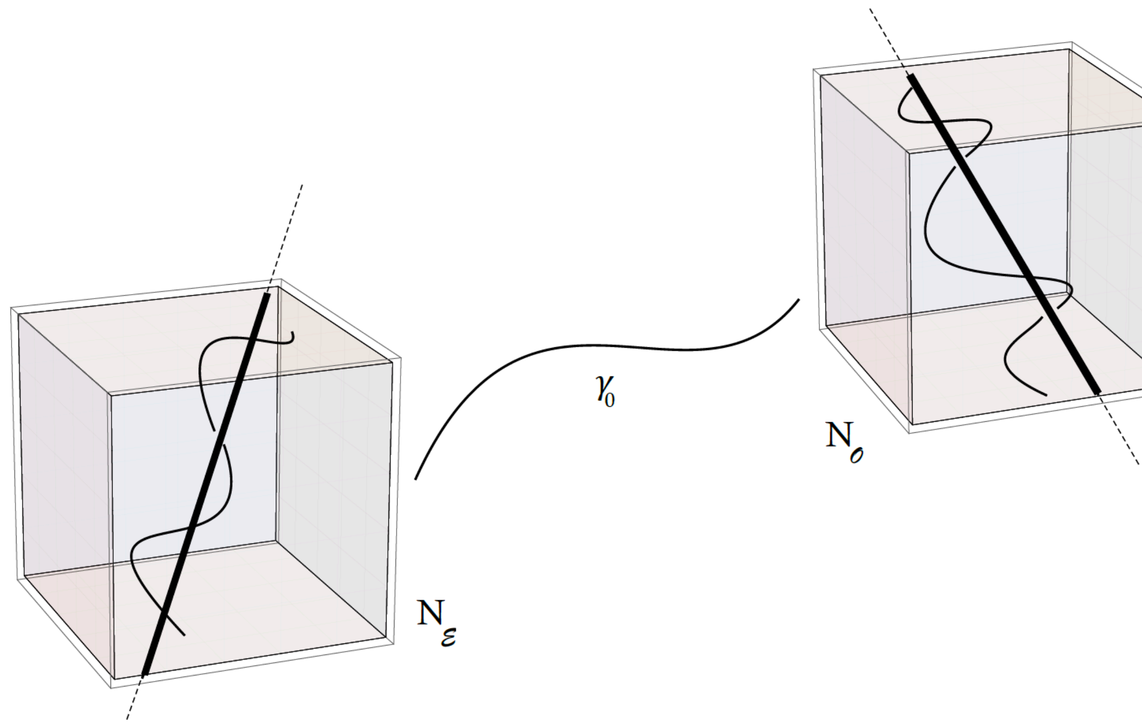


Both observer and emitter in bound systems

Barycenters in free fall

Question: parallax without the aberration effects

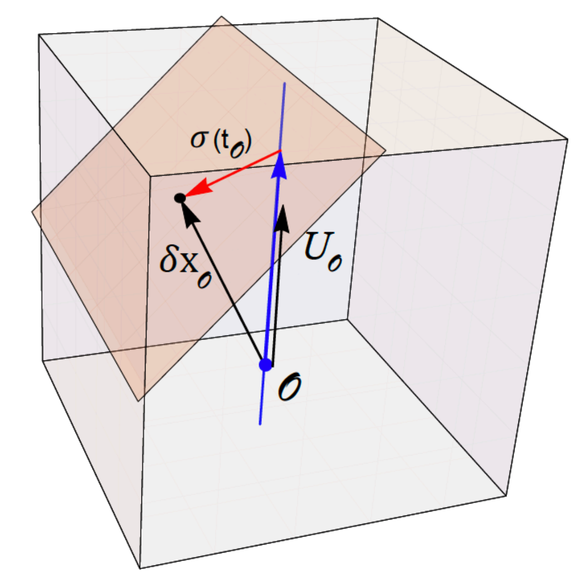
Parallax in a general situation



Both observer and emitter in bound systems

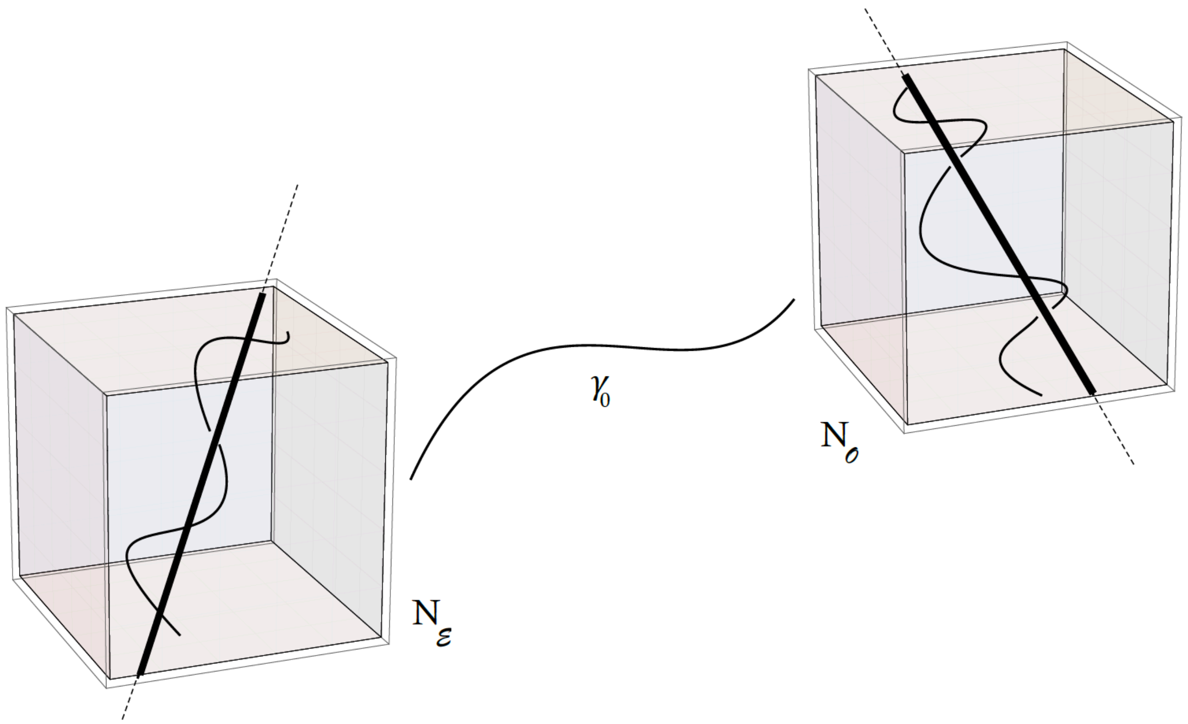
Barycenters in free fall

Question: parallax without the aberration effects



$$\delta x^{\mu}_{\mathscr{O}} = U^{\mu}_{\mathscr{O}}\, t_{\mathscr{O}} + \sigma^{\mu}(t_{\mathscr{O}}) \qquad \sigma^{\mu}\, l_{\mathscr{O}\,\mu} = 0$$

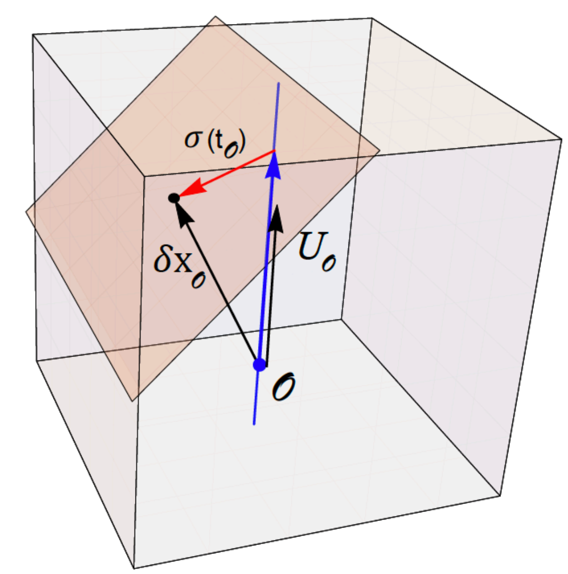
Parallax in a general situation



Both observer and emitter in bound systems

Barycenters in free fall

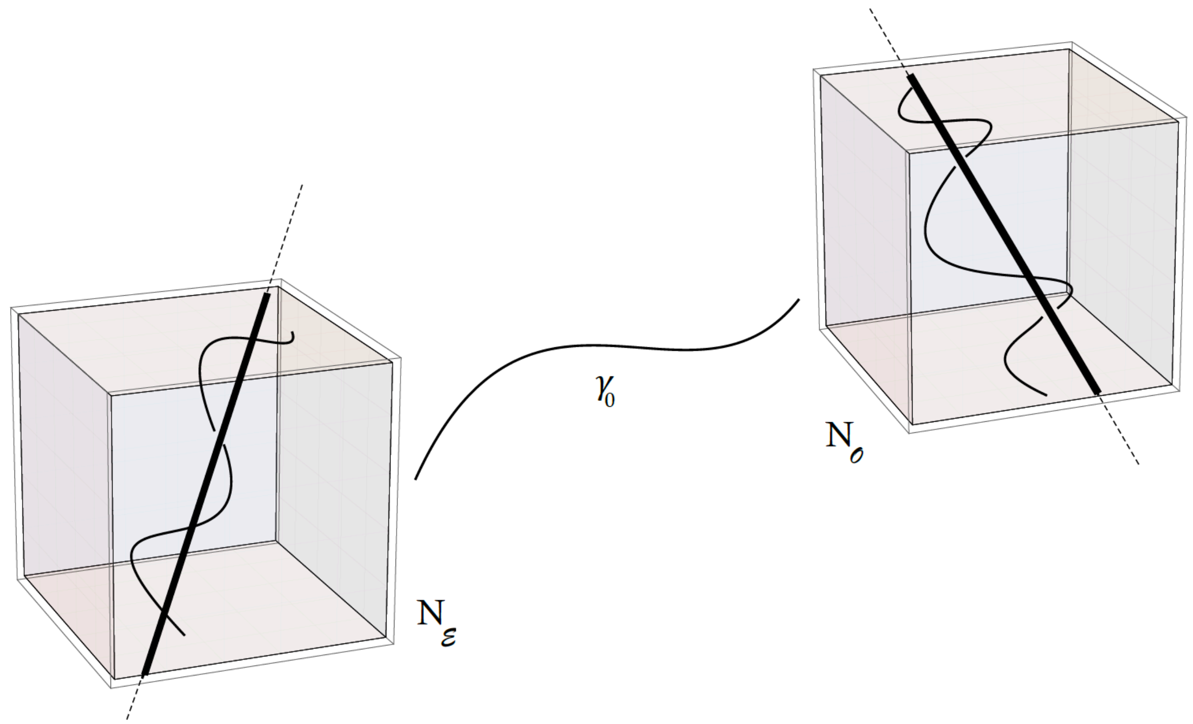
Question: parallax without the aberration effects



$$\delta x^{\mu}_{\mathcal{O}} = U^{\mu}_{\mathcal{O}}\, t_{\mathcal{O}} + \sigma^{\mu}(t_{\mathcal{O}}) \qquad \sigma^{\mu}\, l_{\mathcal{O}\,\mu} = 0$$

$$\delta x^{\mu}_{\mathcal{E}} = U^{\mu}_{\mathcal{E}}\, t_{\mathcal{E}} + \rho^{\mu}(t_{\mathcal{E}}) \qquad \rho^{\mu}\, l_{\mathcal{E}\,\mu} = 0$$

Parallax in a general situation



Both observer and emitter in bound systems

Barycenters in free fall

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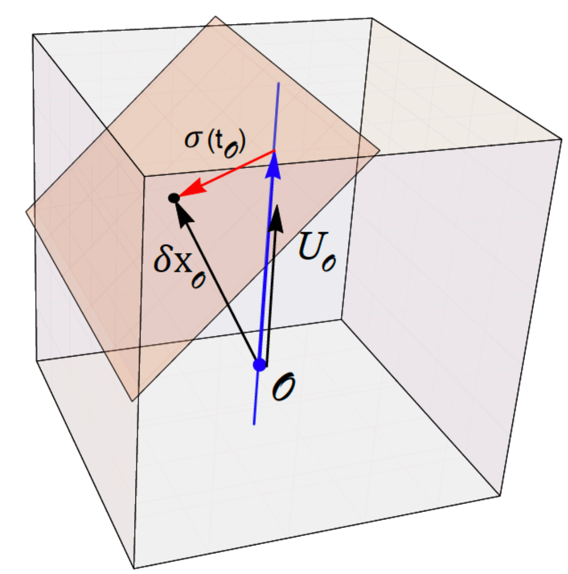
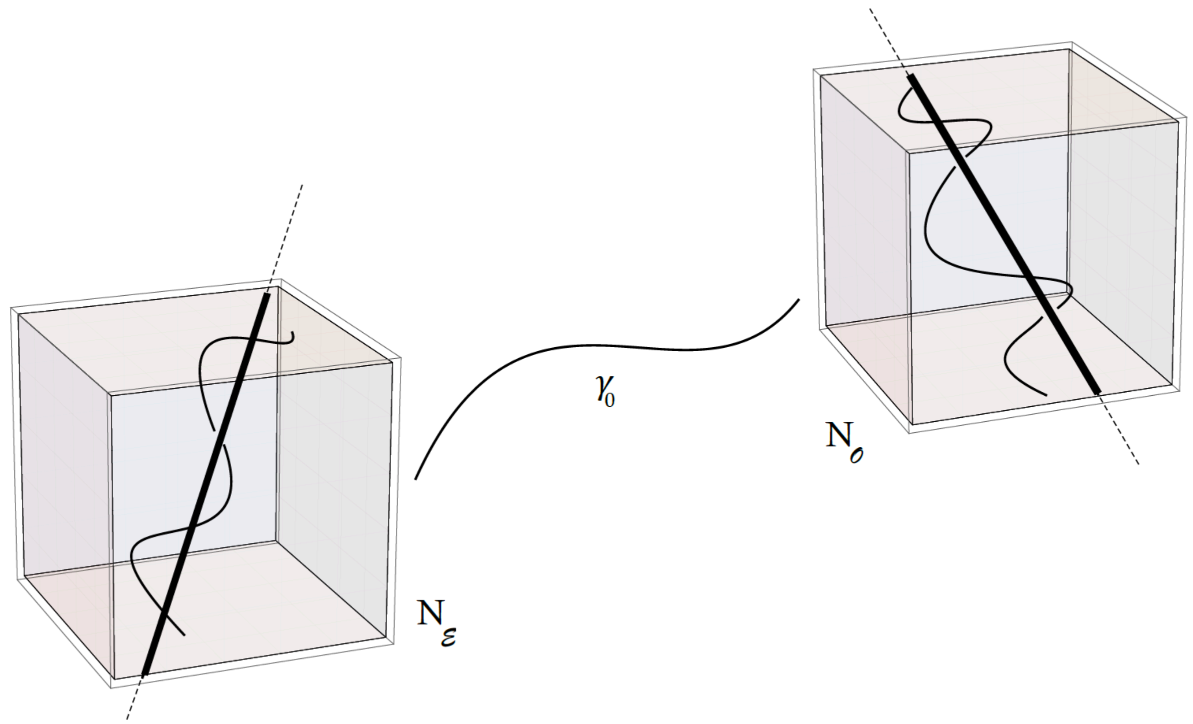


$$\delta x^{\mu}_{\mathcal{O}} = U^{\mu}_{\mathcal{O}}\, t_{\mathcal{O}} + \sigma^{\mu}(t_{\mathcal{O}}) \qquad \sigma^{\mu}\, l_{\mathcal{O}\,\mu} = 0$$

$$\delta x^{\mu}_{\mathscr{E}} = U^{\mu}_{\mathscr{E}}\, t_{\mathscr{E}} + \rho^{\mu}(t_{\mathscr{E}}) \qquad \rho^{\mu}\, l_{\mathscr{E}\,\mu} = 0$$

$$\delta\theta^{A} = \delta_{\mathcal{O}} r^{A}\, t_{\mathcal{O}} + M^{A}{}_{B}\, \rho^{B}\left((1+z)^{-1}\, t_{\mathcal{O}}\right) - \Pi^{A}{}_{B}\, \sigma^{B}(t_{\mathcal{O}})$$

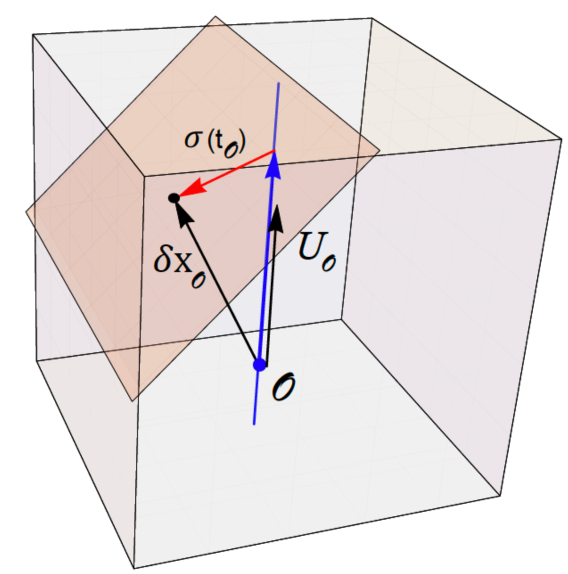
Parallax in a general situation



Both observer and emitter in bound systems

Barycenters in free fall

Question: parallax without the aberration effects



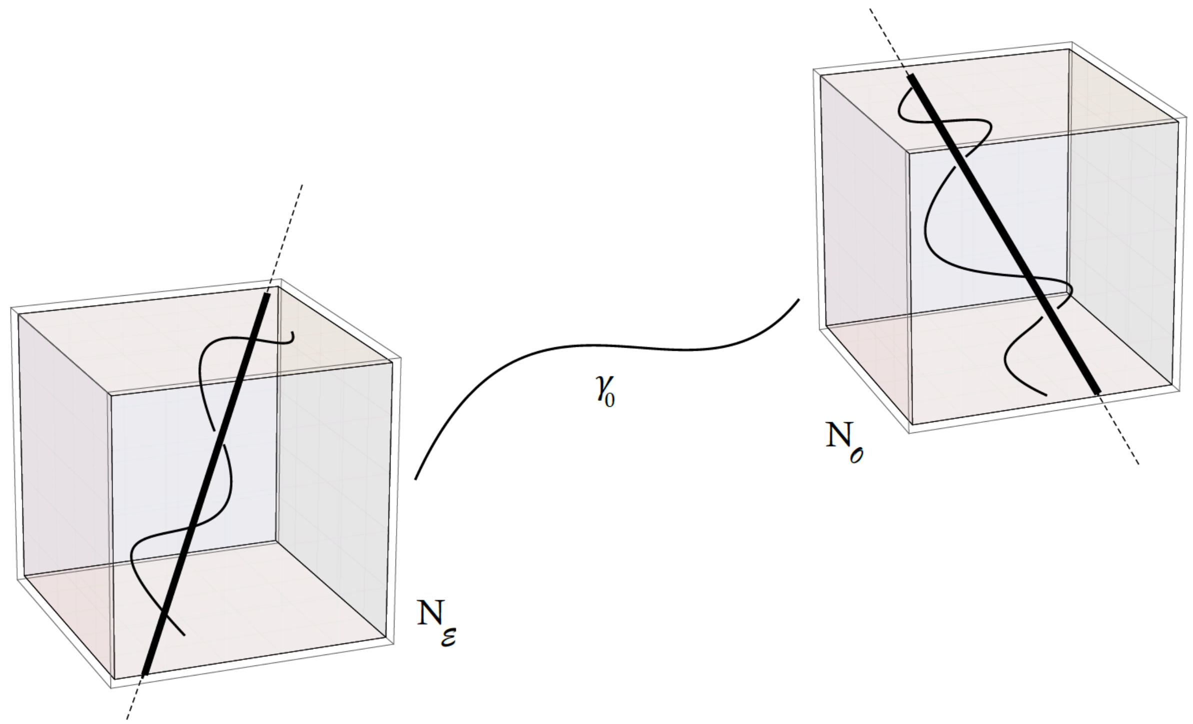
$$\delta x^\mu_{\mathscr{O}} = U^\mu_{\mathscr{O}}\, t_{\mathscr{O}} + \sigma^\mu(t_{\mathscr{O}}) \qquad \sigma^\mu\, l_{\mathscr{O}\,\mu} = 0$$

$$\delta x^\mu_{\mathscr{E}} = U^\mu_{\mathscr{E}}\, t_{\mathscr{E}} + \rho^\mu(t_{\mathscr{E}}) \qquad \rho^\mu\, l_{\mathscr{E}\,\mu} = 0$$

$$\delta\theta^A = \delta_{\mathscr{O}} r^A\, t_{\mathscr{O}} + M^A{}_B\, \rho^B\left((1+z)^{-1}\, t_{\mathscr{O}}\right) - \Pi^A{}_B\, \sigma^B(t_{\mathscr{O}})$$

barycenter drift
(linear)

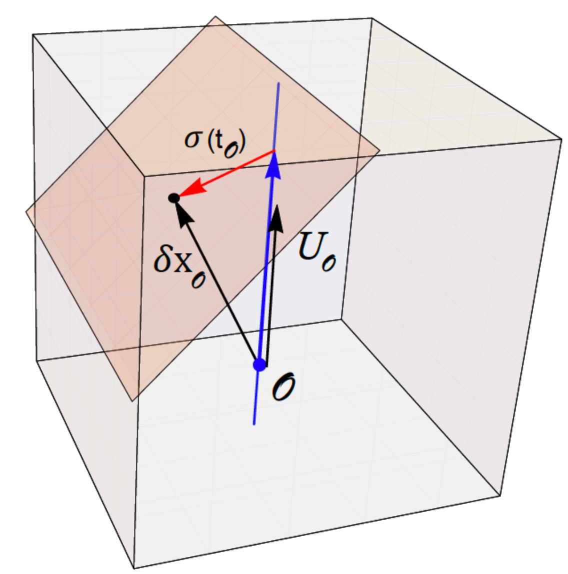
Parallax in a general situation



Both observer and emitter in bound systems

Barycenters in free fall

Question: parallax without the aberration effects



$$\delta x^\mu_\mathcal{O} = U^\mu_\mathcal{O}\, t_\mathcal{O} + \sigma^\mu(t_\mathcal{O}) \qquad \sigma^\mu\, l_{\mathcal{O}\,\mu} = 0$$

$$\delta x^\mu_\mathcal{E} = U^\mu_\mathcal{E}\, t_\mathcal{E} + \rho^\mu(t_\mathcal{E}) \qquad \rho^\mu\, l_{\mathcal{E}\,\mu} = 0$$

$$\delta\theta^A = \delta_\mathcal{O} r^A\, t_\mathcal{O} + M^A{}_B\, \rho^B \left((1+z)^{-1}\, t_\mathcal{O}\right) - \Pi^A{}_B\, \sigma^B(t_\mathcal{O})$$

barycenter drift (linear)

parallax (periodic)

Motions-independent observables

$$z \equiv z(u^\mu_\mathcal{O}, u^\mu_\mathcal{E})$$

$$r^\mu \equiv r^\mu(u^\mu_\mathcal{O})$$

$$M^A{}_B \equiv M^A{}_B \left(R^\mu{}_{\nu\alpha\beta}\Big|_{\gamma_0}, u^\mu_\mathcal{O} \right)$$

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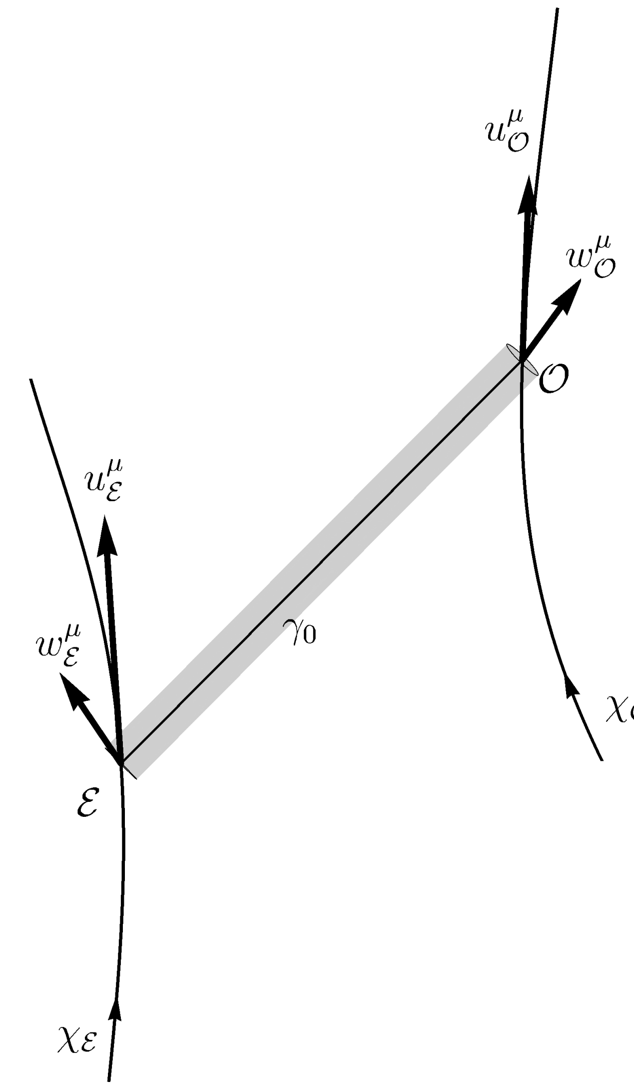
$$\delta_\mathcal{O} r^A \equiv \delta_\mathcal{O} r^A \left(R^\mu{}_{\nu\alpha\beta}\Big|_{\gamma_0}, u^\mu_\mathcal{O}, u^\mu_\mathcal{E}, w^\mu_\mathcal{O} \right)$$

$$\frac{dz}{d\tau_\mathcal{O}} \equiv \frac{dz}{d\tau_\mathcal{O}} \left(R^\mu{}_{\nu\alpha\beta}\Big|_{\gamma_0}, u^\mu_\mathcal{O}, u^\mu_\mathcal{E}, w^\mu_\mathcal{O}, w^\mu_\mathcal{E} \right)$$



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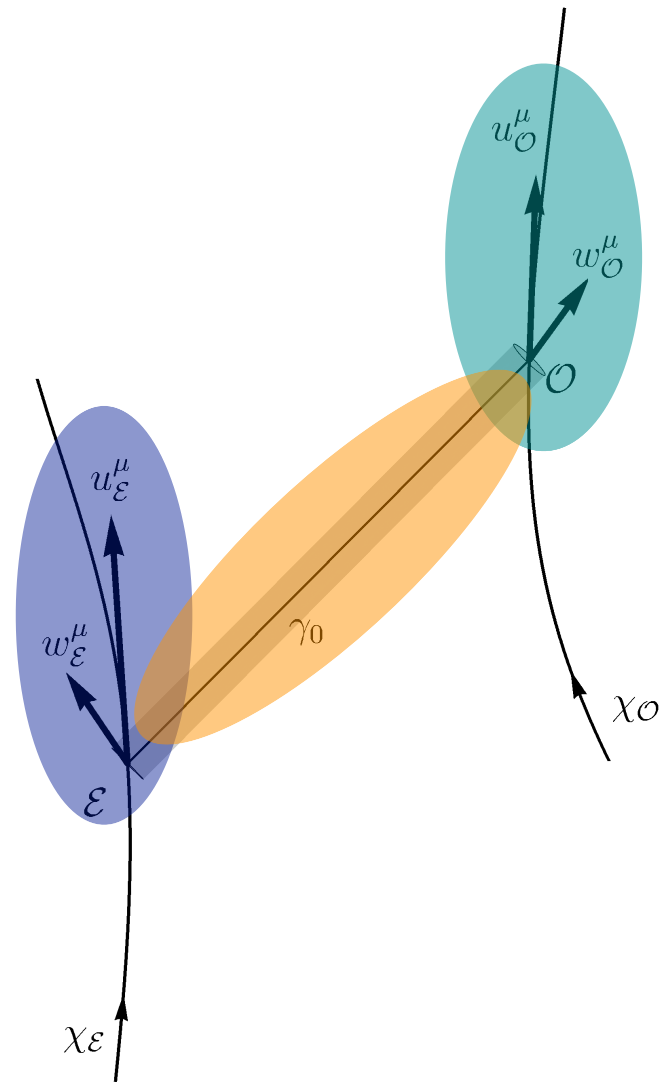

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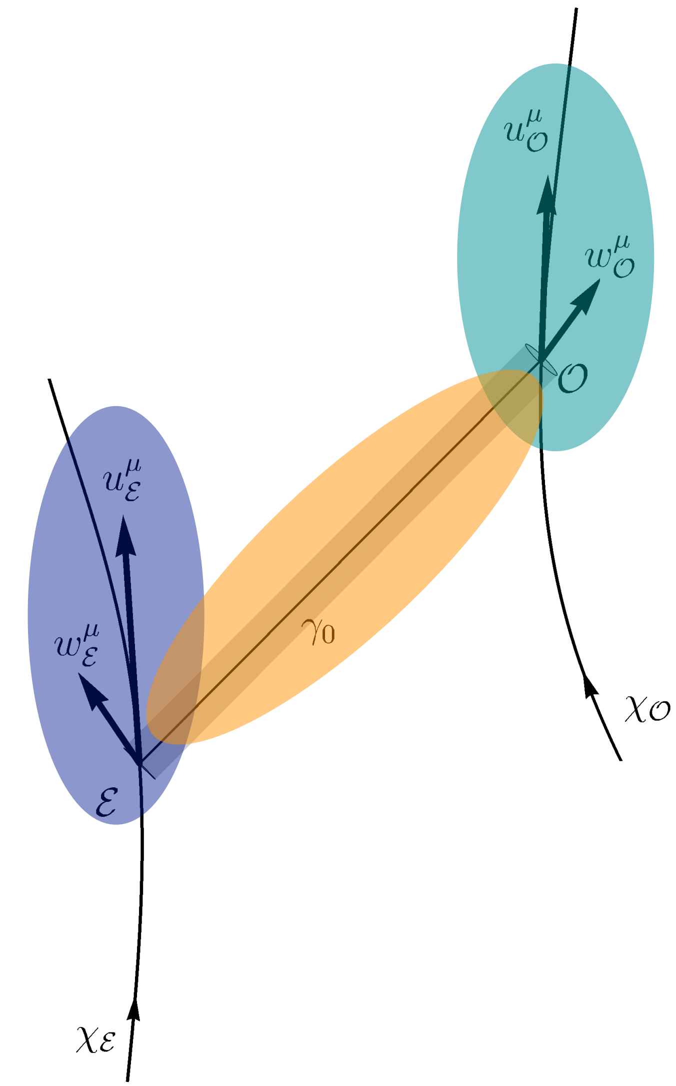

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$w_\perp{}^A{}_B = M^{-1A}{}_C\, \Pi^C{}_B$

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$$M^A{}_B = \frac{1}{u^\sigma_{\mathcal{O}}\, l_{\mathcal{O}\,\sigma}} \mathscr{D}^{-1A}{}_B$$

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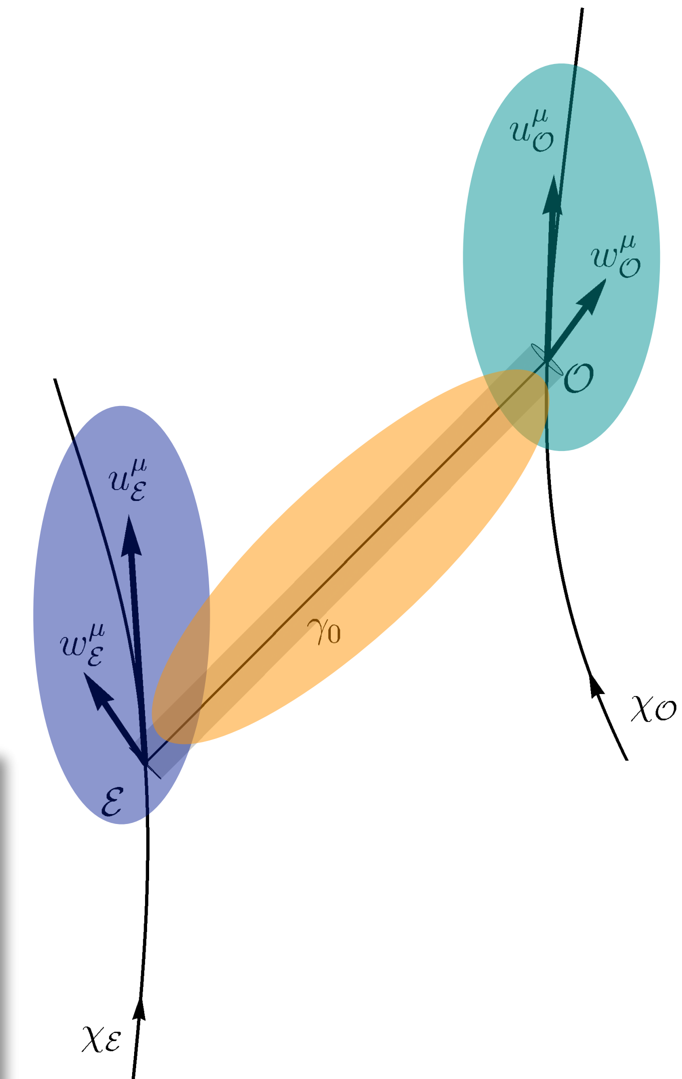

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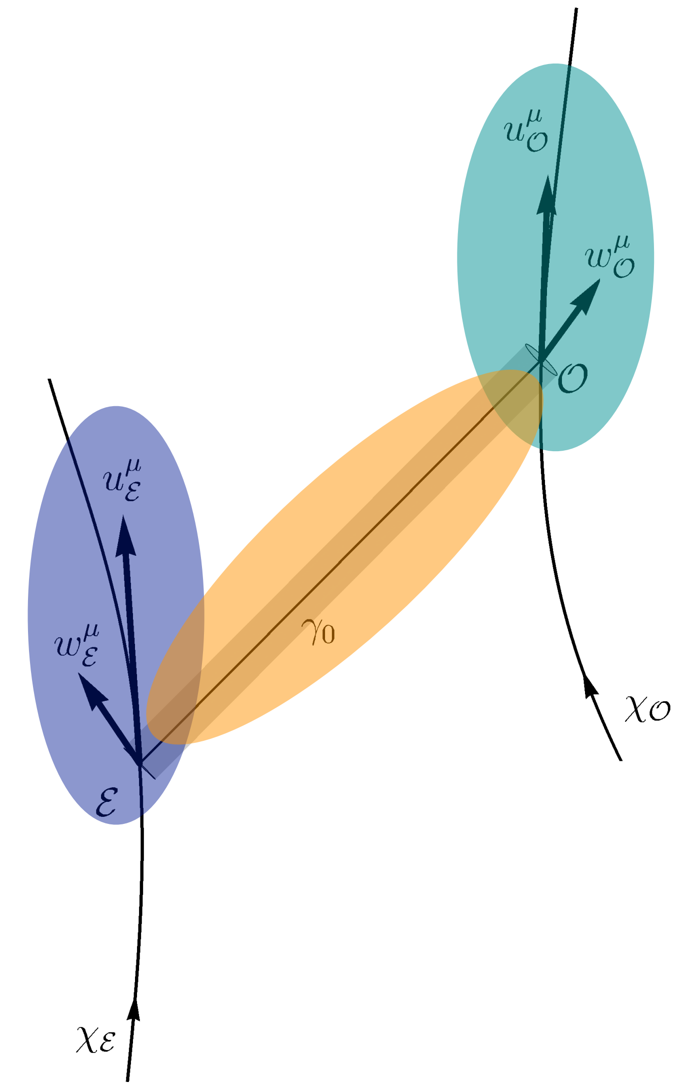

$$\delta_{\mathcal{O}} r^A \equiv \delta_{\mathcal{O}} r^A\left(R^\mu{}_{\nu\alpha\beta}\Big|_{\gamma_0}, u^\mu_{\mathcal{O}}, u^\mu_{\mathcal{E}}, w^\mu_{\mathcal{O}}\right)$$

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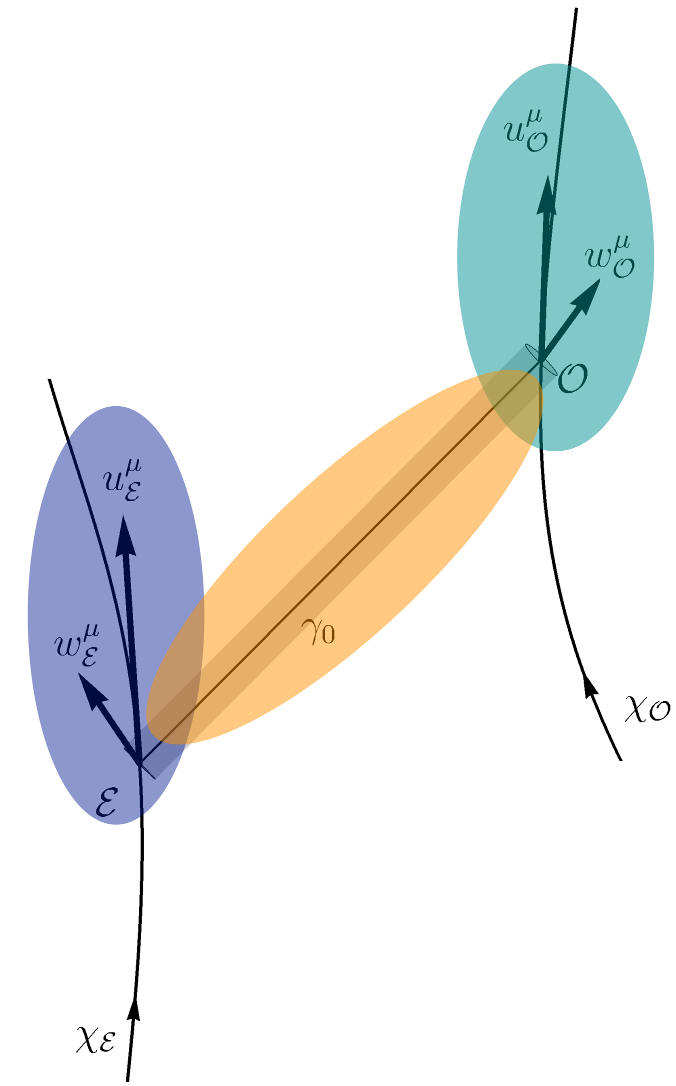

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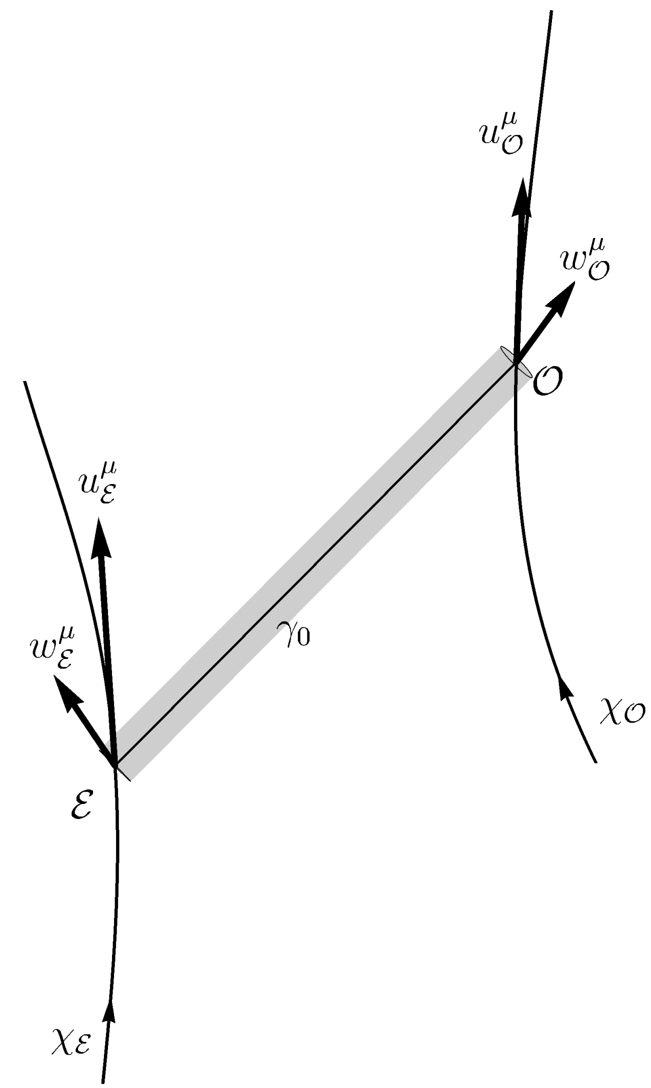
Parameter μ

$$\mu = 1 - \det w_\perp{}^A{}_B = 1 - \frac{\det \Pi^A{}_B}{\det M^A{}_B}$$



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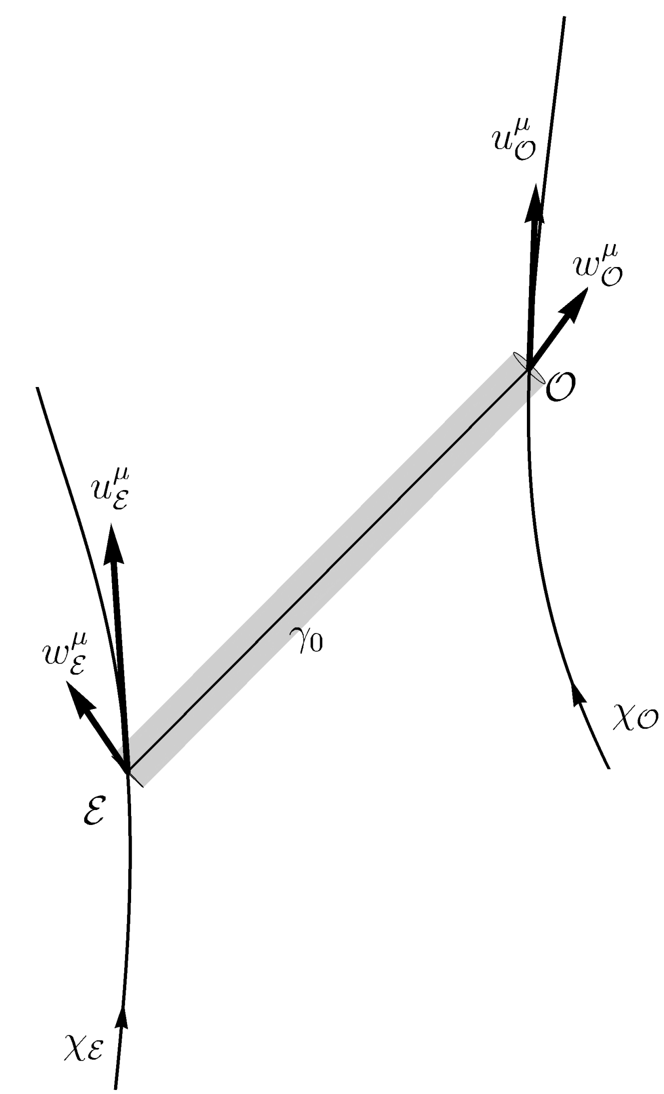
$$\mu = 1 - \det w_{\perp}{}^{A}{}_{B} = 1 - \frac{\det \Pi^{A}{}_{B}}{\det M^{A}{}_{B}}$$

- scalar, dimensionless



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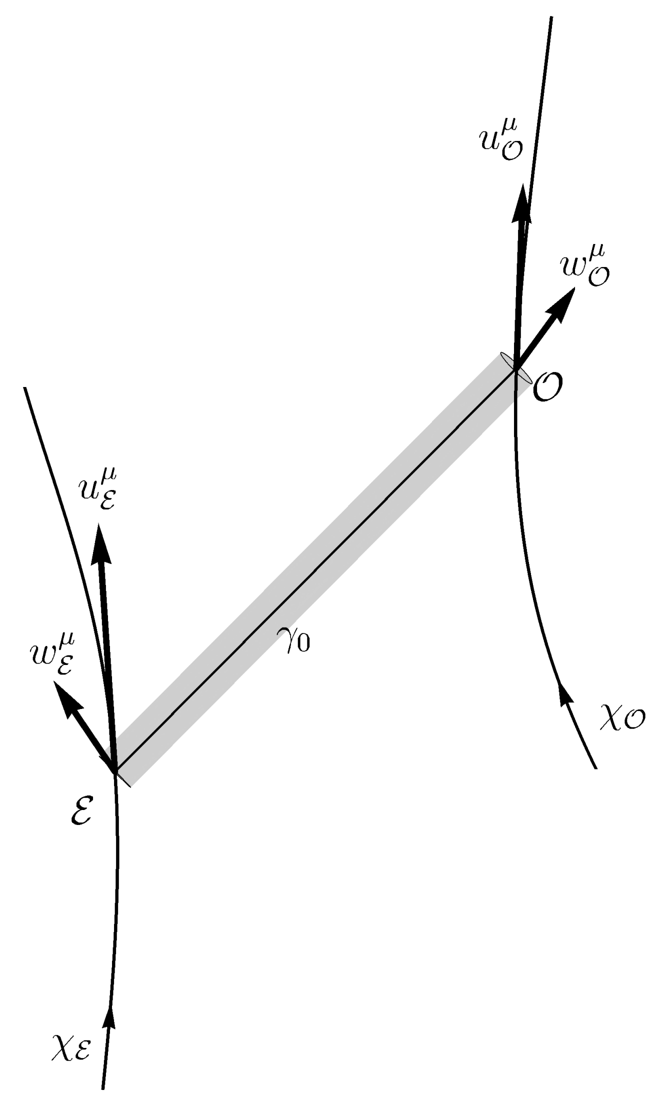
- momentary motions-independent



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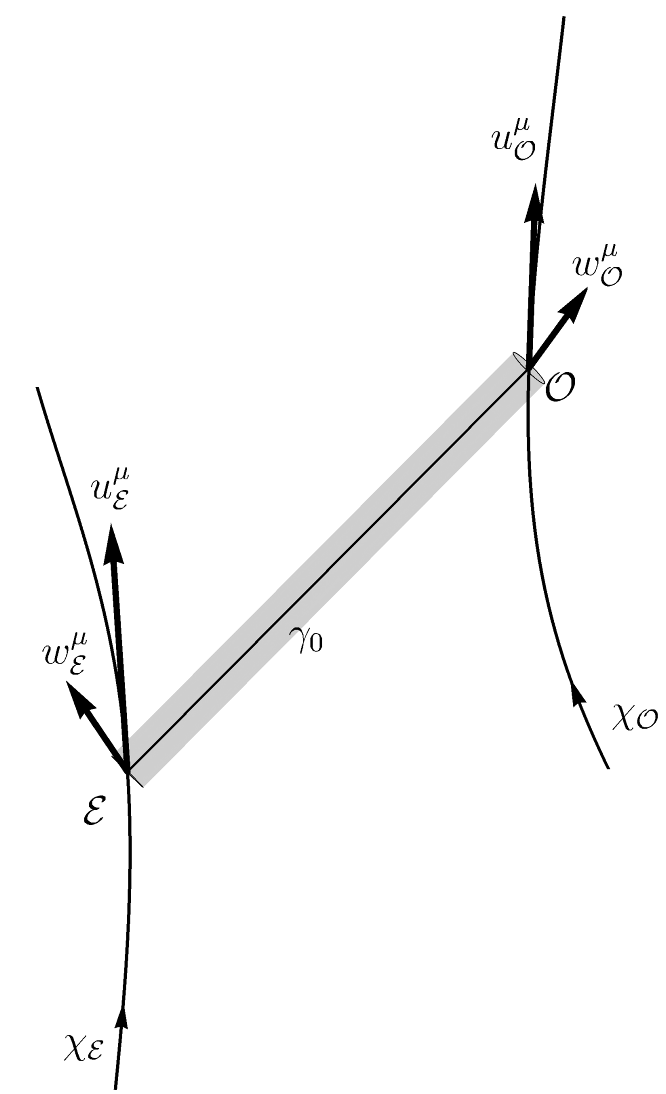
- scalar, dimensionless
- momentary motions-independent
- (theoretically) simple to measure

$$\mu = 1 \mp \frac{D^2_{ang}}{D^2_{par}}$$



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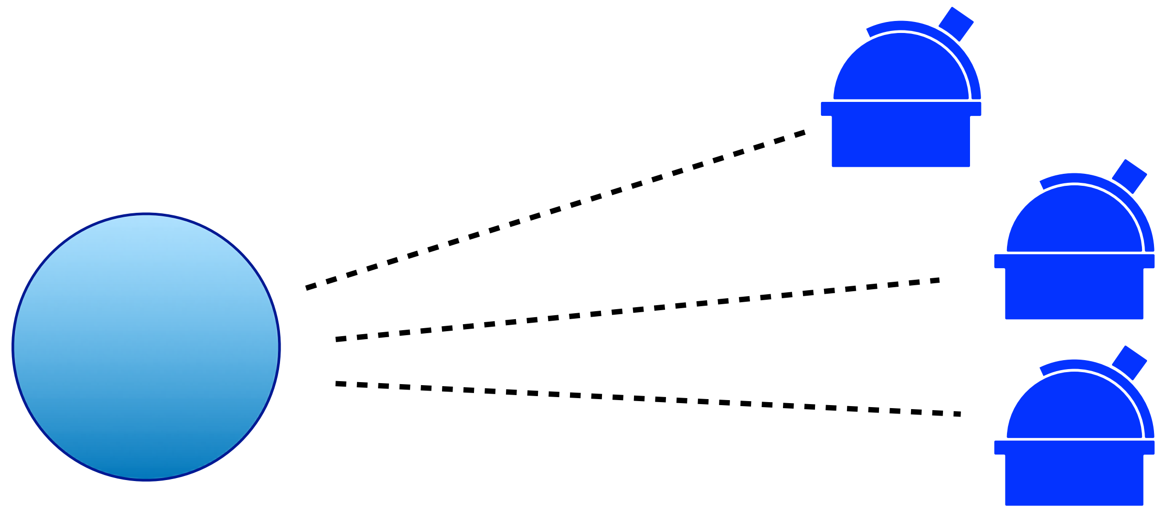

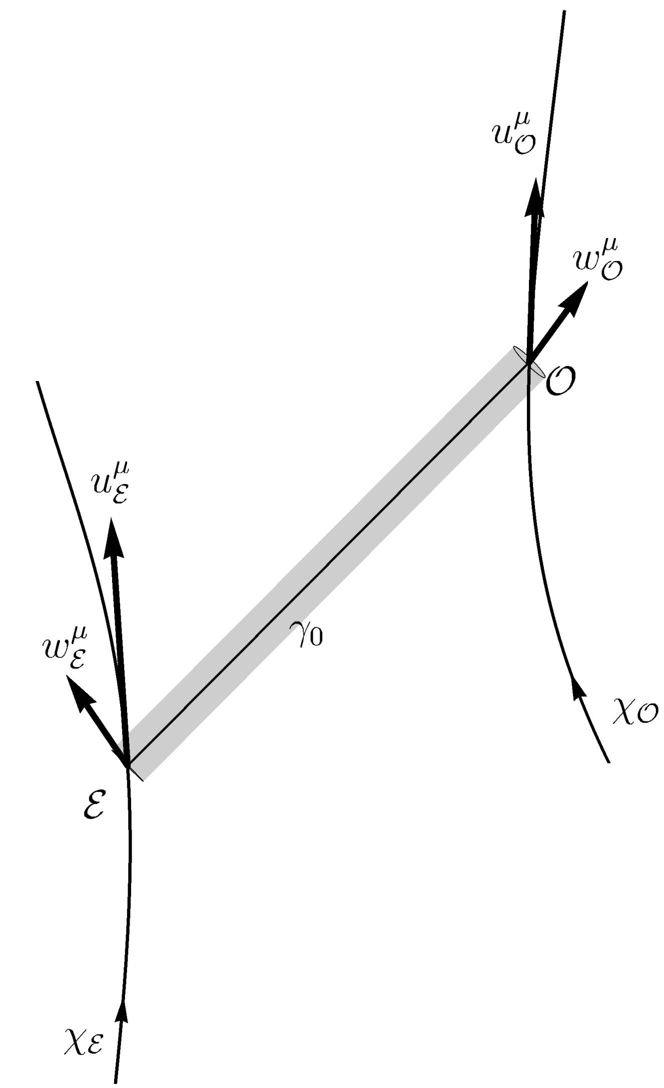

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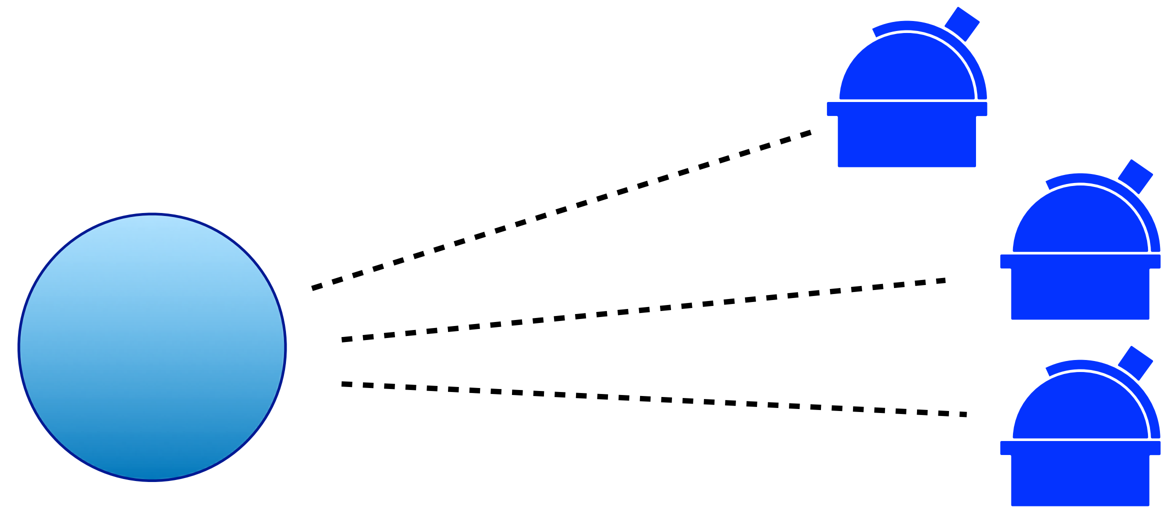

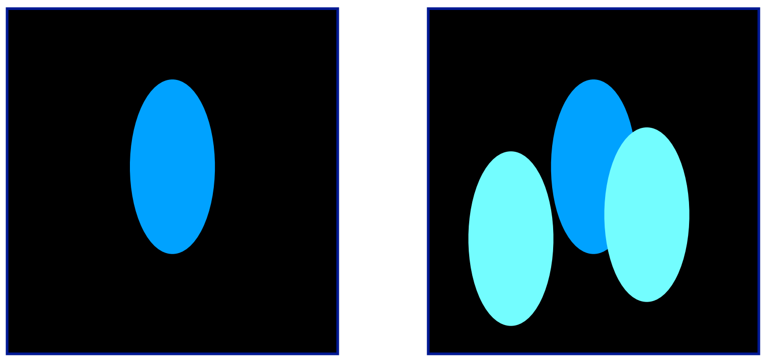

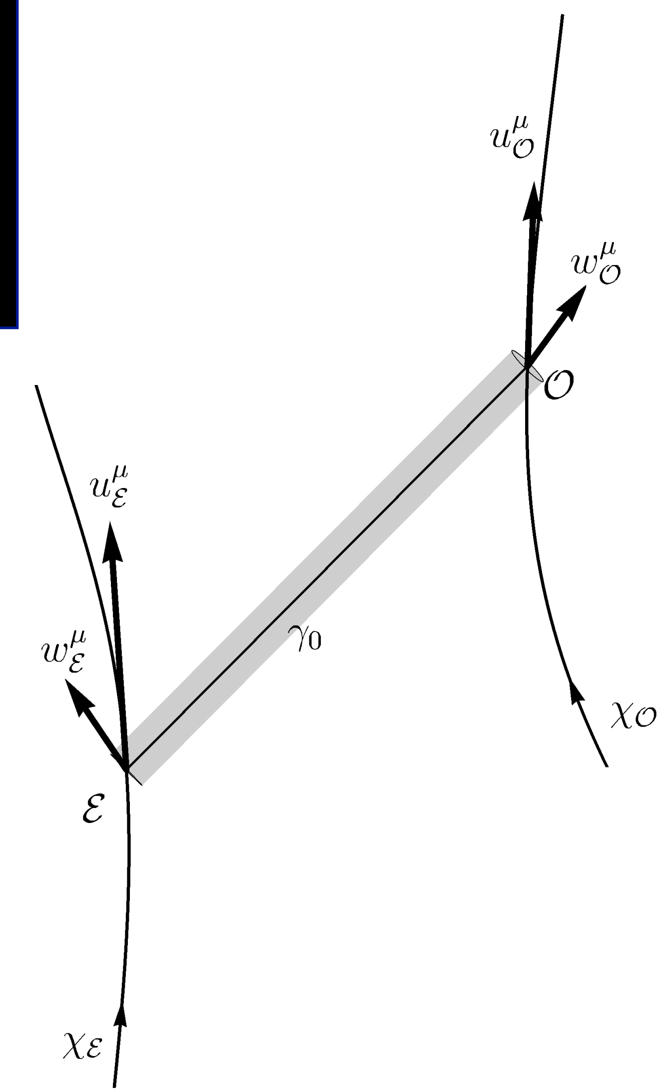

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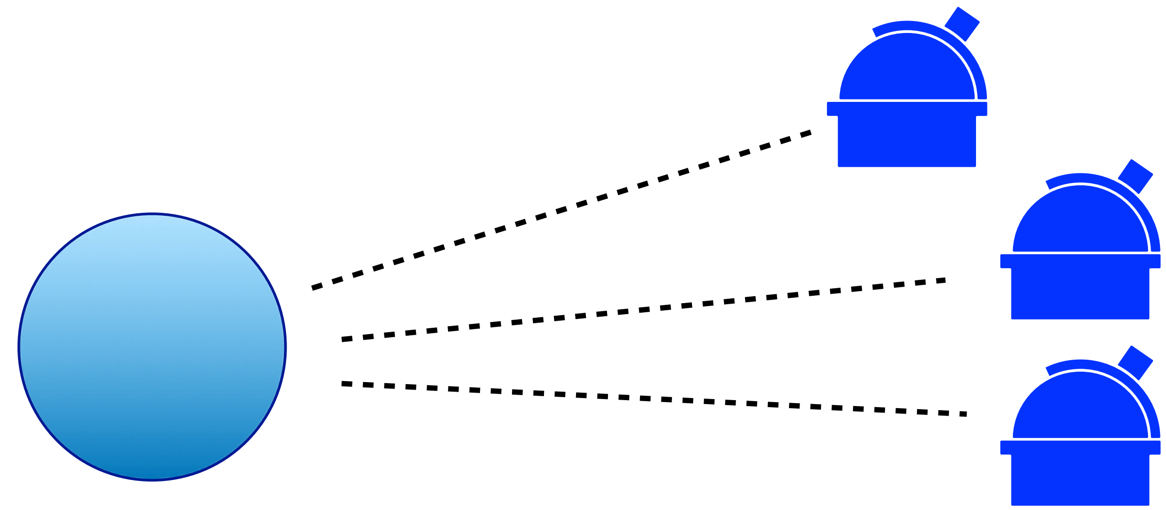
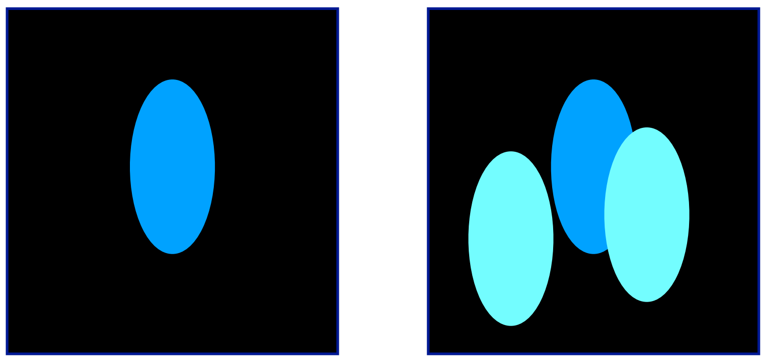
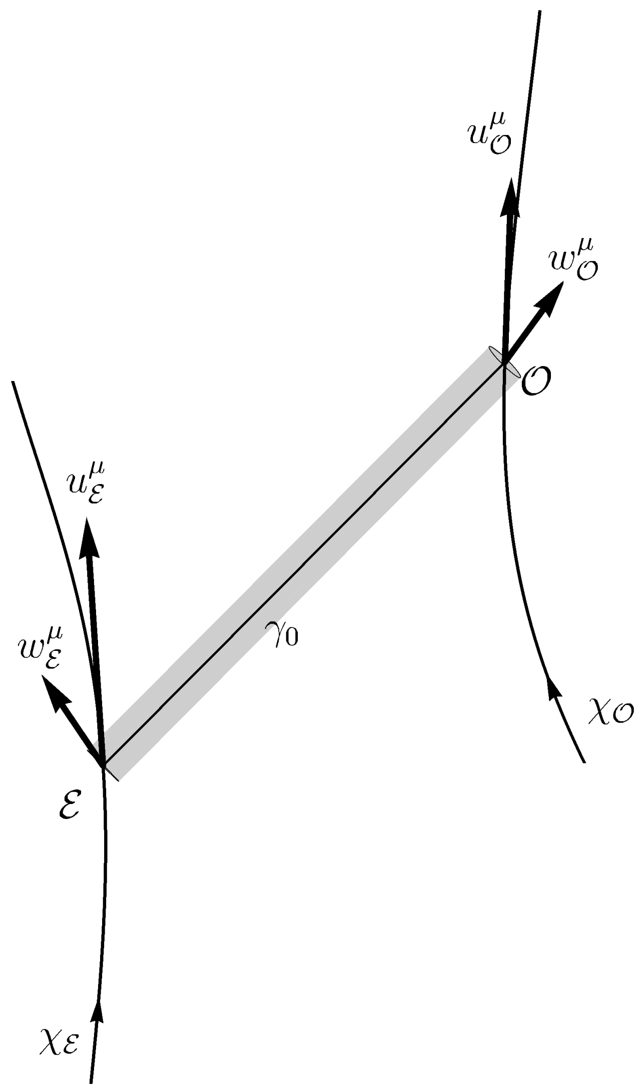




- no need to measure the parallel transport independently

$$m_{\perp}{}^{A}{}_{B} = w_{\perp}{}^{A}{}_{B} - \delta^{A}{}_{B}$$

parallel transport of perpendicular vectors



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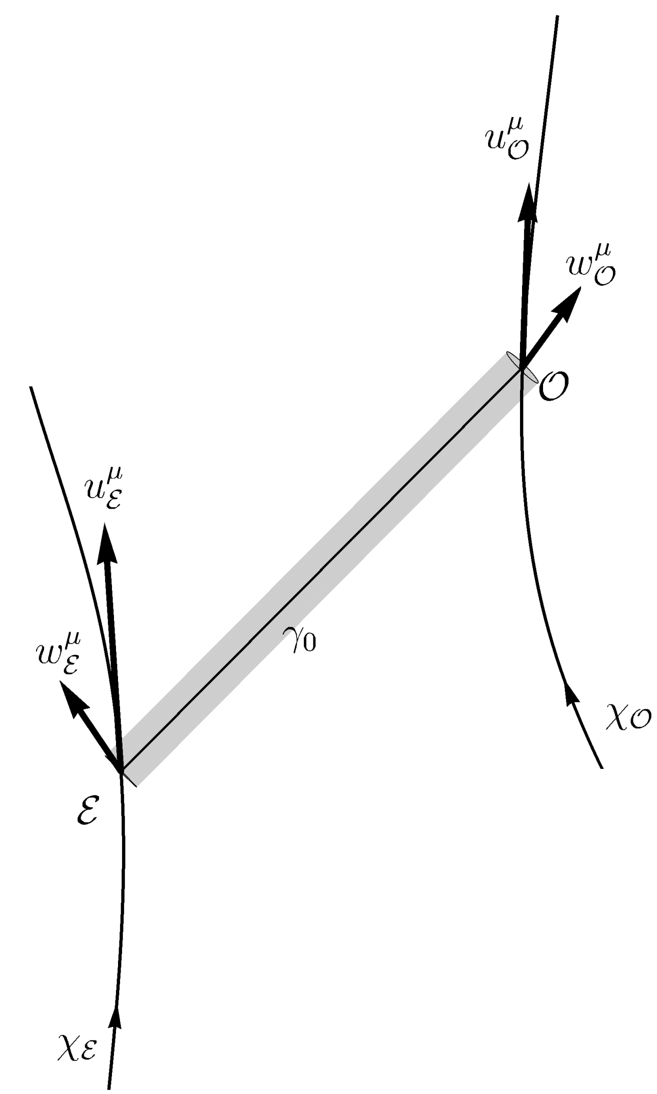

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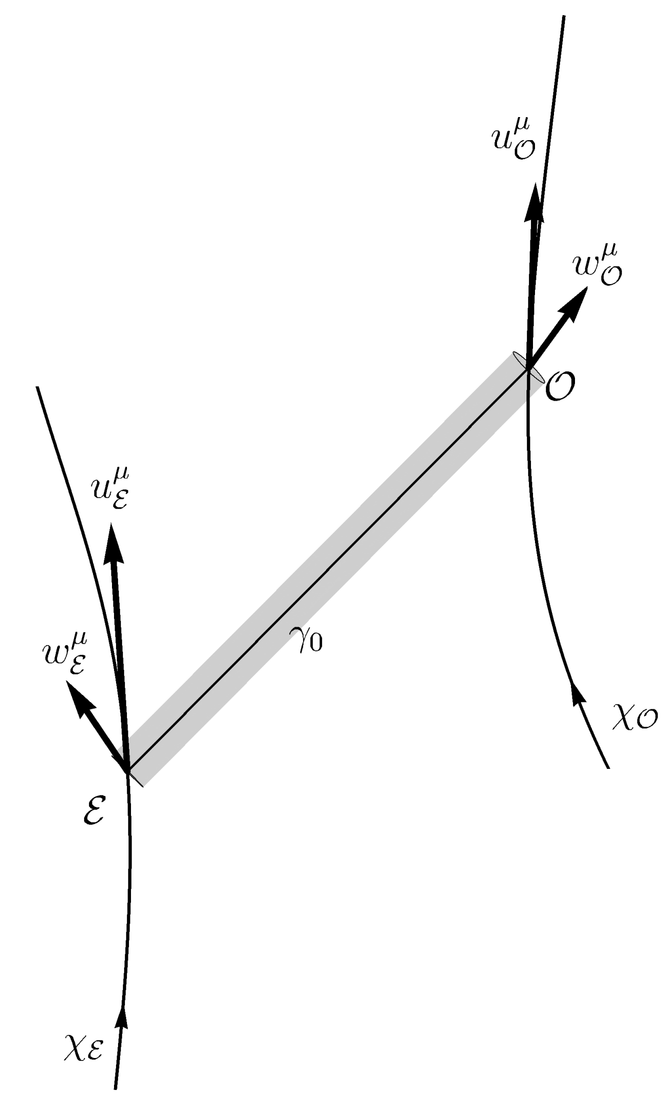
- curvature detector

$$\mu = 1 - \det\left(\delta^A{}_B + m_\perp{}^A{}_B\right)$$

$R^\mu{}_{\nu\alpha\beta}\Big|_{\gamma_0}$



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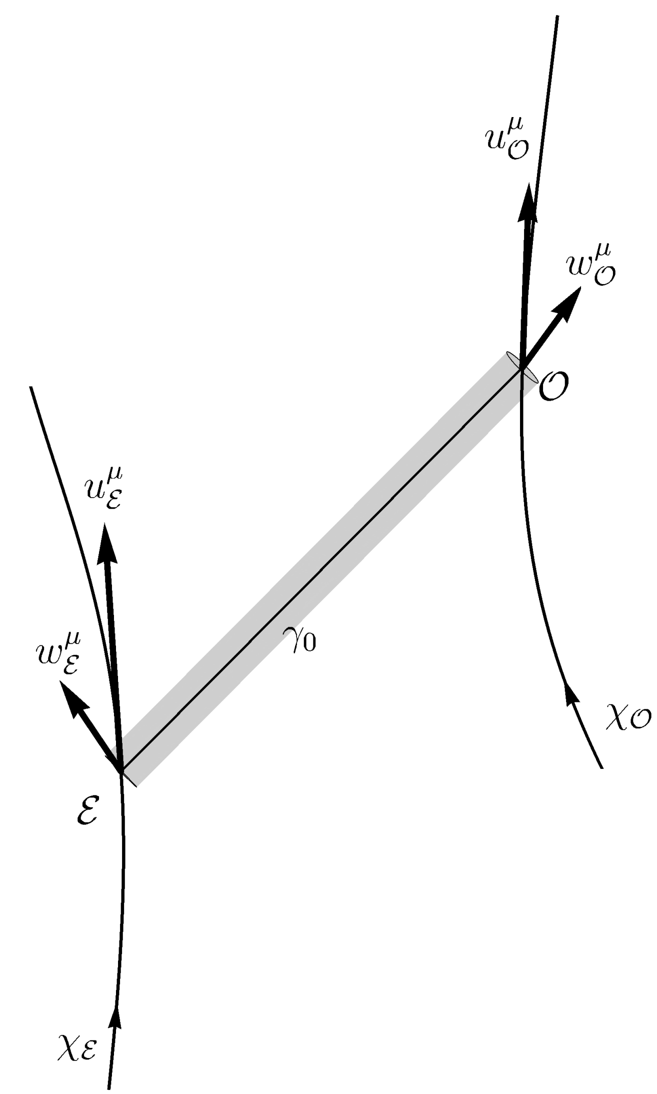
flat space:

$$D_{ang} = D_{par} = D_{\mathcal{O}}$$

$$\mu = 0$$



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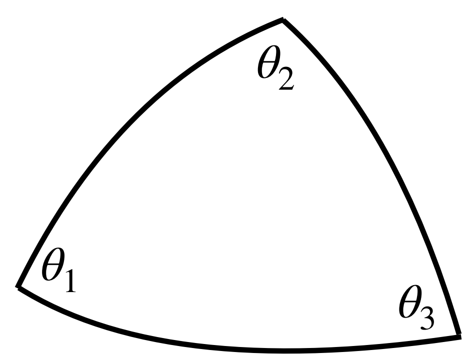
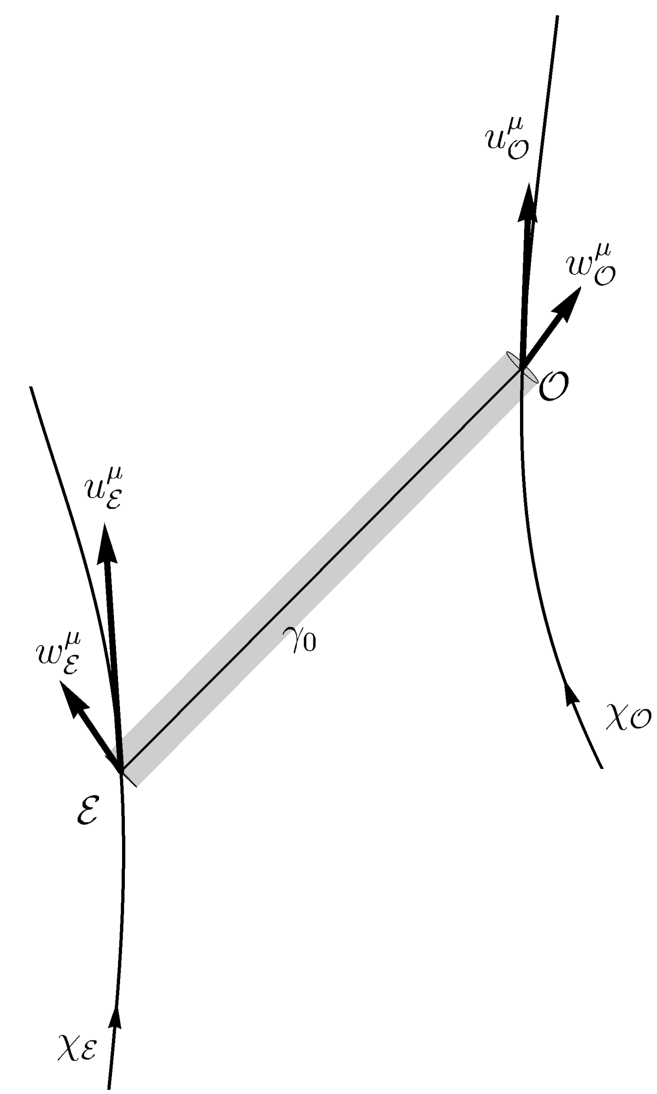




compare the angular deficit formula

$$\theta_1 + \theta_2 + \theta_3 - \pi = \int K\, d^2 A$$

Parameter μ

spacetime geometry $g_{\mu\nu}(x), \Gamma^{\mu}{}_{\alpha\beta}(x)$

observation and emission points along a null geodesic $\gamma_0^{\mu}(\lambda), \mathscr{O}, \mathscr{E}$

curvature along the line of sight $R^{\mu}{}_{\nu\alpha\beta}\big|_{\gamma_0}$

(null) geodesic deviation equation

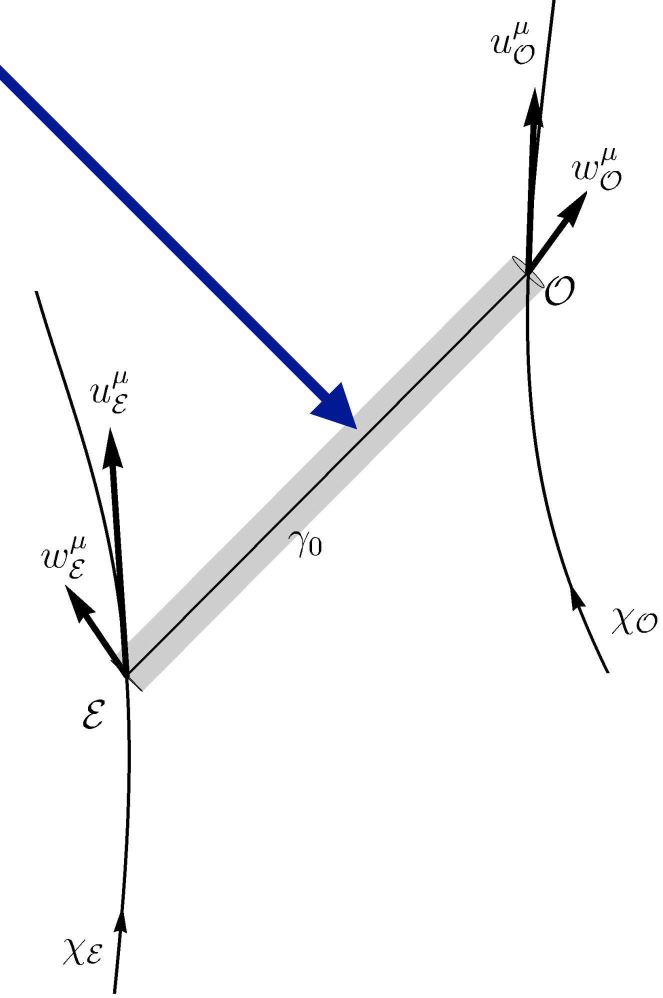
bilocal geodesic operators $W_{XX}, W_{XL}, W_{LL}, W_{LX}$

covariant expressions for observables

momentary positions and motions of the observer and emitter $\delta x^{\mu}_{\mathscr{O}}, \delta x^{\mu}_{\mathscr{E}}, u^{\mu}_{\mathscr{O}}, u^{\mu}_{\mathscr{E}}, w^{\mu}_{\mathscr{O}}, w^{\mu}_{\mathscr{E}}$

observables $r^{\mu}, z, \delta_{\mathscr{O}} r^{A}, \delta_{\mathscr{O}} z, \ldots$

$g_{\mu\nu} =$ **[flat]** $+$ **[curvature corrections]** $\left(R^{\mu}{}_{\nu\alpha\beta}\big|_{\gamma_0}\right) + h.o.t.$



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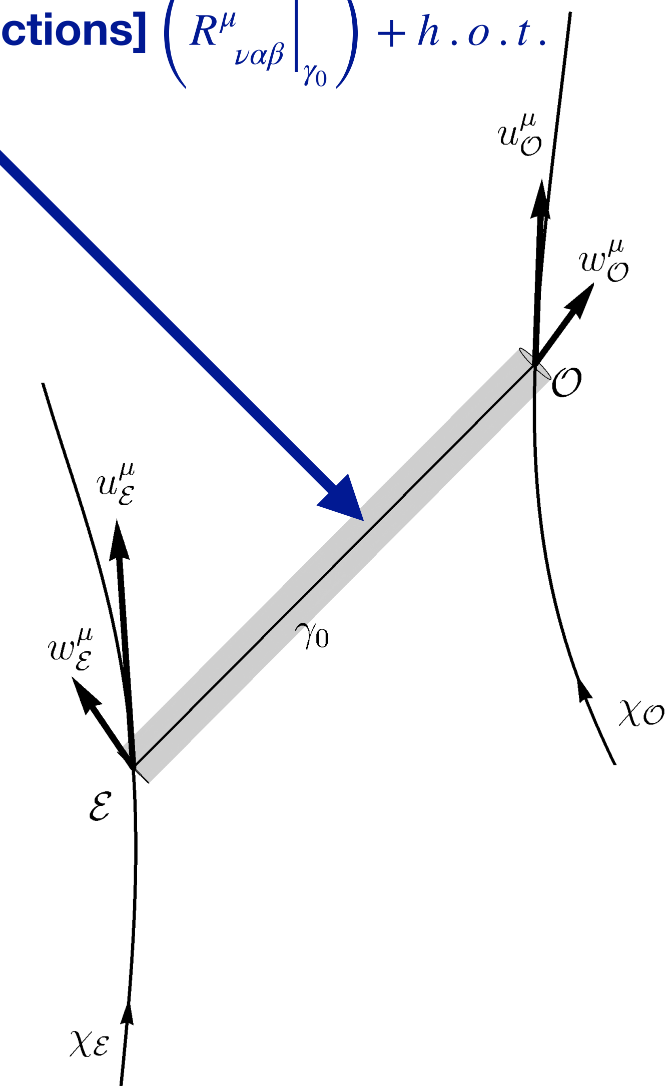
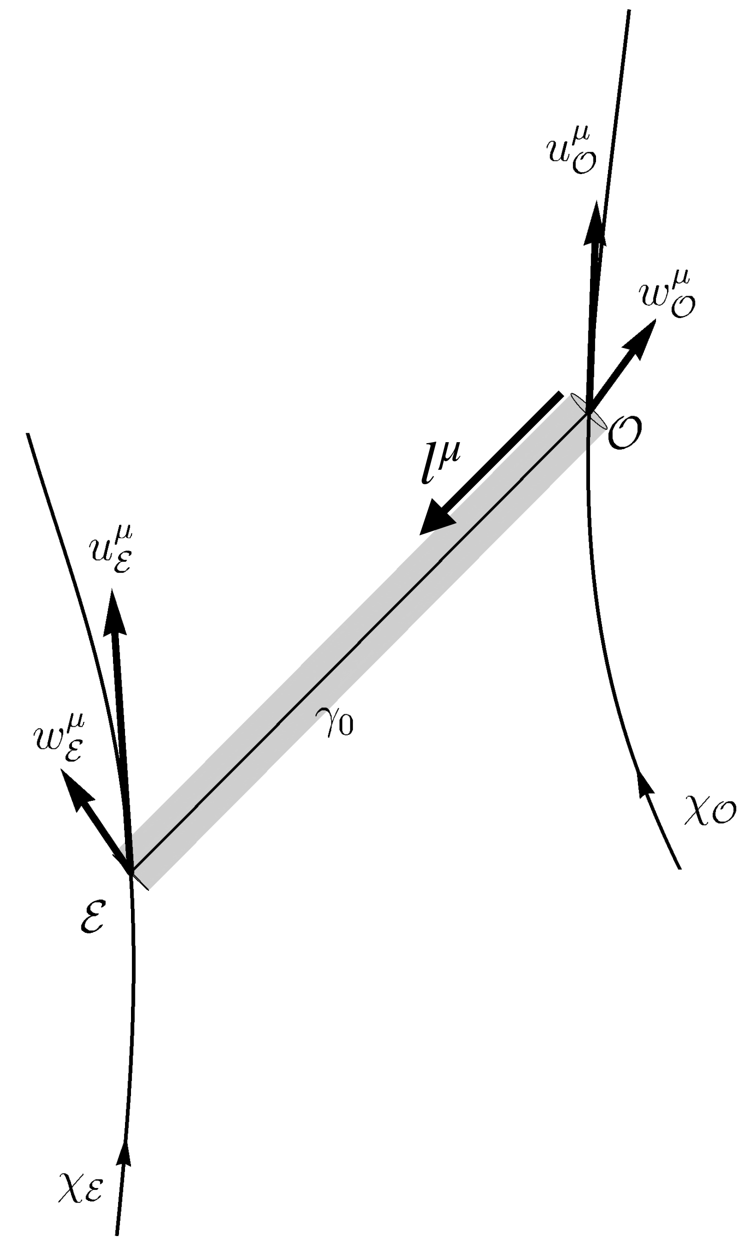


Parameter μ

Approximate formula for short distances/small curvature

Linearisation in the curvature of all expressions…



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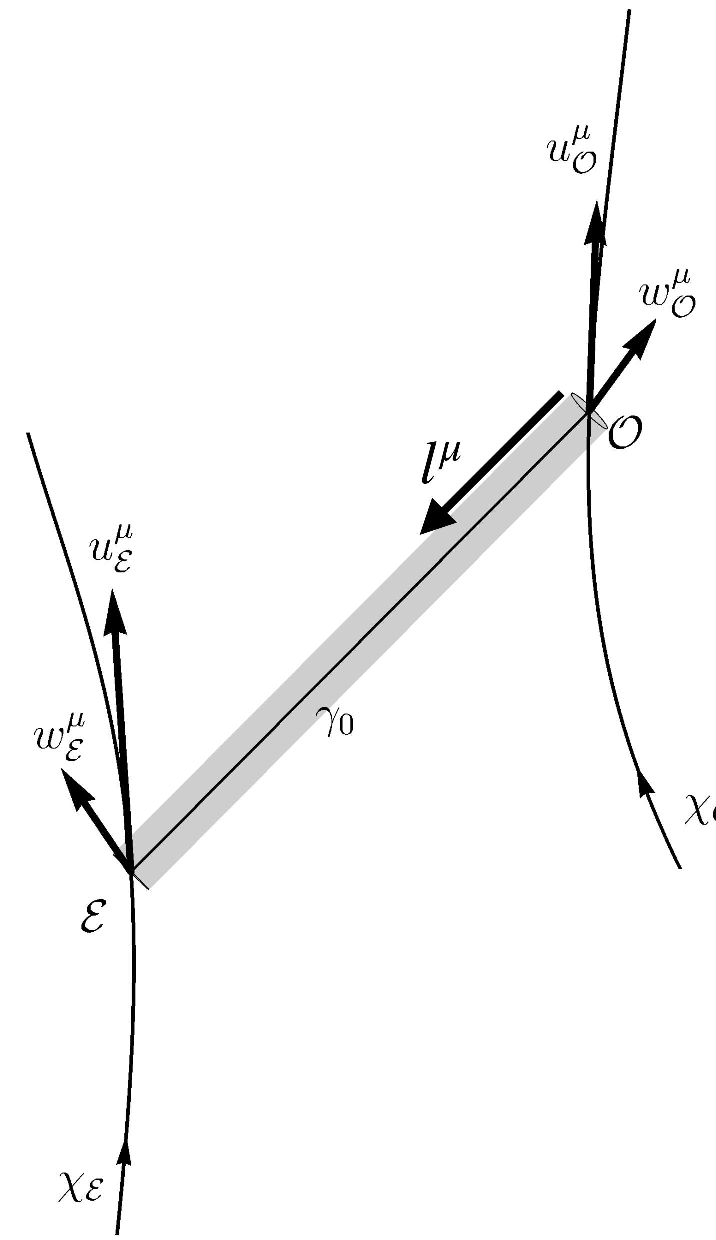
Linearisation in the curvature of all expressions…

$$\mu \approx -\,{m_\perp}^{A}{}_{A} \approx -\int_{\lambda_\mathcal{O}}^{\lambda_\mathcal{E}} R^{A}{}_{llA}(\lambda)\,(\lambda_\mathcal{E}-\lambda)\,d\lambda$$



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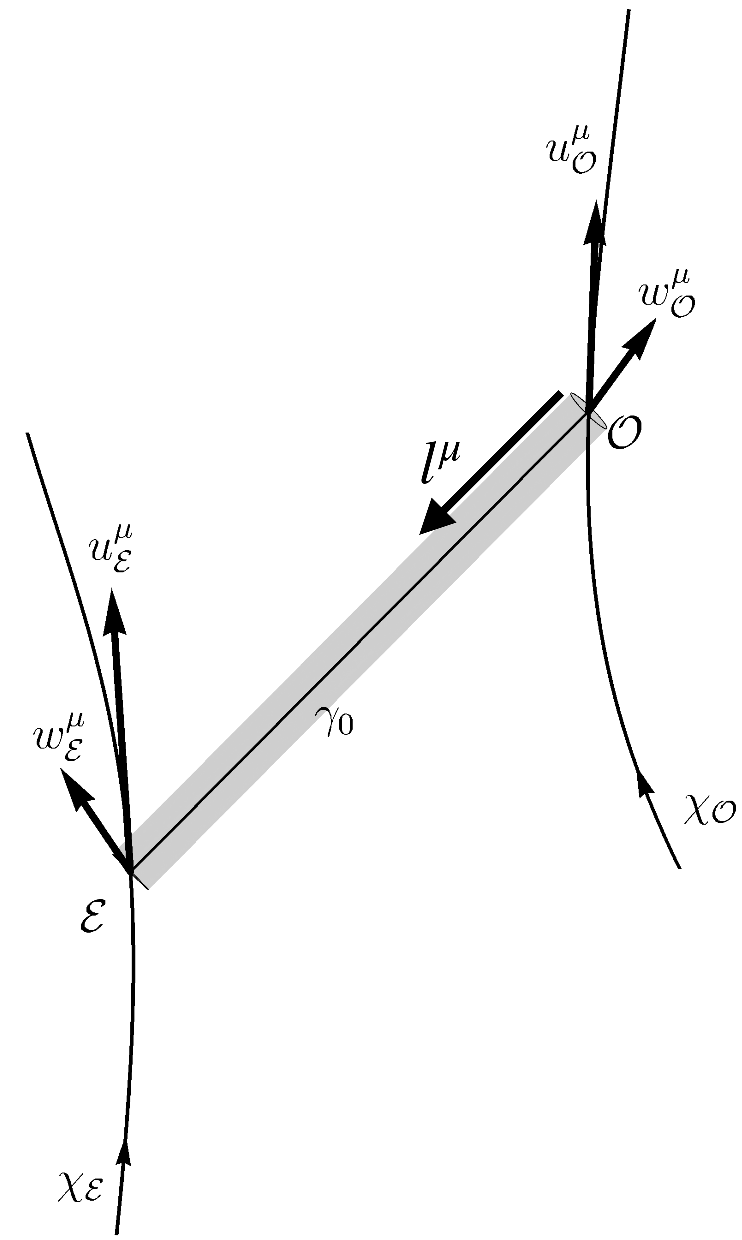
a bit of tensor manipulation…



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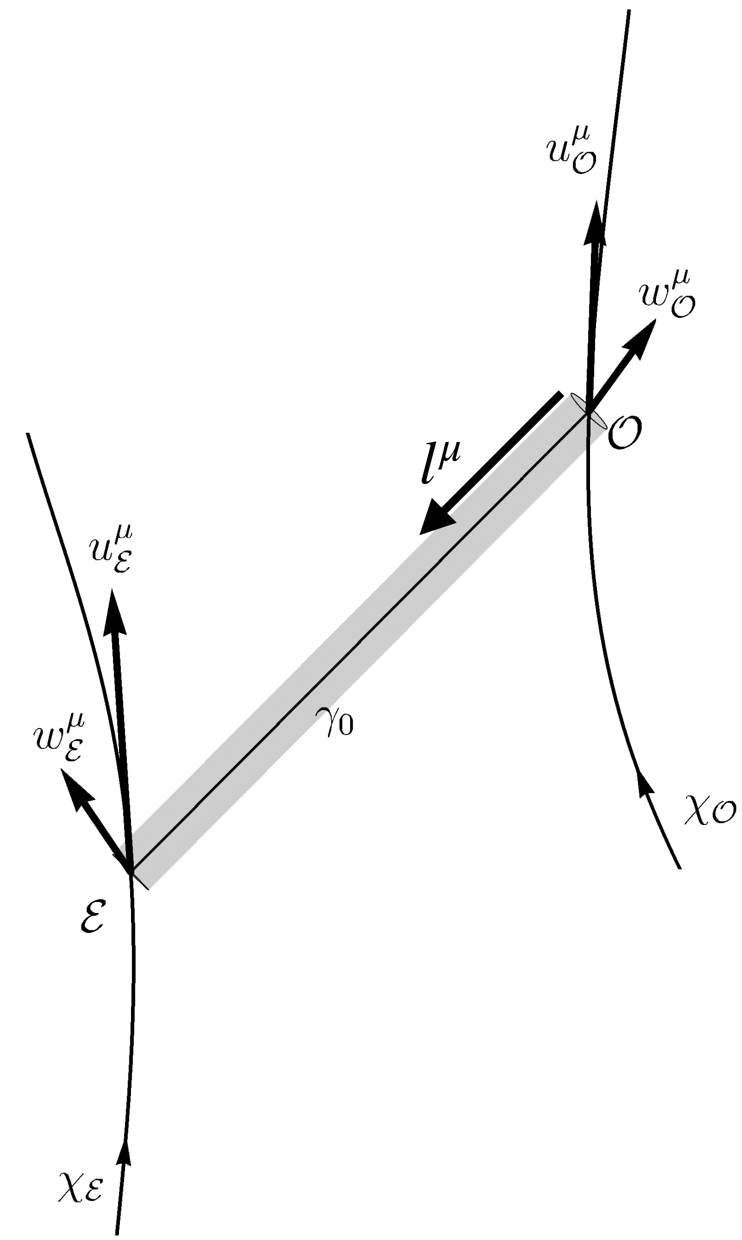
Approximate formula for short distances/small curvature

Linearisation in the curvature of all expressions…

$$\mu \approx -\, {m_\perp}^{A}{}_{A} \approx -\int_{\lambda_{\mathscr{O}}}^{\lambda_{\mathscr{E}}} R^{A}{}_{llA}(\lambda)\,(\lambda_{\mathscr{E}} - \lambda)\,d\lambda$$

a bit of tensor manipulation…

$$\mu \approx \frac{8\pi G}{c^4}\int_{\lambda_{\mathscr{O}}}^{\lambda_{\mathscr{E}}} T_{ll}(\lambda)\,(\lambda_{\mathscr{E}} - \lambda)\,d\lambda$$



Weighing spacetime along the line of sight

tidal forces, gravitational waves (non-local)

Weyl tensor $C^{\mu}{}_{\nu\alpha\beta}\big|_{\gamma_0}$

Ricci decomposition

matter content along the line of sight (local) $T_{\mu\nu}\big|_{\gamma_0}$

Einstein equations

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(null) geodesic deviation equation

bilocal geodesic operators $W_{XX}, W_{XL}, W_{LL}, W_{LX}$

covariant expressions for observables

momentary positions and motions of the observer and emitter $\delta x^{\mu}_{\mathcal{O}}, \delta x^{\mu}_{\mathcal{E}}, u^{\mu}_{\mathcal{O}}, u^{\mu}_{\mathcal{E}}, w^{\mu}_{\mathcal{O}}, w^{\mu}_{\mathcal{E}}$

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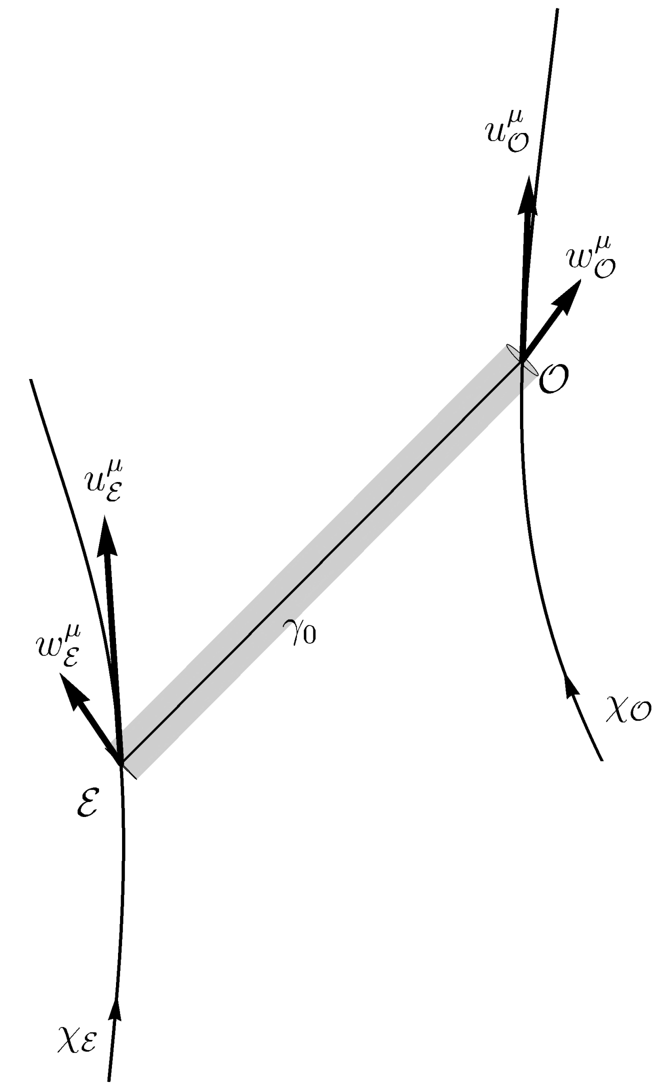

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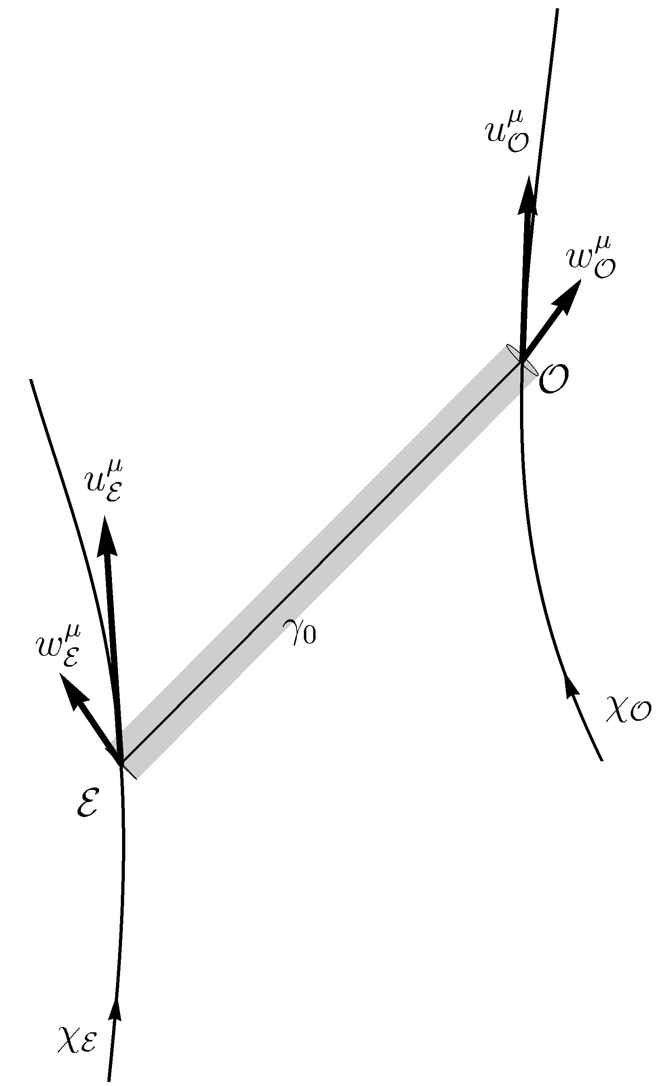

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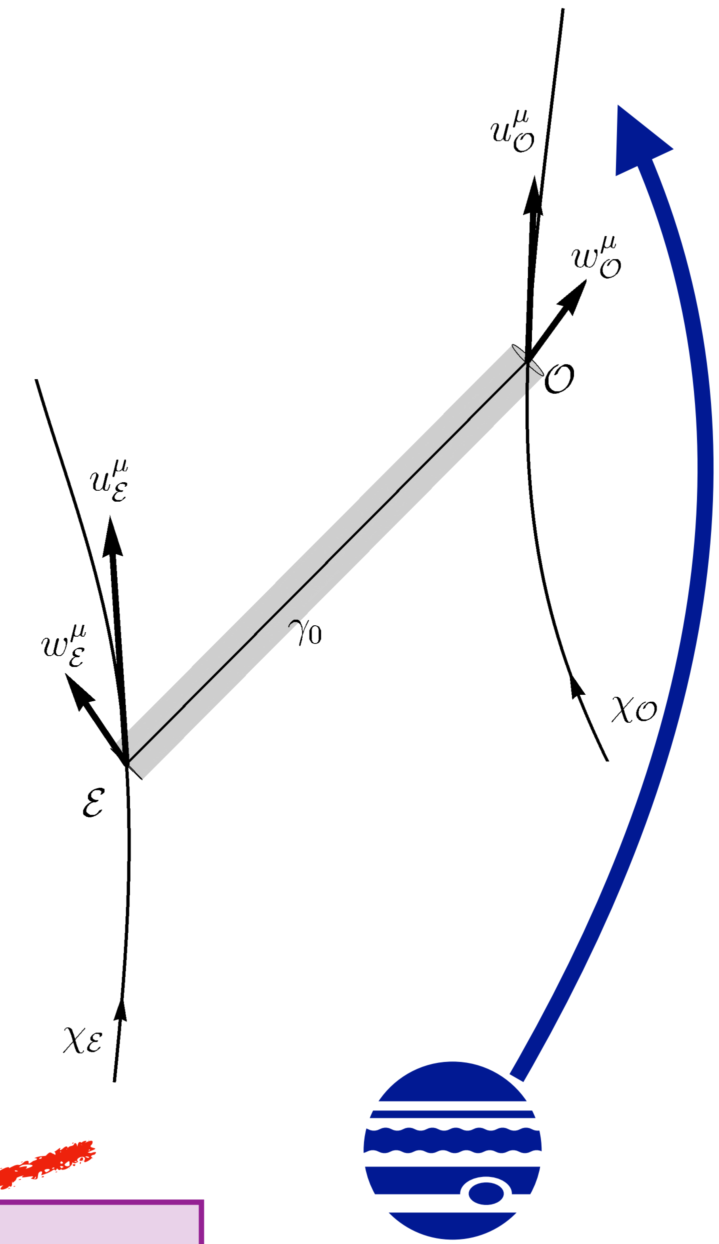

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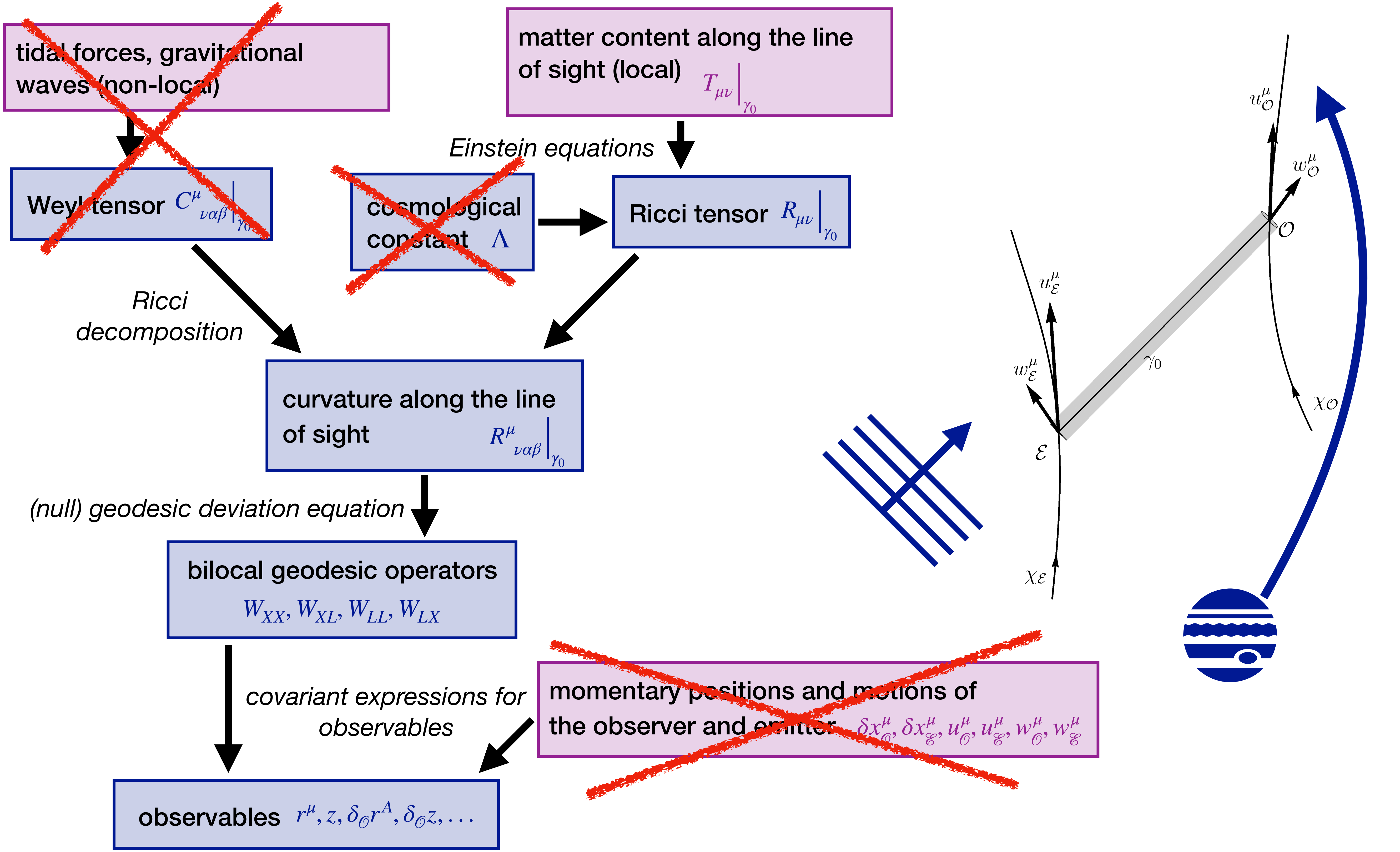

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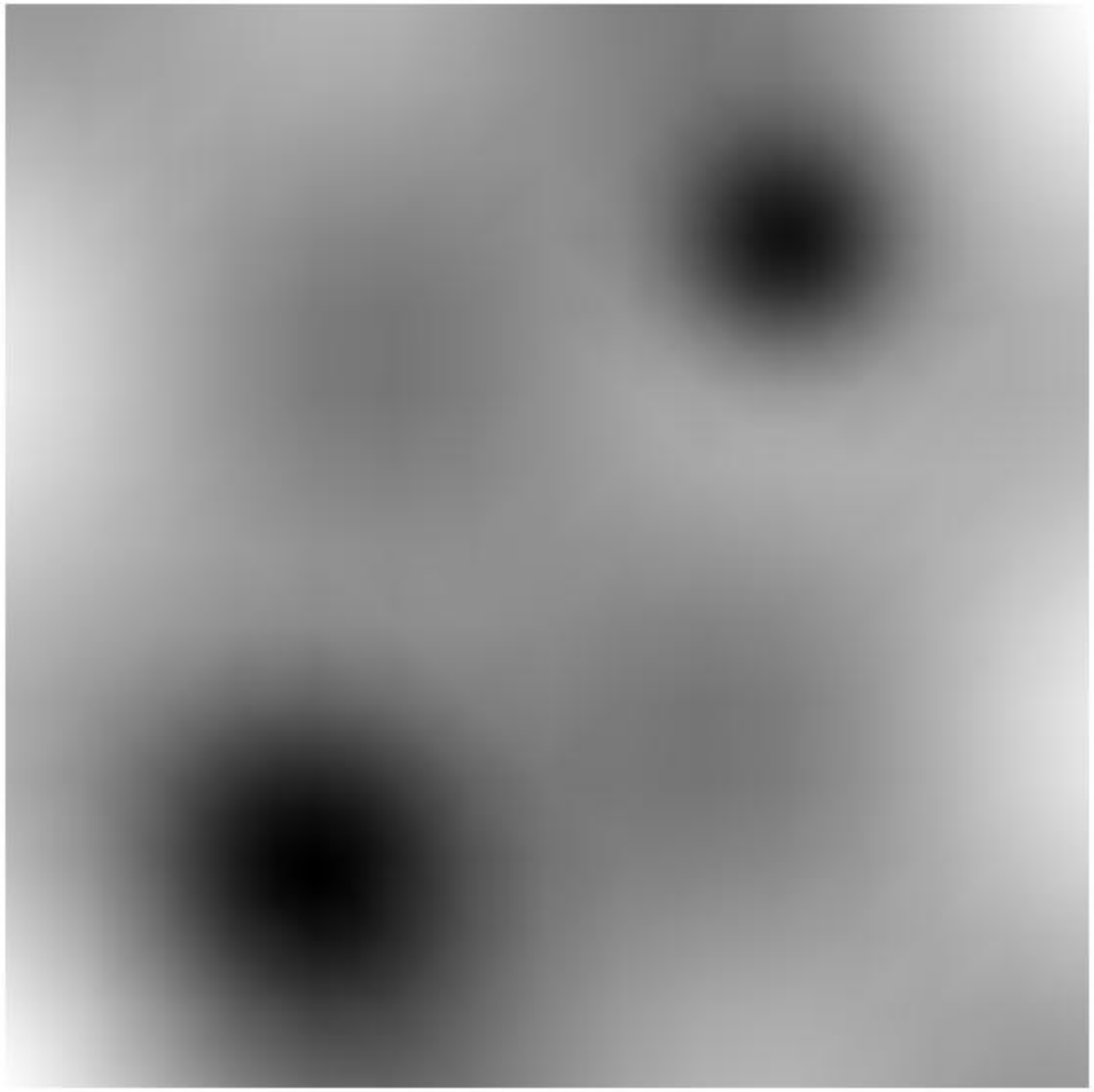

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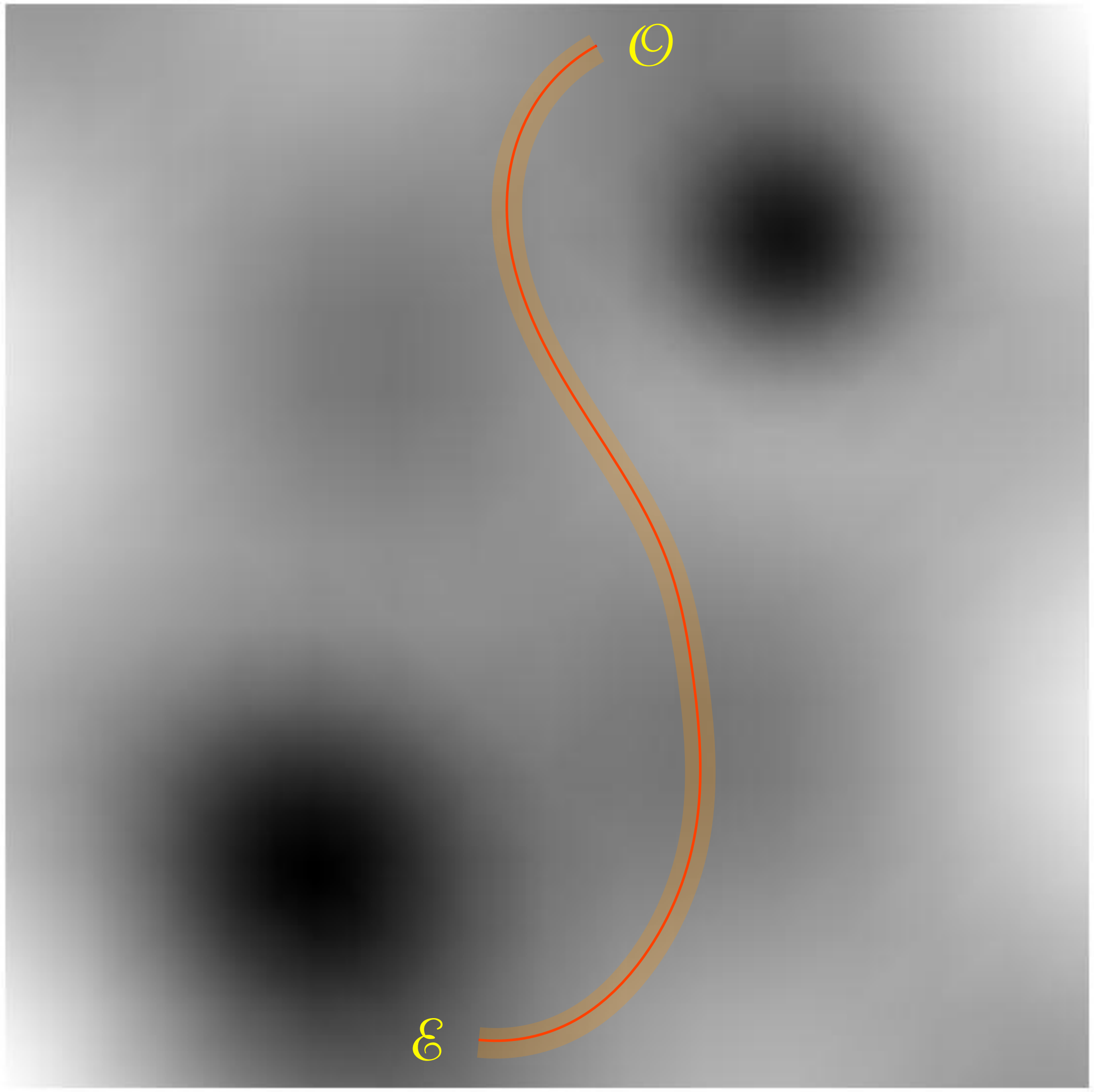

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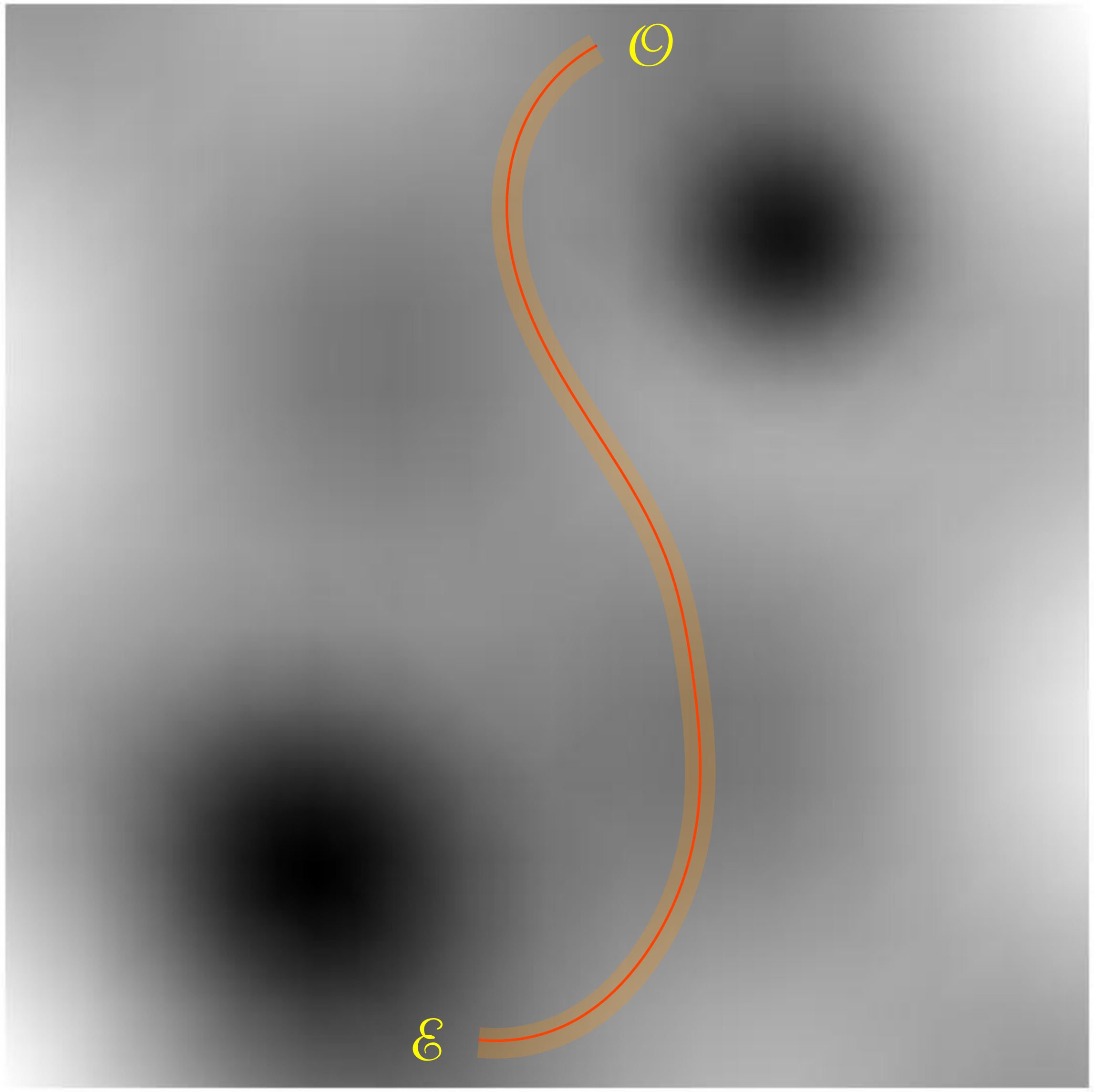

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…but we may use the motion of the LC wrt CMB frame in the future [Kardashev 1986, Räsänen 2014, Quercellini *et al* 2012, Marcori *et al* 2018], effects borderline visible

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Conceptually simple

The only observable so insensitive to external gravitational perturbations
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Tomography-like measurement

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Similar ideas before:

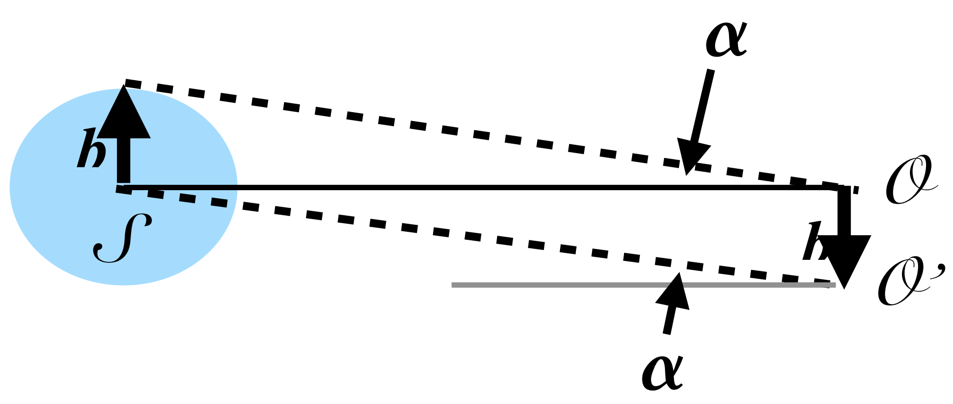
McCrea 1935 - parallax distance in FLRW metric carries additional information

Weinberg 1970 - parallax distance in FLRW metric determines k = 0,1,-1

Kasai 1988, Rosquist 1988 - parallax distance in FLRW (+ perturbations)

Räsänen 2014 - parallax distance vs. luminosity distance as consistency test of FLRW

Summary



α
h
$\mathscr{O}$
$\mathscr{S}$
h
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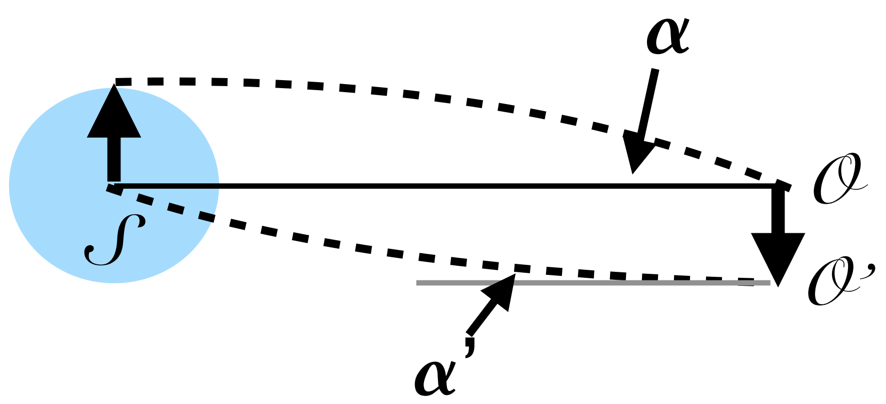
α



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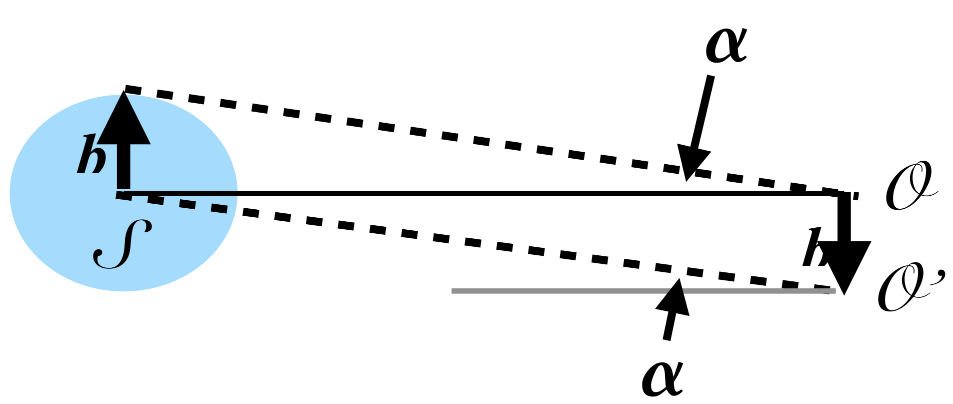

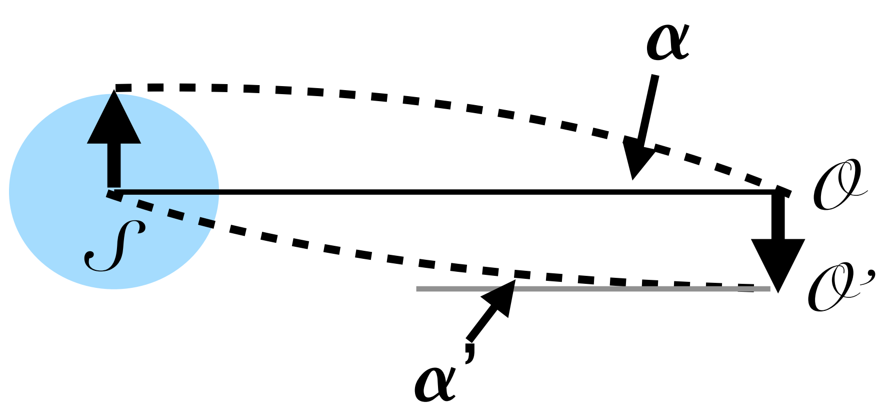

New method of curvature determination/weighing matter along the line of sight

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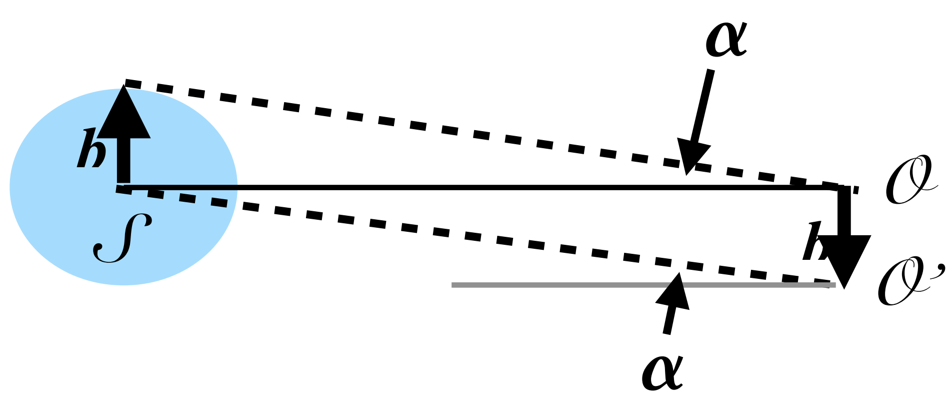

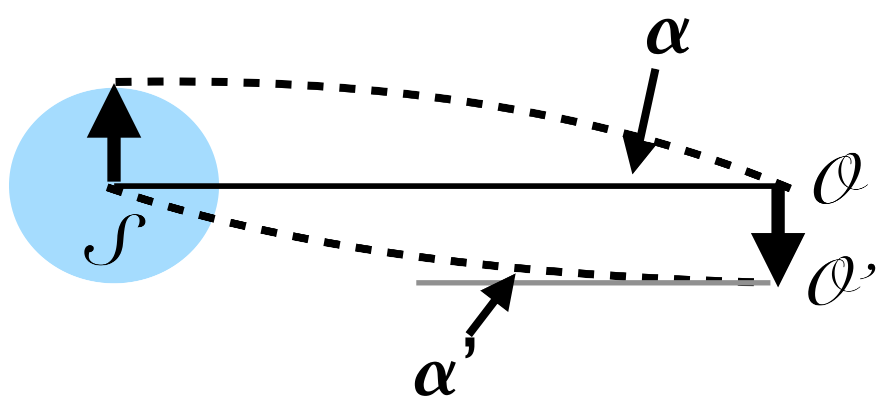

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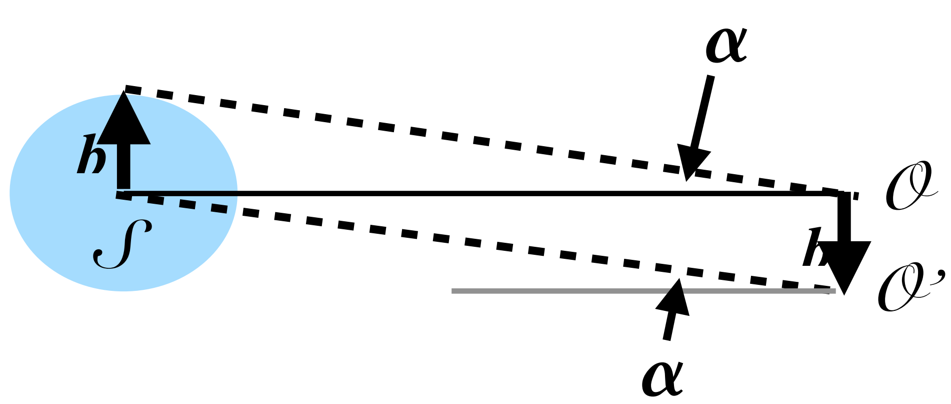

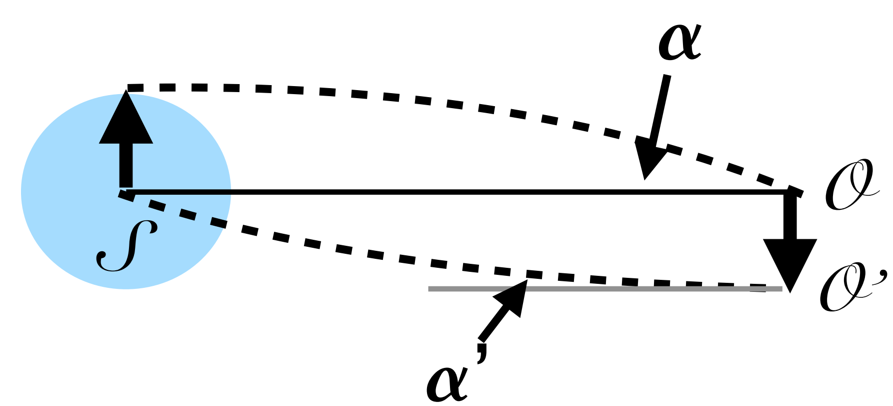

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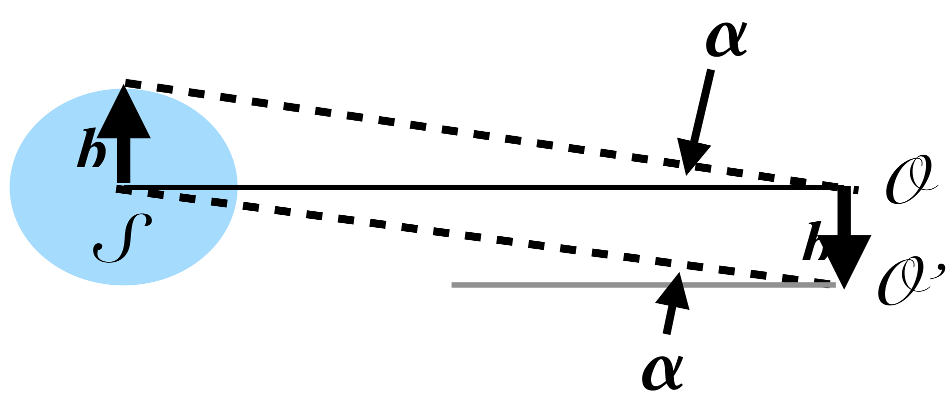

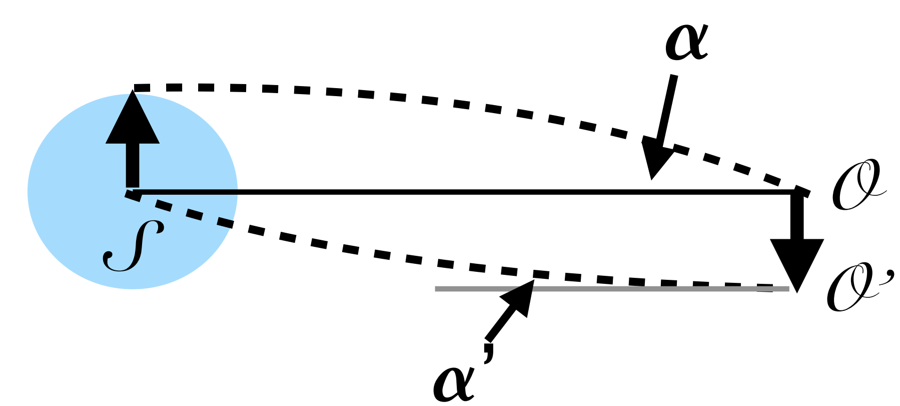

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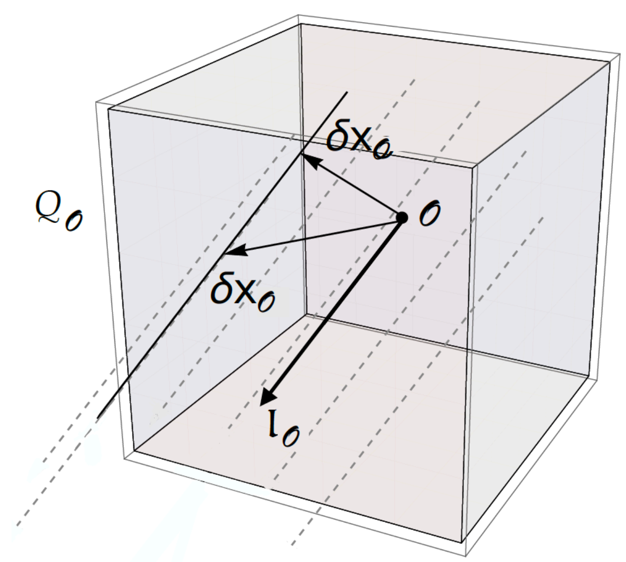

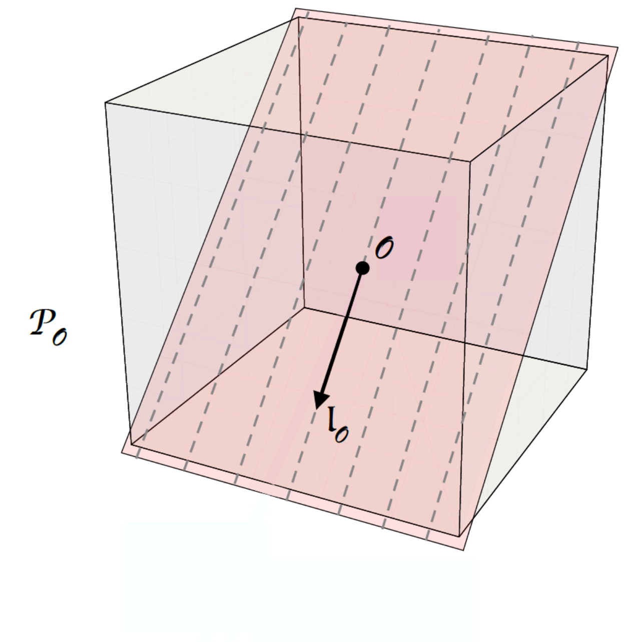

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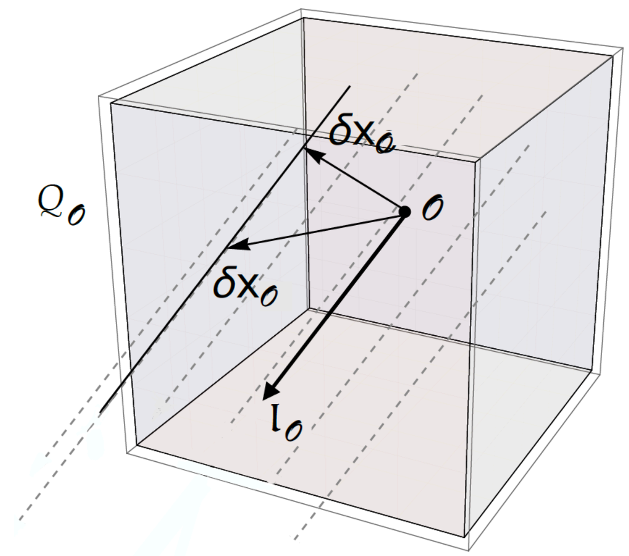

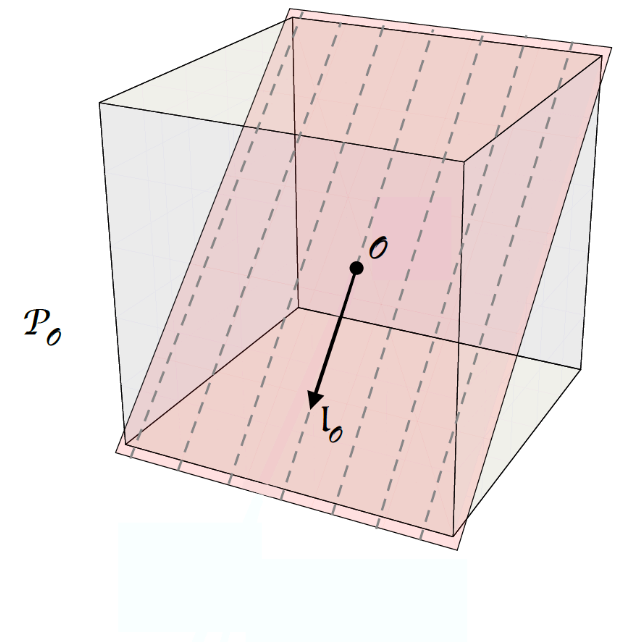

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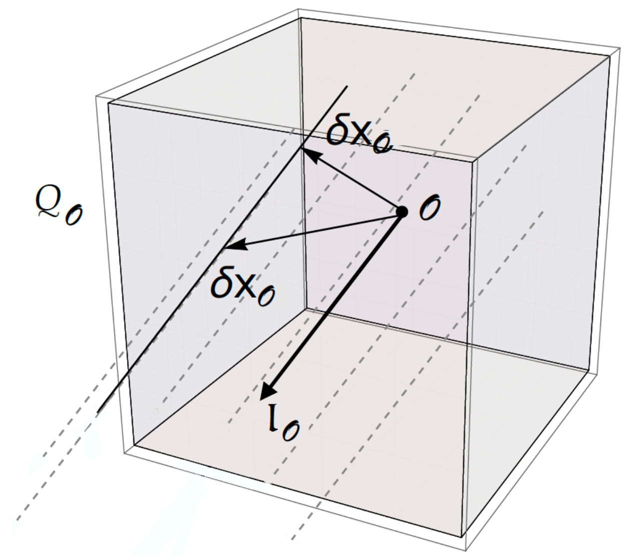

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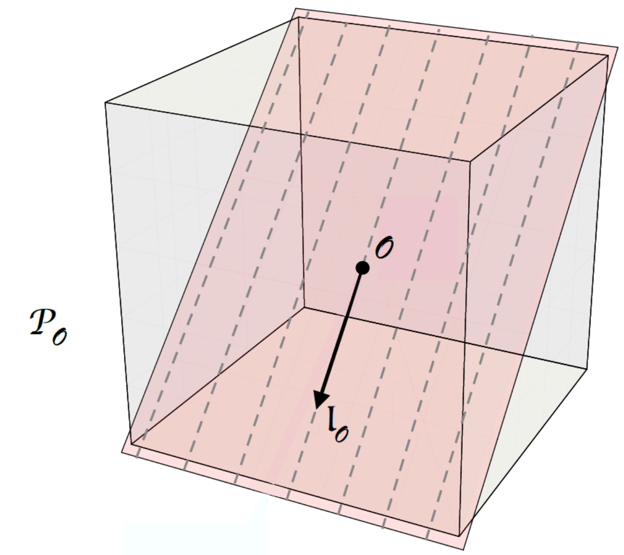

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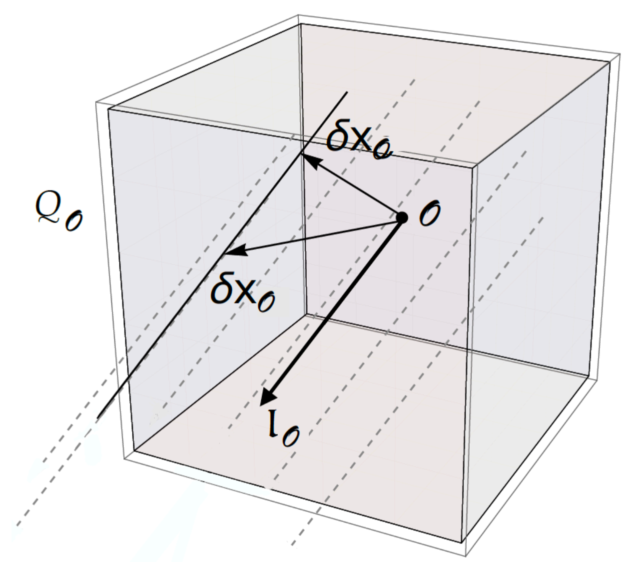

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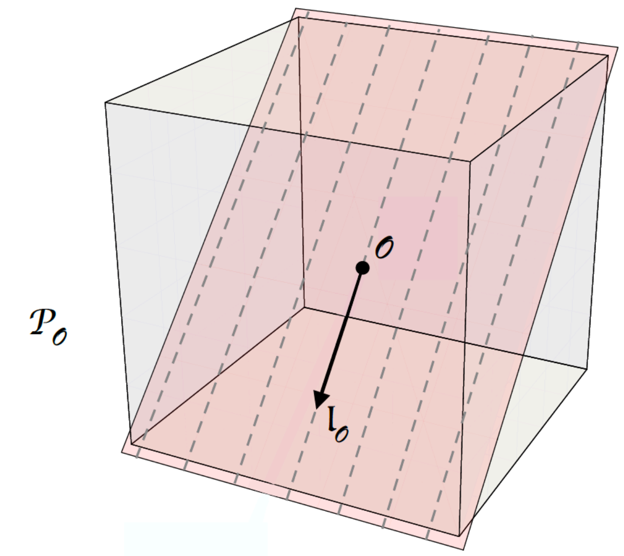

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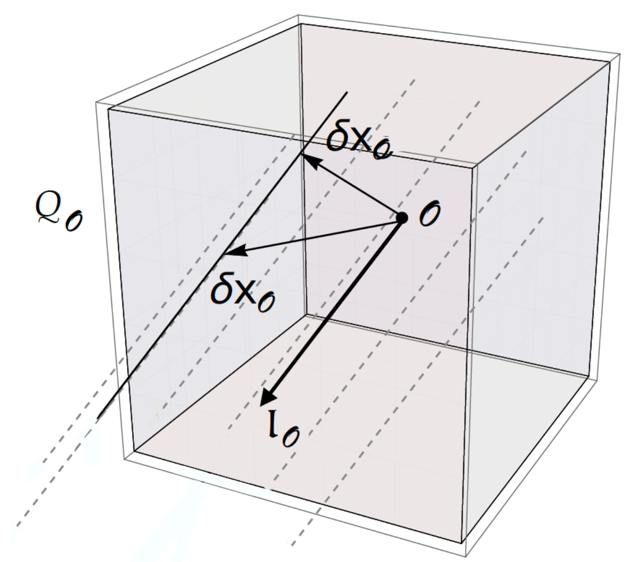

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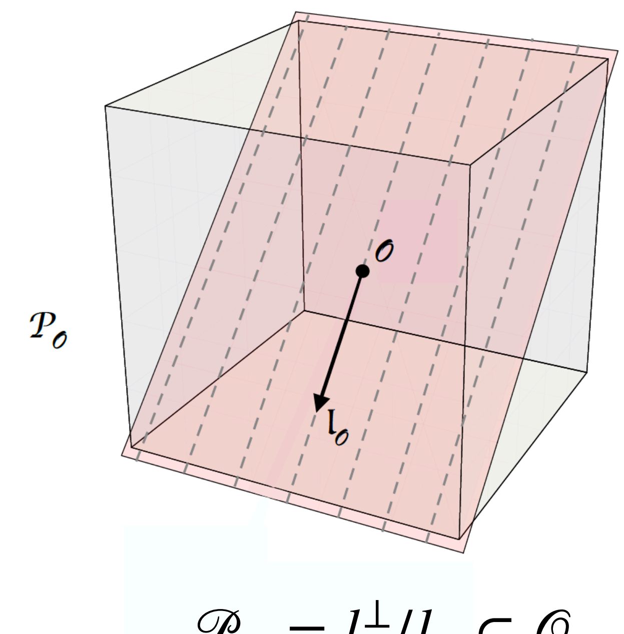

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Helps also with observer invariance of the formalism:

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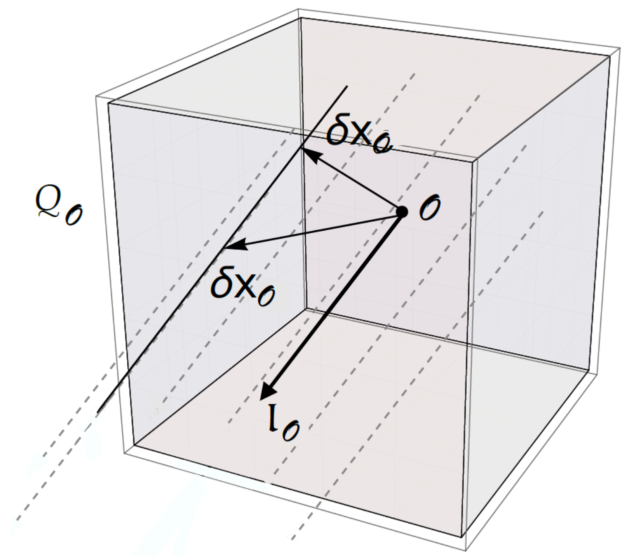

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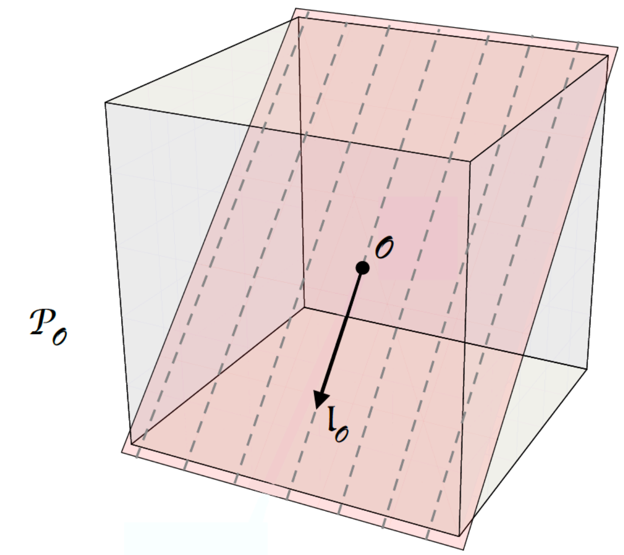


$$\mathcal{P}_{\mathcal{O}} = l^\perp_{\mathcal{O}} / l_{\mathcal{O}} \subset \mathcal{Q}_{\mathcal{O}}$$

$$[\Delta l_{\mathcal{O}}] \in \mathcal{P}_{\mathcal{O}}$$

$$\dim \mathcal{P}_{\mathcal{O}} = 2$$

Helps also with observer invariance of the formalism:

$\mathcal{P}_{\mathcal{O}}$ inherits a positive-definite metric from g

Quotient spaces

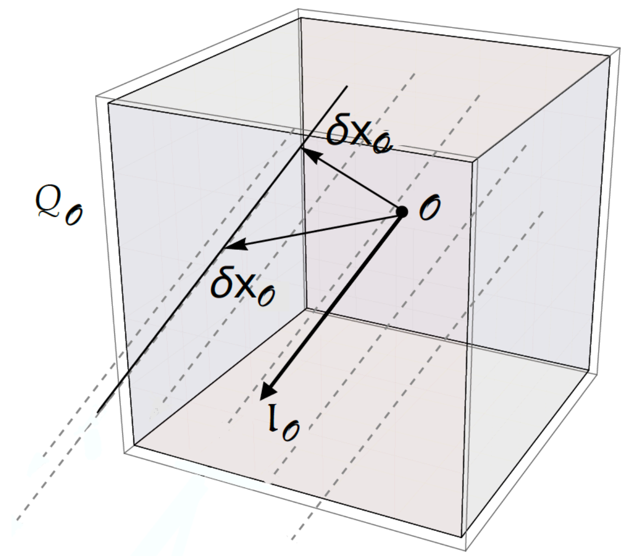
FLA & PRA - components proportional to l^μ drop out

$$\delta r^A(\Delta l_{\mathcal{O}} + C\, l_{\mathcal{O}}) = \delta r^A(\Delta l_{\mathcal{O}})$$

$$(\delta x^\mu_{\mathcal{O}} + D\, l^\mu_{\mathcal{O}})\, l_{\mathcal{O}\,\mu} = \delta x^\mu_{\mathcal{O}}\, l_{\mathcal{O}\,\mu}$$

Components along l^μ = gauge

Forgetting the l^μ component (gauge)



$$\mathcal{Q}_{\mathcal{O}} = T_{\mathcal{O}}M / l_{\mathcal{O}}$$

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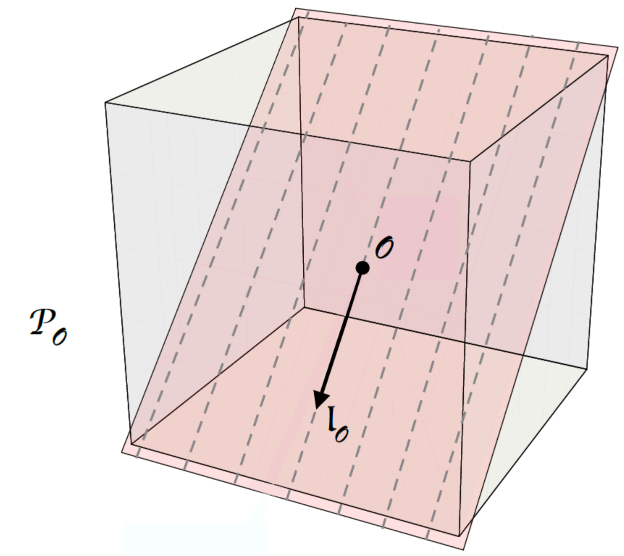

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Distances and angles measured by any observer on his or her screen space (Sachs shadow theorem)

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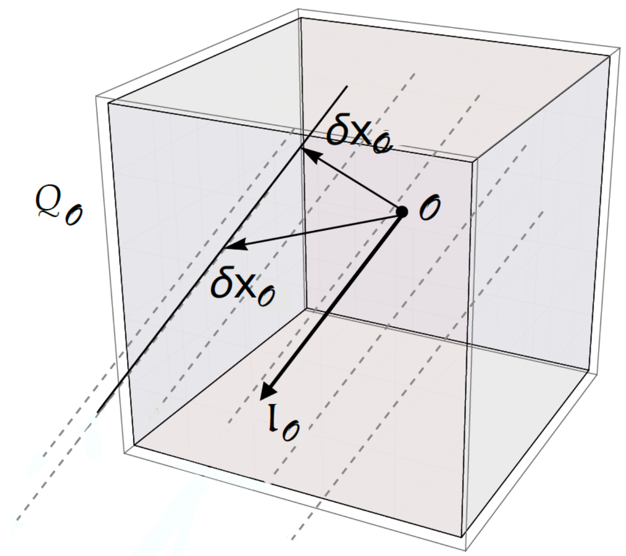

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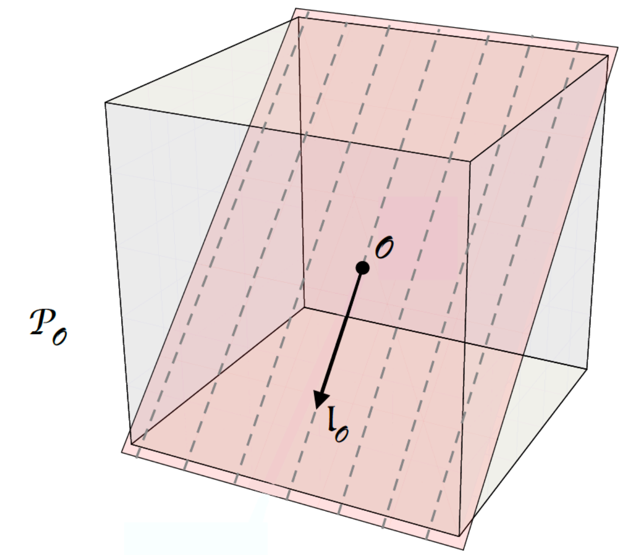

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Helps also with observer invariance of the formalism:

$\mathcal{P}_{\mathcal{O}}$ inherits a positive-definite metric from g

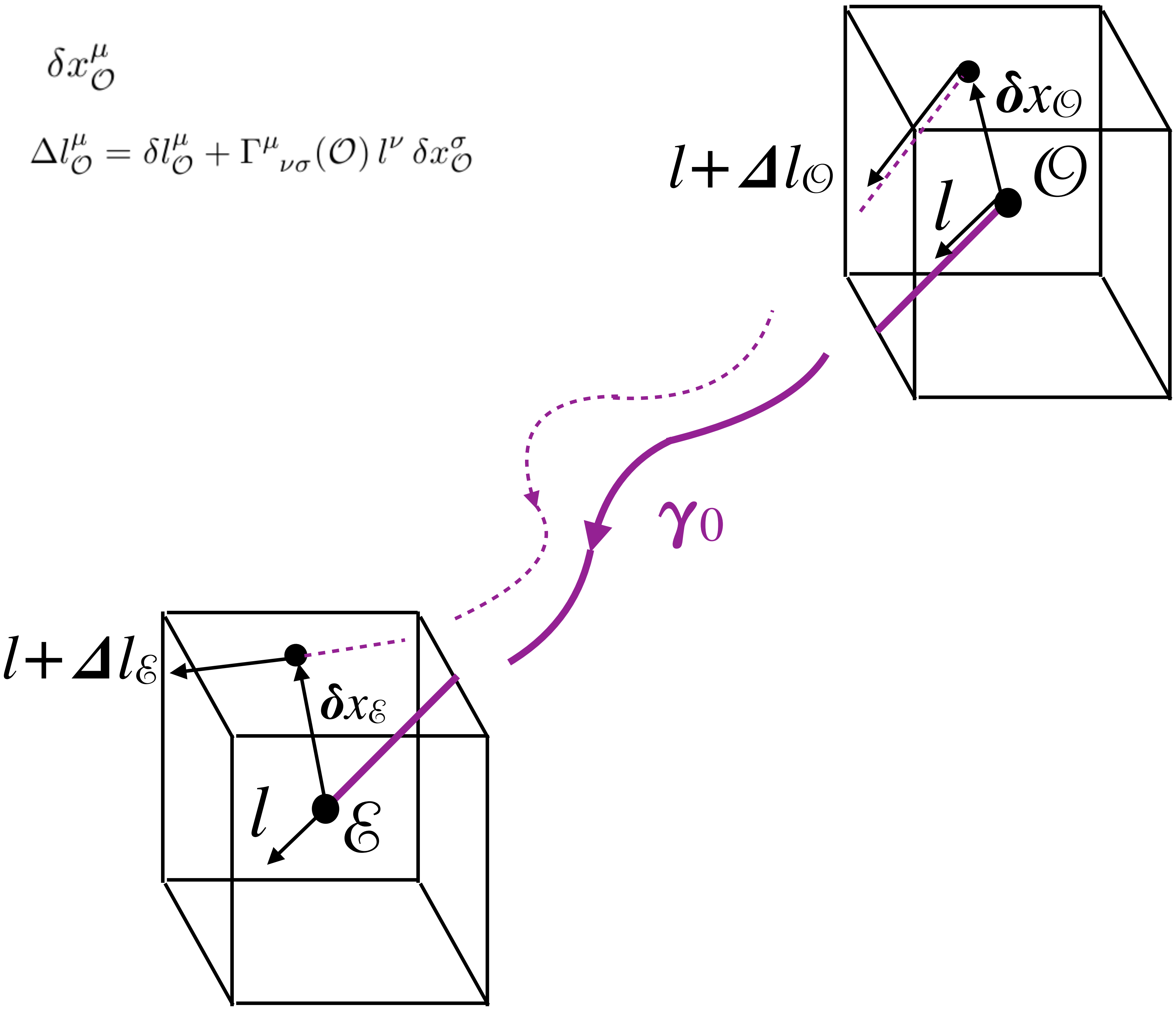
Distances and angles measured by any observer on his or her screen space (Sachs shadow theorem)

Mappings between the quotient spaces have an observer-invariant geometric meaning

Light propagation effects

$\delta x^\mu_\mathcal{O}$

$$\Delta l^\mu_\mathcal{O} = \delta l^\mu_\mathcal{O} + \Gamma^\mu{}_{\nu\sigma}(\mathcal{O})\, l^\nu\, \delta x^\sigma_\mathcal{O}$$



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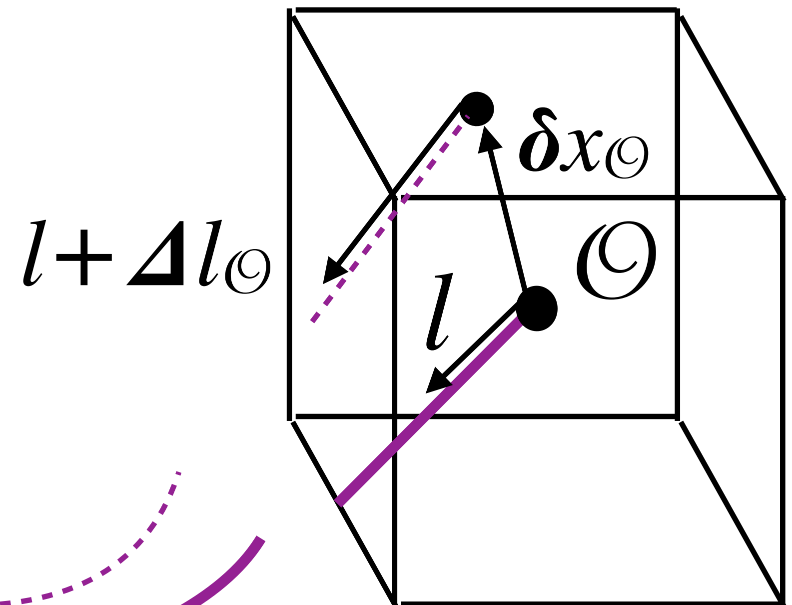

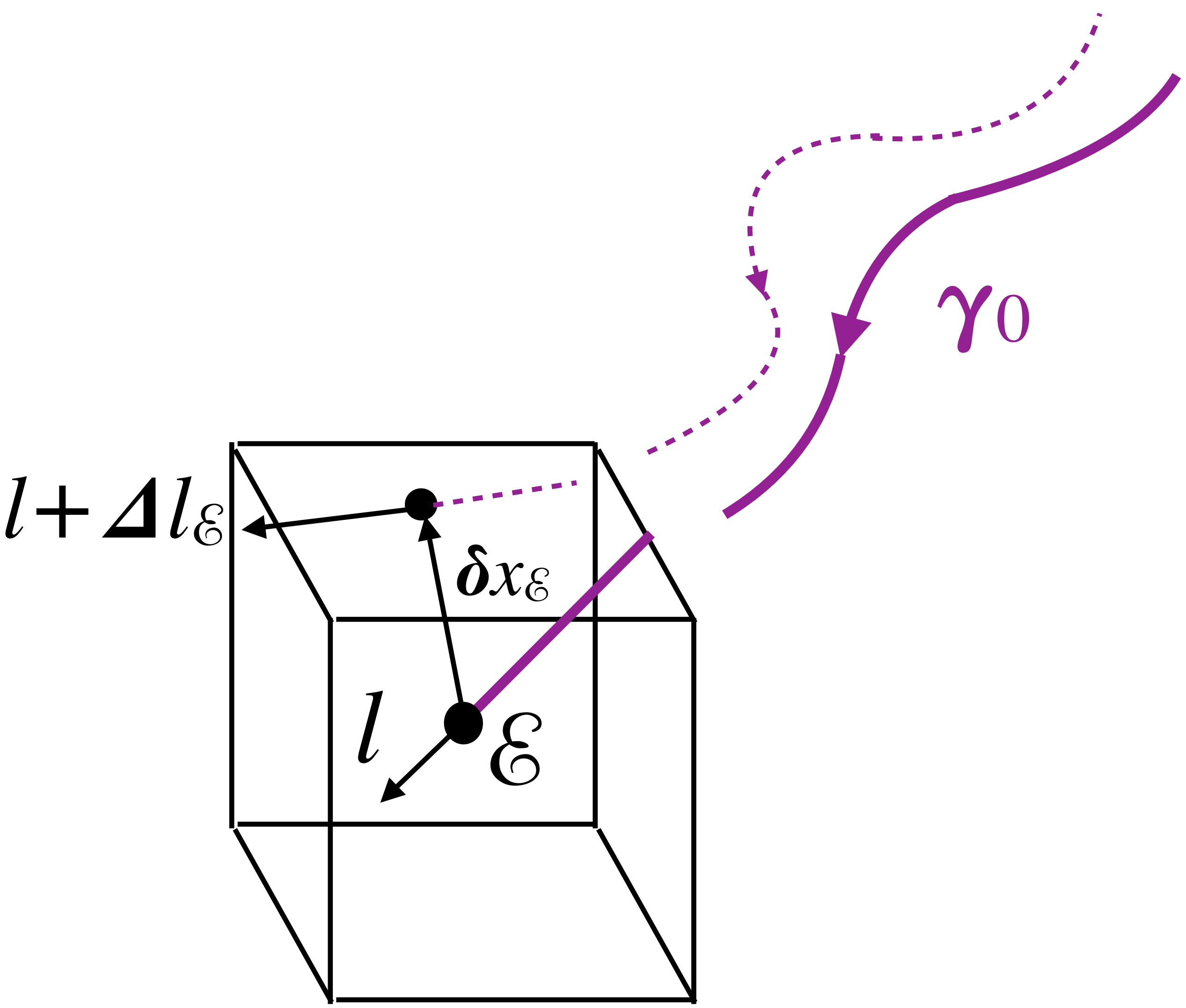

Can be combined into a single object

$$\begin{pmatrix} \delta x^\mu_{\mathcal{E}} \\ \Delta l_{\mathcal{E}\,\nu} \end{pmatrix} = \underbrace{\begin{pmatrix} W_{XX}{}^\mu{}_\rho & W_{XL}{}^{\mu\sigma} \\ W_{LX\,\nu\rho} & W_{LL\,\nu}{}^\sigma \end{pmatrix}}_{\mathcal{W}} \begin{pmatrix} \delta x^\rho_{\mathcal{O}} \\ \Delta l_{\mathcal{O}\,\sigma} \end{pmatrix}$$

Light propagation effects

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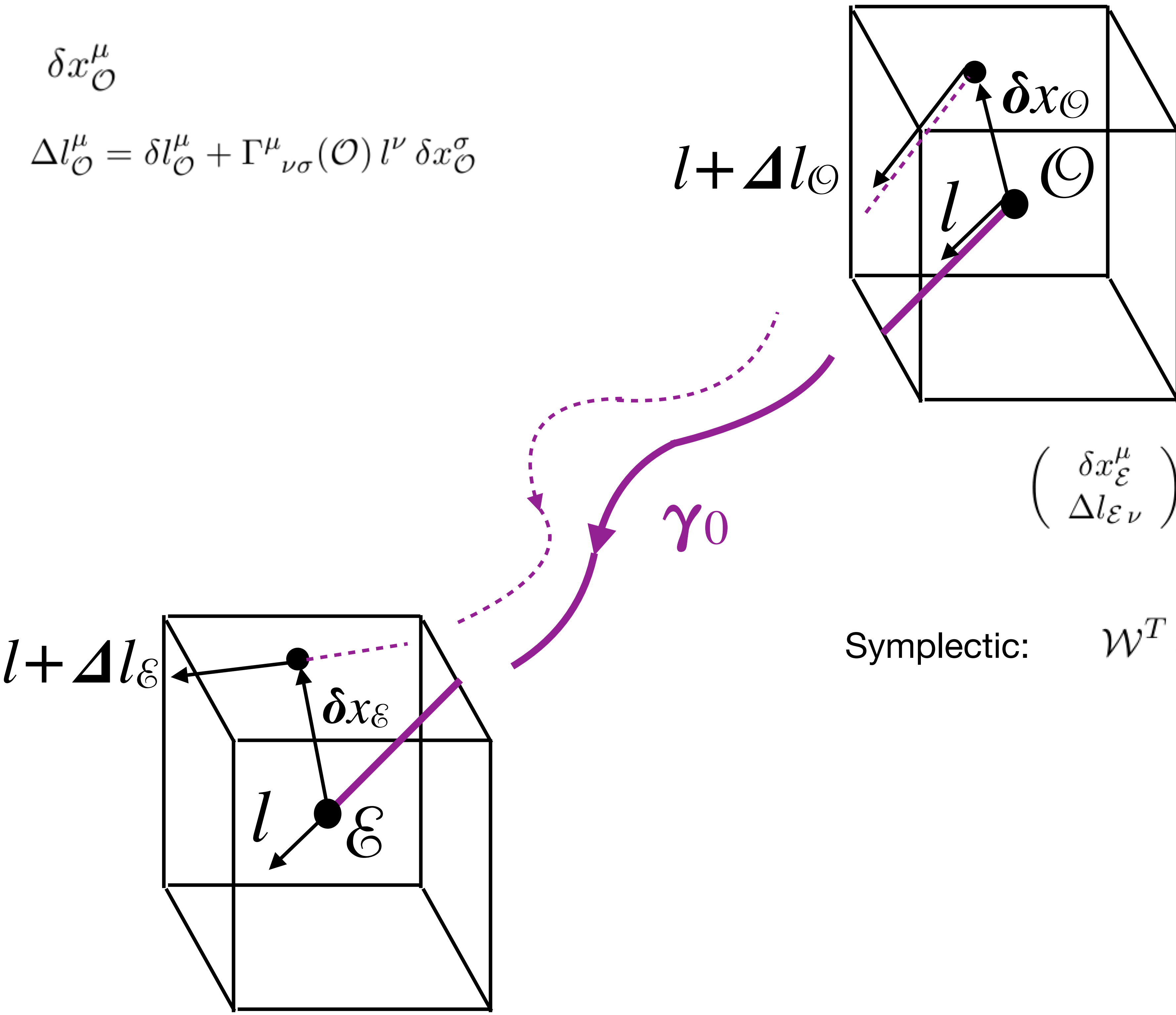

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Symplectic: $\mathcal{W}^T\,\Omega\,\mathcal{W} = \Omega$

Light propagation effects

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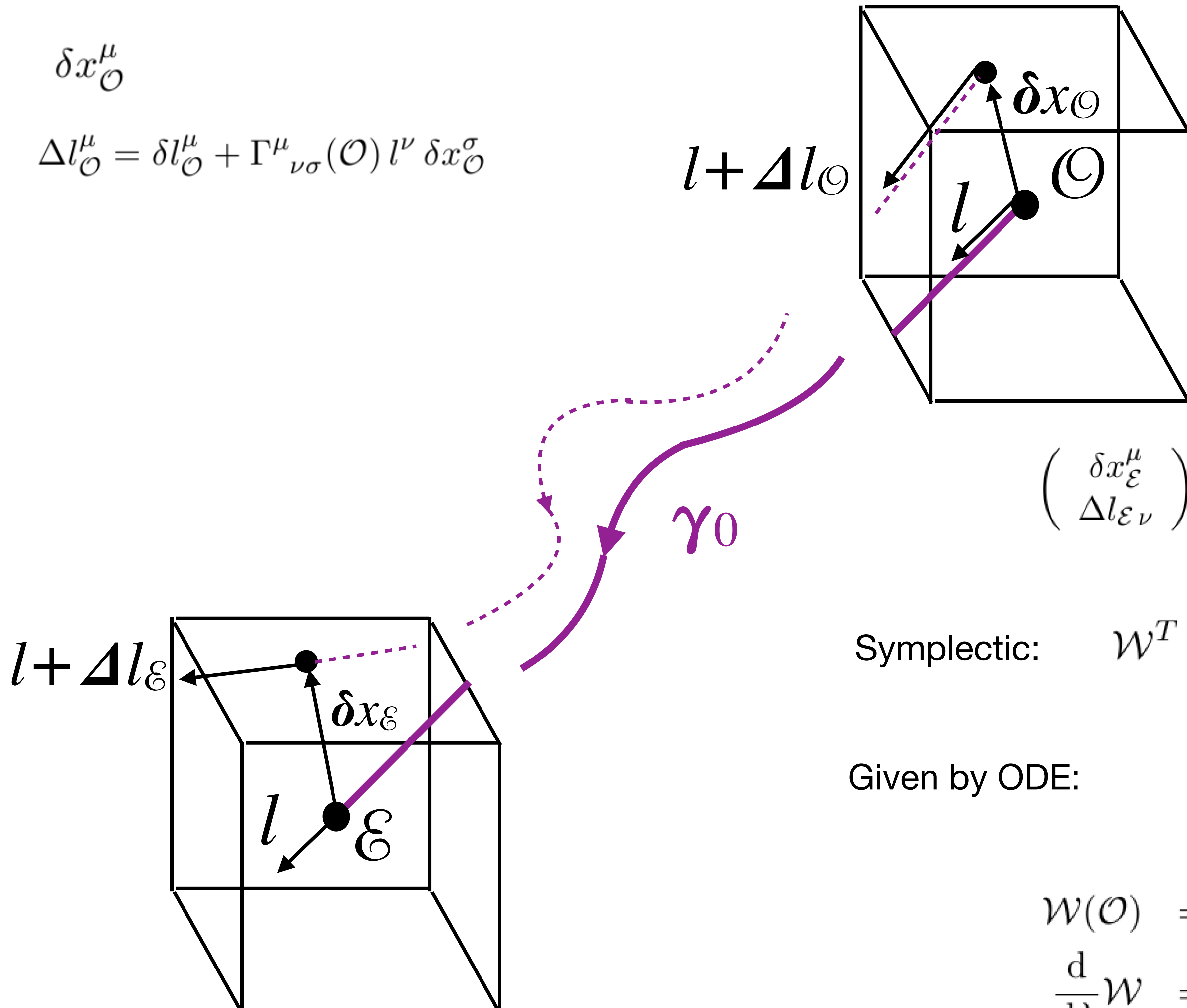

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Symplectic: $\mathcal{W}^{T}\,\Omega\,\mathcal{W} = \Omega$

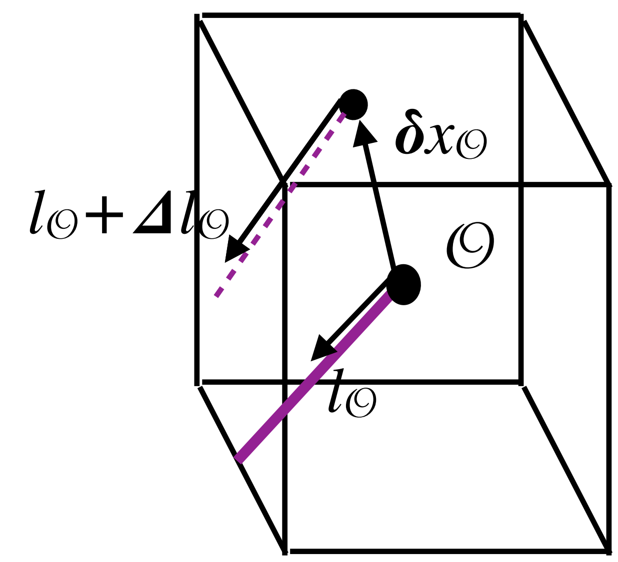
Given by ODE:

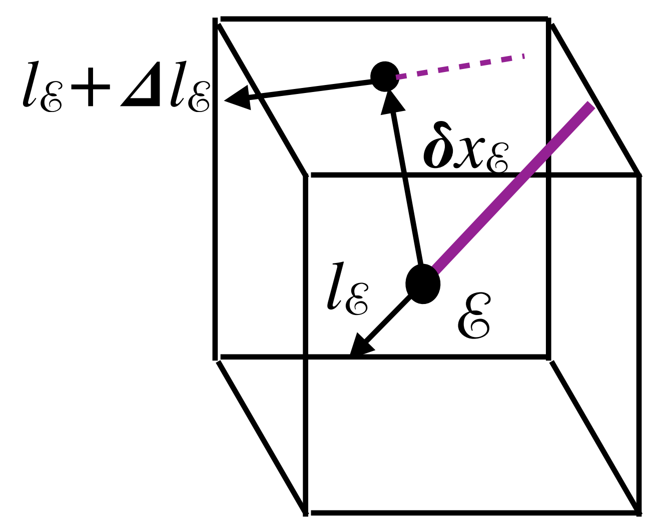
$$\begin{aligned} \mathcal{W}(\mathcal{O}) &= \mathbf{1} \\ \frac{\mathrm{d}}{\mathrm{d}\lambda}\mathcal{W} &= \begin{pmatrix} 0 & g^{\mu\sigma} \\ R_{\nu\alpha\beta\rho}\, l^{\alpha}\, l^{\beta} & 0 \end{pmatrix} \mathcal{W} \end{aligned}$$

Displacement formulas

$$\mathscr{D}([\Delta l_{\mathscr{O}}]) = [\delta x_{\mathscr{E}} - \delta\hat{x}_{\mathscr{O}}] - m([\delta x_{\mathscr{O}}])$$

$$g([\delta x_{\mathscr{O}}], l_{\mathscr{O}}) = g([\delta x_{\mathscr{E}}], l_{\mathscr{E}})$$





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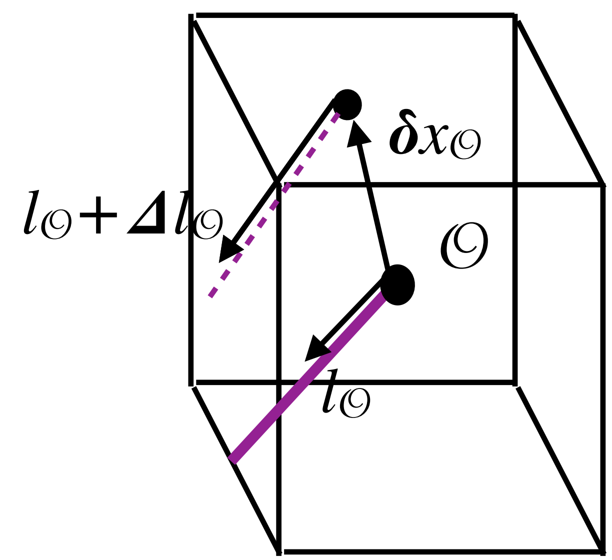
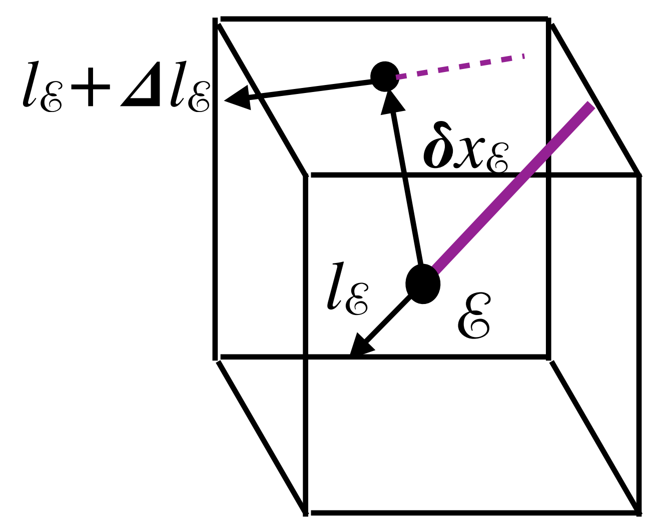


Two parametrizations of displaced geodesics:

endpoint positions	$[\delta x_{\mathscr{O}}]$	$[\delta x_{\mathscr{E}}]$
initial data	$[\delta x_{\mathscr{O}}]$	$[\Delta l_{\mathscr{O}}]$



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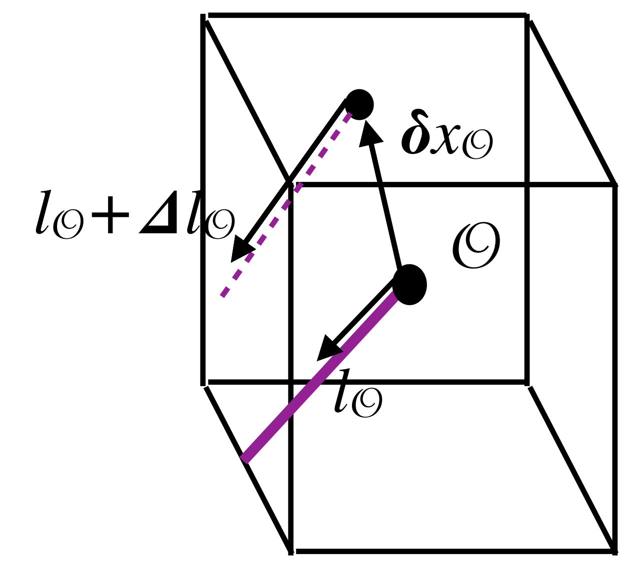
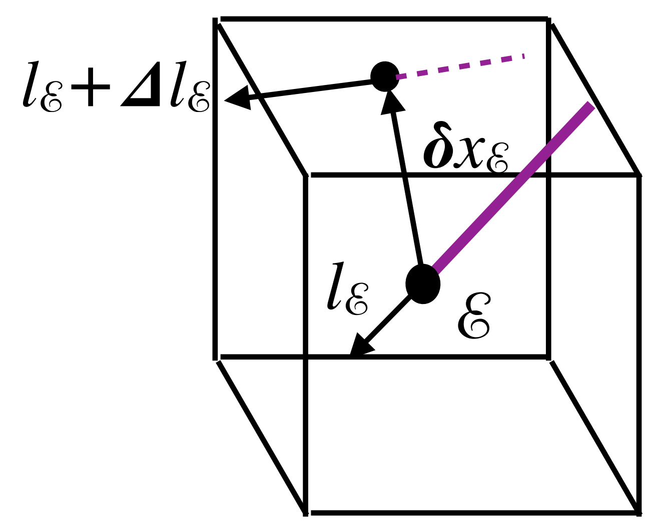


Two parametrizations of displaced geodesics:

endpoint positions $[\delta x_{\mathscr{O}}]$ $[\delta x_{\mathscr{E}}]$

initial data $[\delta x_{\mathscr{O}}]$ $[\Delta l_{\mathscr{O}}]$



Dimensions:

$[\delta x_{\mathscr{O}}] \in \mathscr{Q}_{\mathscr{O}} \Rightarrow 3$ $\qquad$ $[\delta x_{\mathscr{O}}] \in \mathscr{Q}_{\mathscr{O}} \Rightarrow 3$

$[\delta x_{\mathscr{E}}] \in \mathscr{Q}_{\mathscr{E}} \Rightarrow 3$ $\qquad$ $[\Delta l_{\mathscr{O}}] \in \mathscr{P}_{\mathscr{O}} \Rightarrow 2$

time lapse condition -1

Displacement formulas

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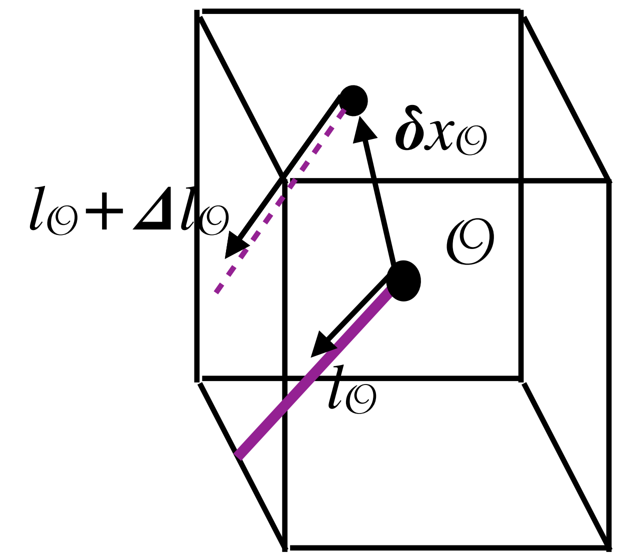

Two parametrizations of displaced geodesics:

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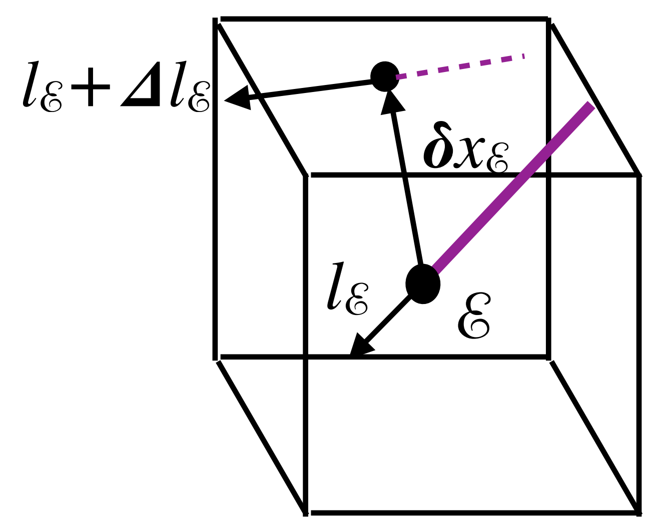

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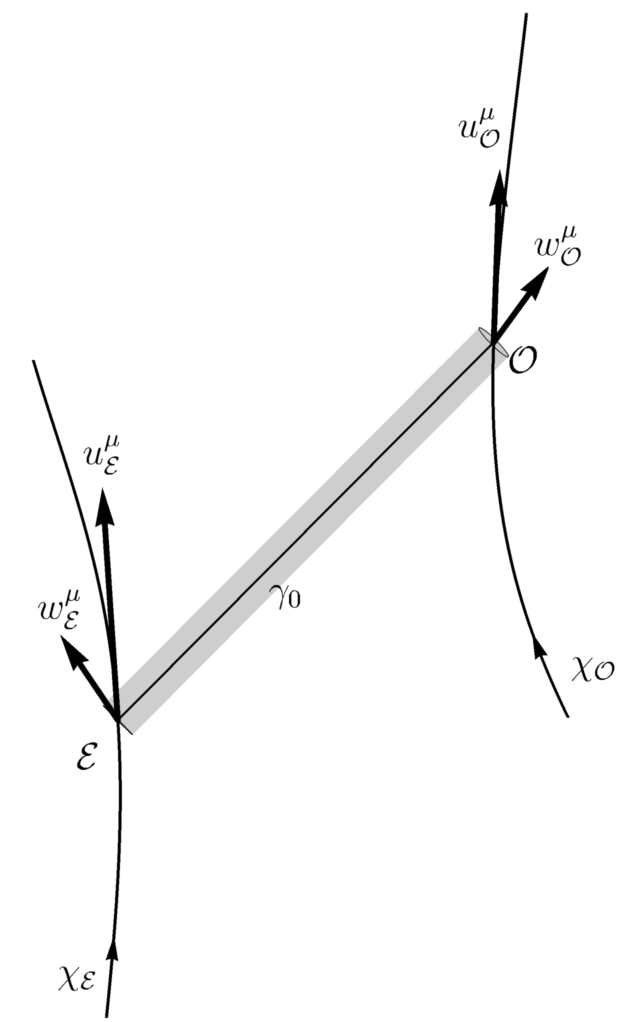
time lapse condition -1

total dimension = 5

Position drift (proper motion)



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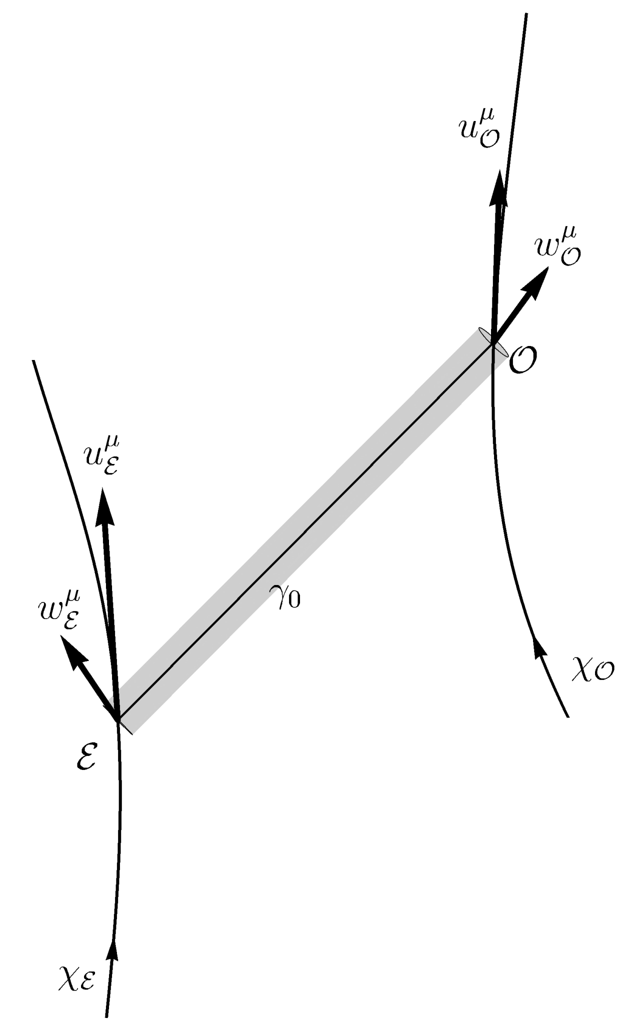

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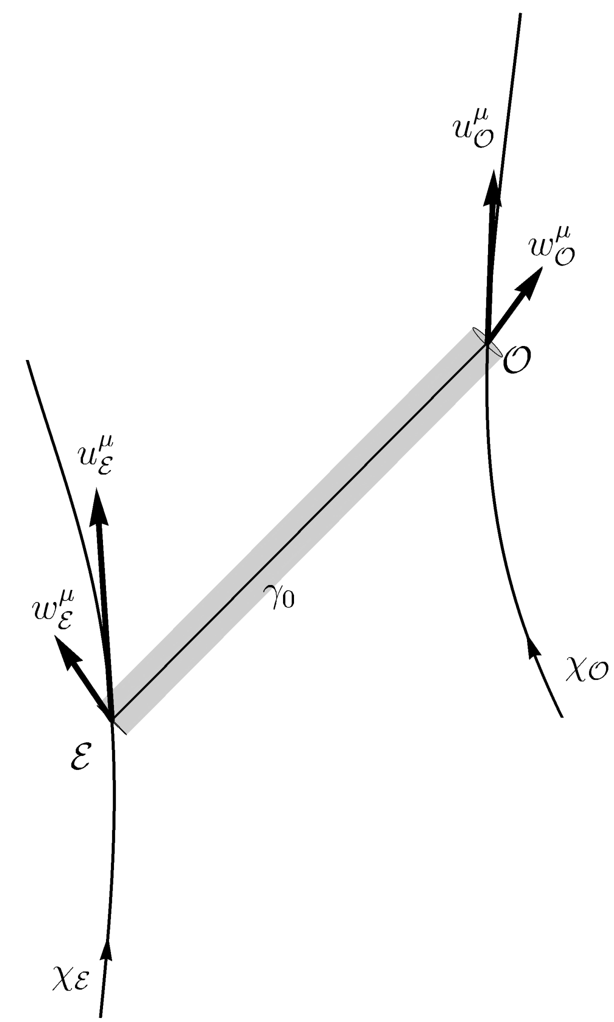
$$\frac{\mathrm{d}\tau_{\mathcal{O}}}{\mathrm{d}\tau_{\mathcal{E}}} = \frac{l_\sigma\, u_{\mathcal{E}}^\sigma}{l_\rho\, u_{\mathcal{O}}^\rho} = \frac{1}{1+z}$$

$$\frac{\Delta l_{\mathcal{O}}^A}{\mathrm{d}\tau_{\mathcal{O}}} = {\mathcal{D}^{-1}}^A{}_B \left(\left(\frac{1}{1+z} u_{\mathcal{E}} - \hat{u}_{\mathcal{O}} \right)^B - m^B{}_\mu\, u_{\mathcal{O}}^\mu \right)$$



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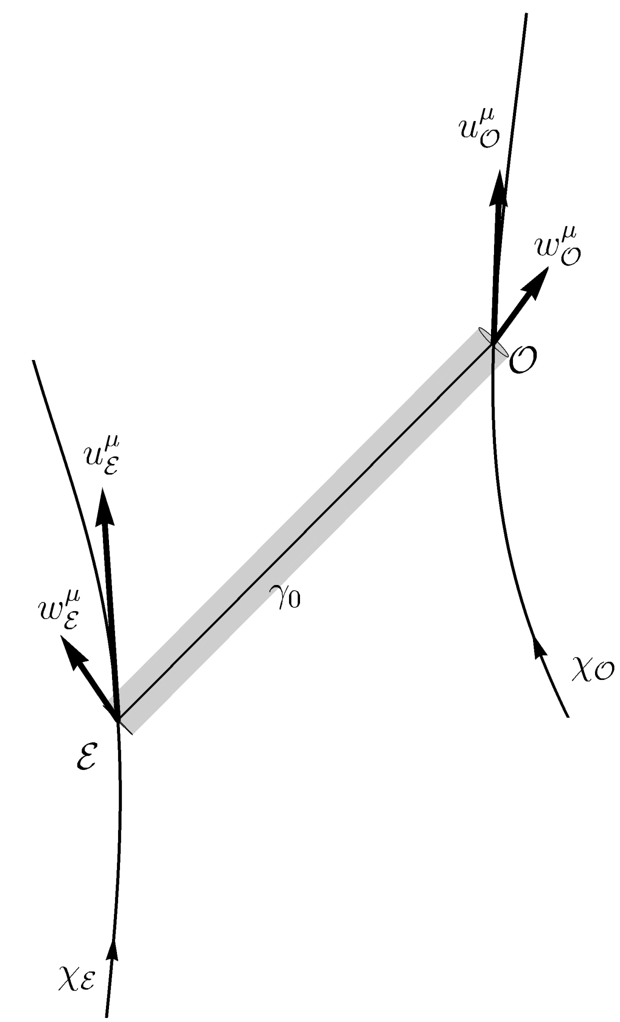
$$\delta_{\mathcal{O}} r^A = w^A_{\mathcal{O}} + \frac{1}{l_\sigma\, u^\sigma_{\mathcal{O}}} {\mathcal{D}^{-1}}^A{}_B \left(\left(\frac{1}{1+z} u_{\mathcal{E}} - \hat{u}_{\mathcal{O}} \right)^B - m^B{}_\mu\, u^\mu_{\mathcal{O}} \right)$$



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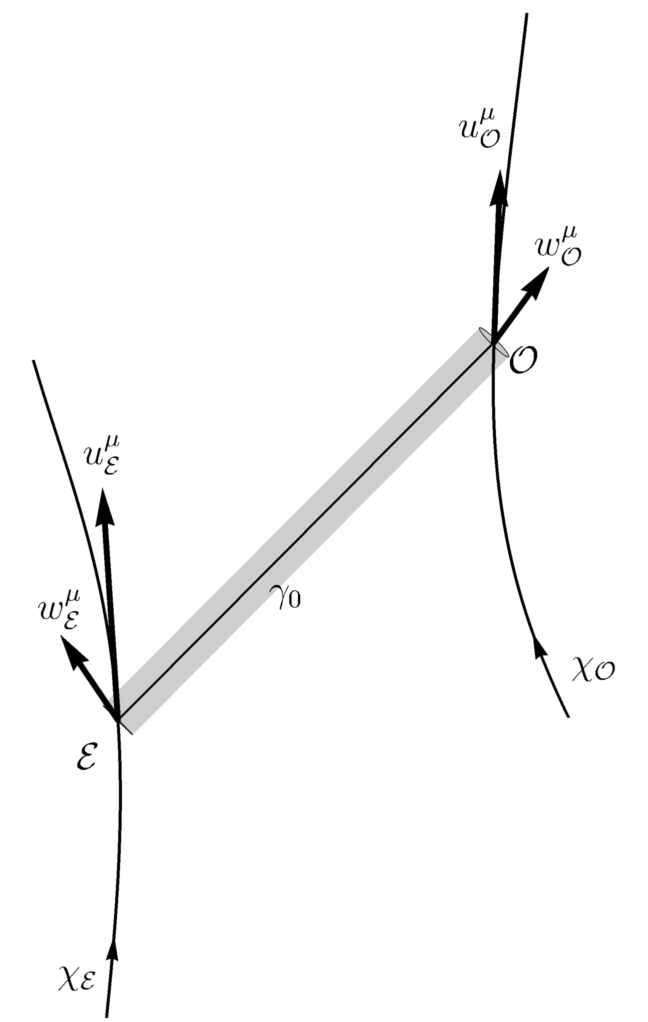

An exact formula for the position drift rate in any curved space

Position drift (proper motion)

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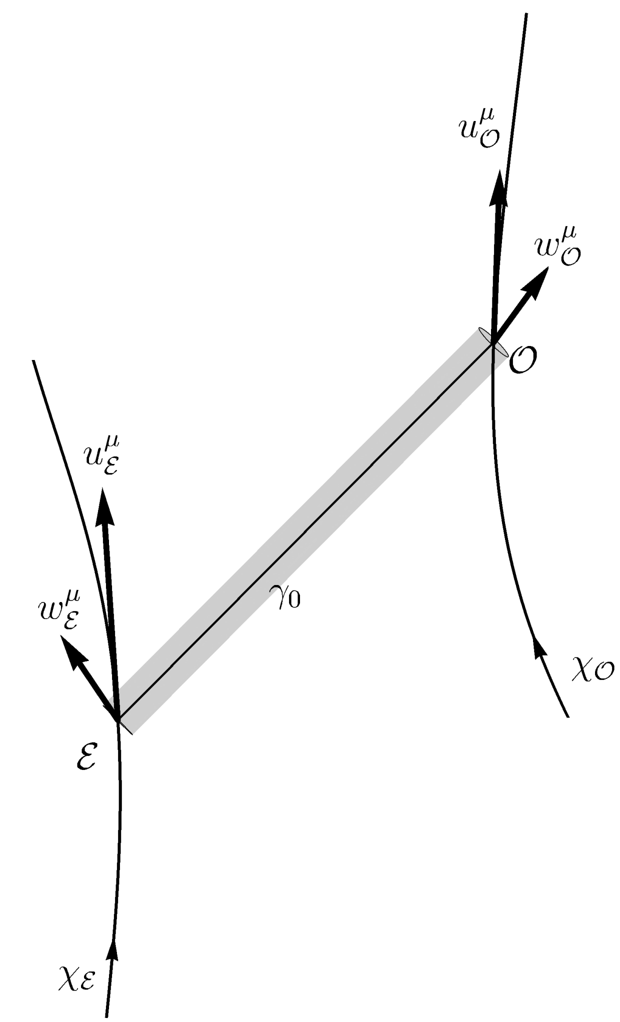

An exact formula for the position drift rate in any curved space

Data: curvature along the line of sight + kinematical variables

Position drift (proper motion)

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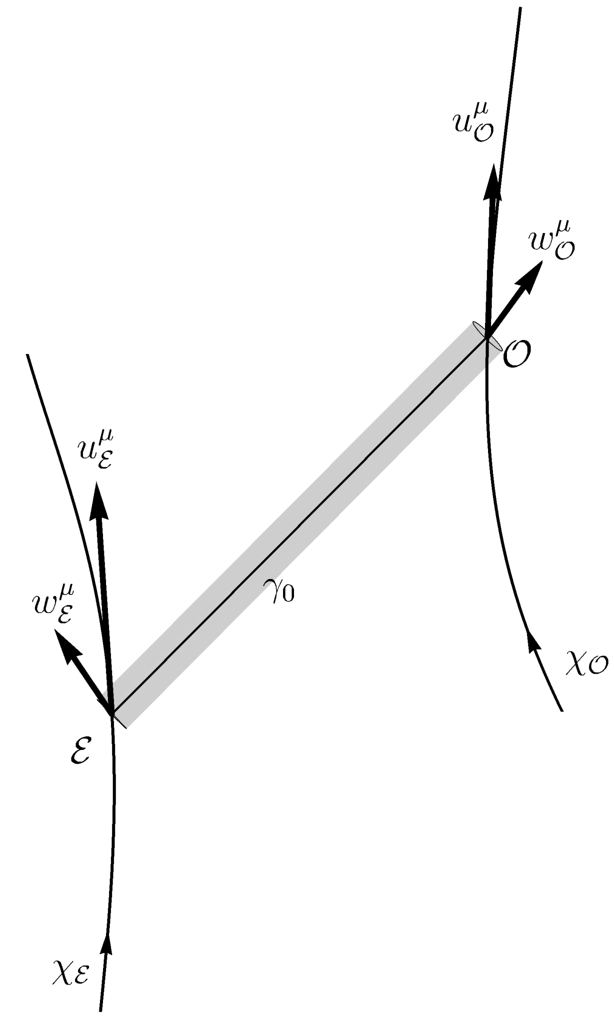

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An exact formula for the position drift rate in any curved space

Data: curvature along the line of sight + kinematical variables

Interesting physical consequences…

Position drift (proper motion)

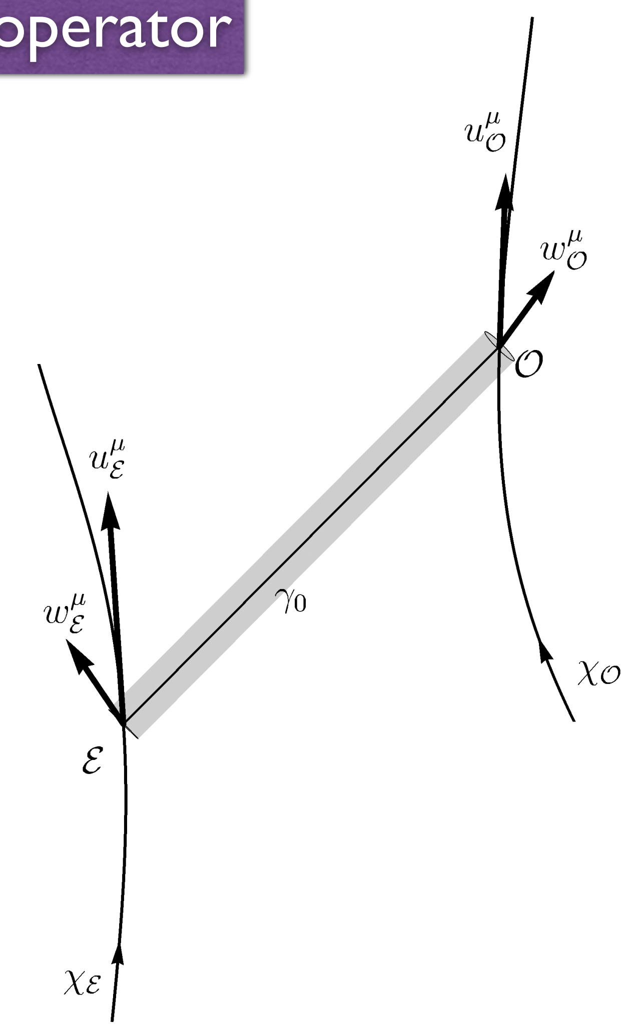
aberration drift

lensing

transverse
4-velocity difference

E-O asymmetry operator

$$\delta_{\mathcal{O}} r^A = w_{\mathcal{O}}^A + \frac{1}{l_{\mathcal{O}\,\sigma}\, u_{\mathcal{O}}^{\sigma}}\, {\mathcal{D}^{-1}}^A{}_B \left(\left(\frac{1}{1+z}\, u_{\mathcal{E}} - \hat{u}_{\mathcal{O}} \right)^B - m^B{}_\mu\, u_{\mathcal{O}}^\mu \right)$$



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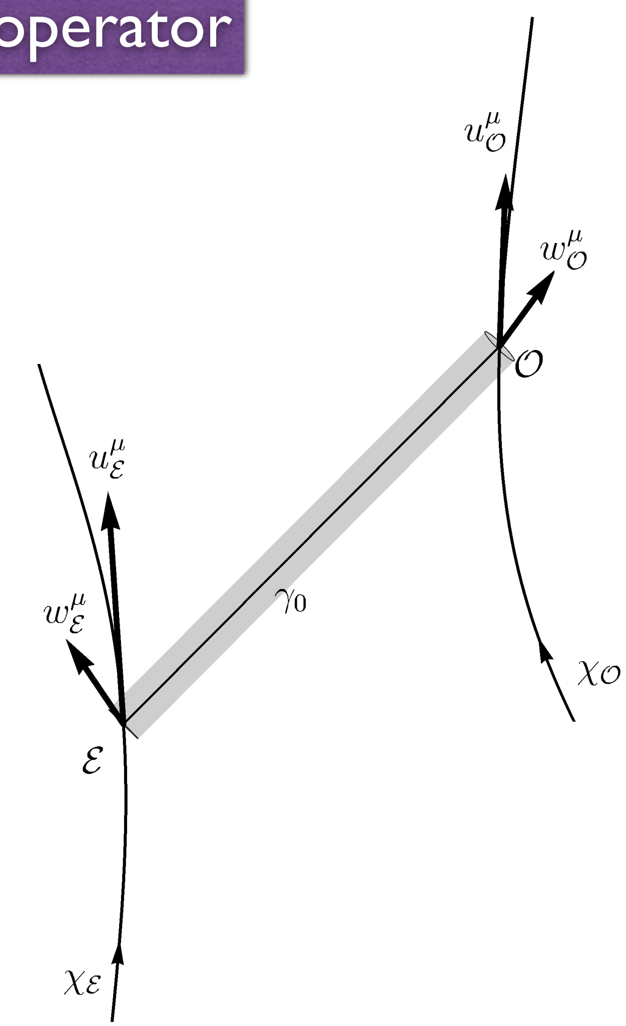
automatically orthogonal to $l_{\mathscr{E}}$

$\in \mathscr{P}_{\mathscr{E}}$



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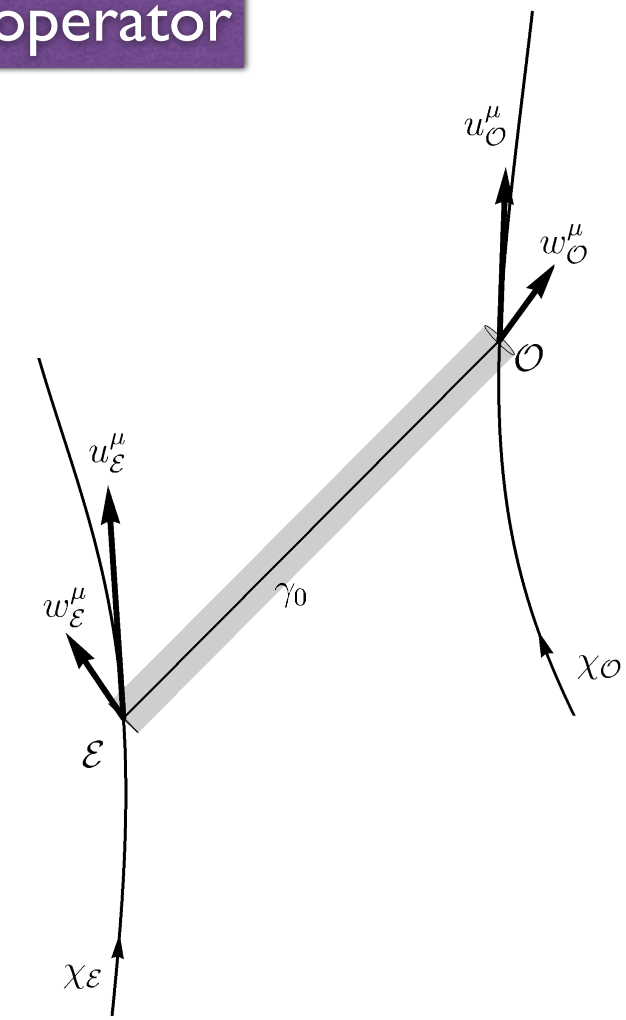

Compare with the flat case

$$\delta_{\mathcal{O}} r^A = w^A_{\mathcal{O}} + \frac{1}{D_{\mathcal{O}}} \left(\frac{1}{1+z}\, u_{\mathcal{E}} - u_{\mathcal{O}} \right)^A$$

Position drift (proper motion)

aberration drift

lensing

transverse 4-velocity difference

E-O asymmetry operator

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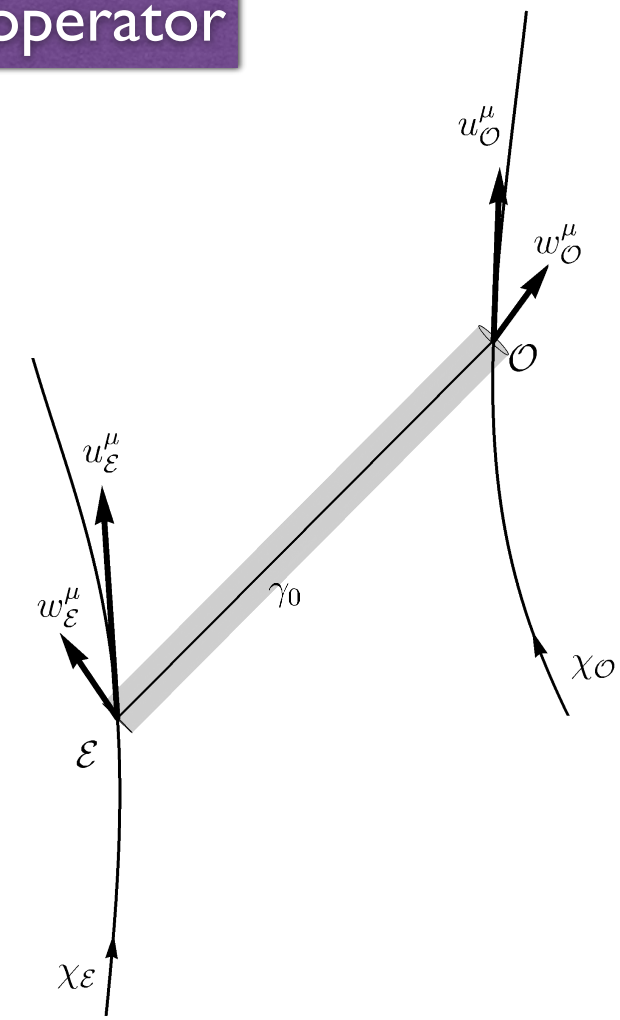

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- Apparent superluminal motions

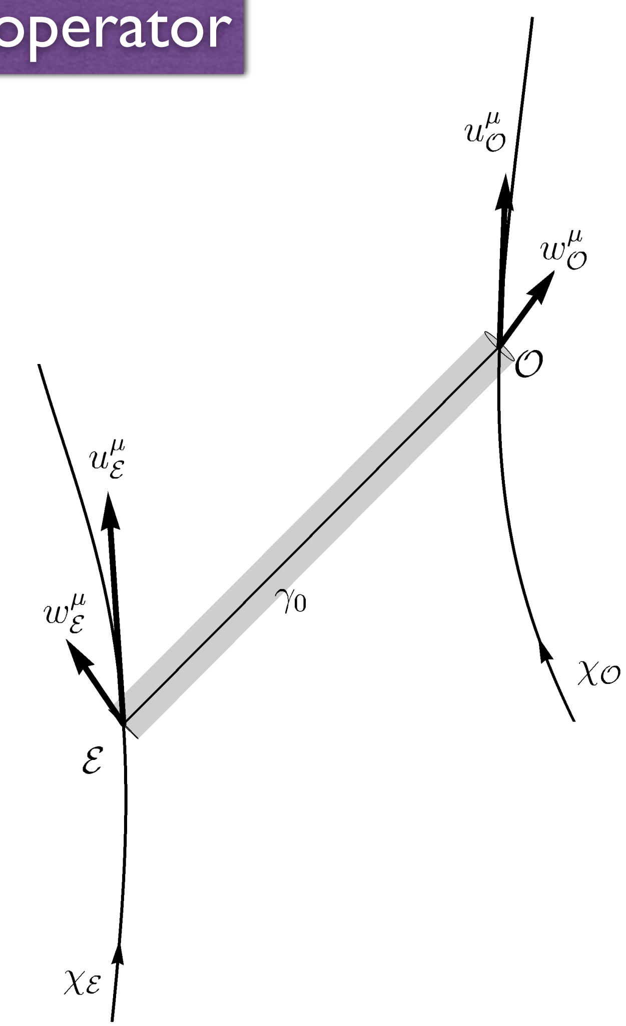
Position drift (proper motion)

aberration drift | lensing | transverse 4-velocity difference | E-O asymmetry operator

$$\delta_{\mathcal{O}} r^A = w^A_{\mathcal{O}} + \frac{1}{l_{\mathcal{O}\,\sigma}\, u^\sigma_{\mathcal{O}}}\, \mathcal{D}^{-1\,A}{}_B \left(\left(\frac{1}{1+z}\, u_{\mathcal{E}} - \hat{u}_{\mathcal{O}} \right)^B - m^B{}_\mu\, u^\mu_{\mathcal{O}} \right)$$

automatically orthogonal to $l_{\mathcal{E}}$

$\in \mathcal{P}_{\mathcal{E}}$



Compare with the flat case

$$\delta_{\mathcal{O}} r^A = w^A_{\mathcal{O}} + \frac{1}{D_{\mathcal{O}}} \left(\frac{1}{1+z}\, u_{\mathcal{E}} - u_{\mathcal{O}} \right)^A$$

- Apparent superluminal motions

$\dfrac{1}{1+z}$ large (strong blueshift)

Position drift (proper motion)

aberration drift | lensing | transverse 4-velocity difference | E-O asymmetry operator

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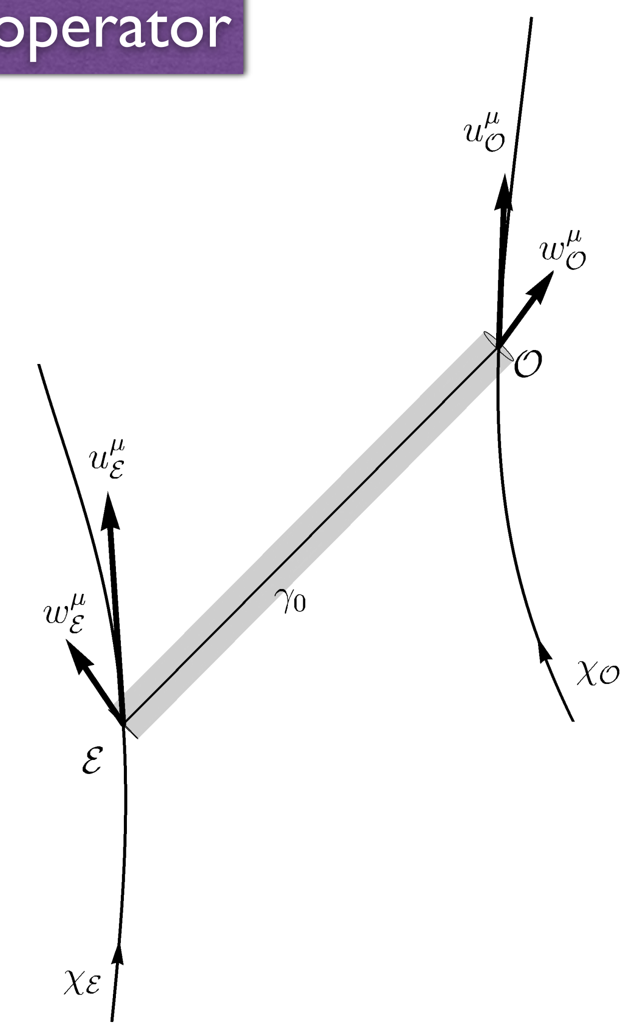

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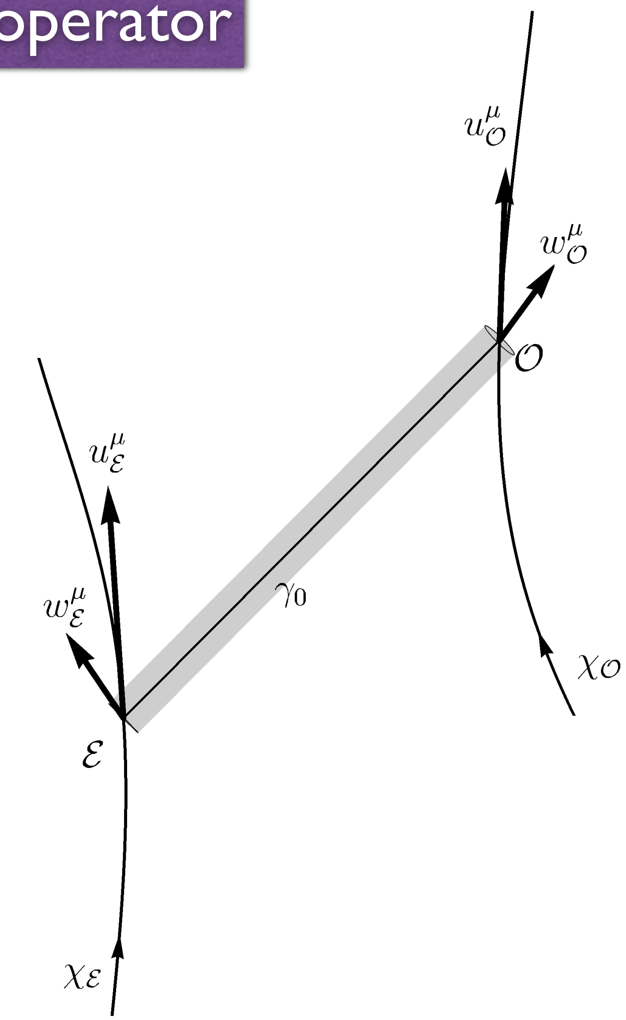

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$$\delta_{\mathcal{O}} r^A = w^A_{\mathcal{O}} + \frac{1}{D_{\mathcal{O}}} \left(\frac{1}{1+z}\, u_{\mathcal{E}} - u_{\mathcal{O}} \right)^A$$

- Apparent superluminal motions

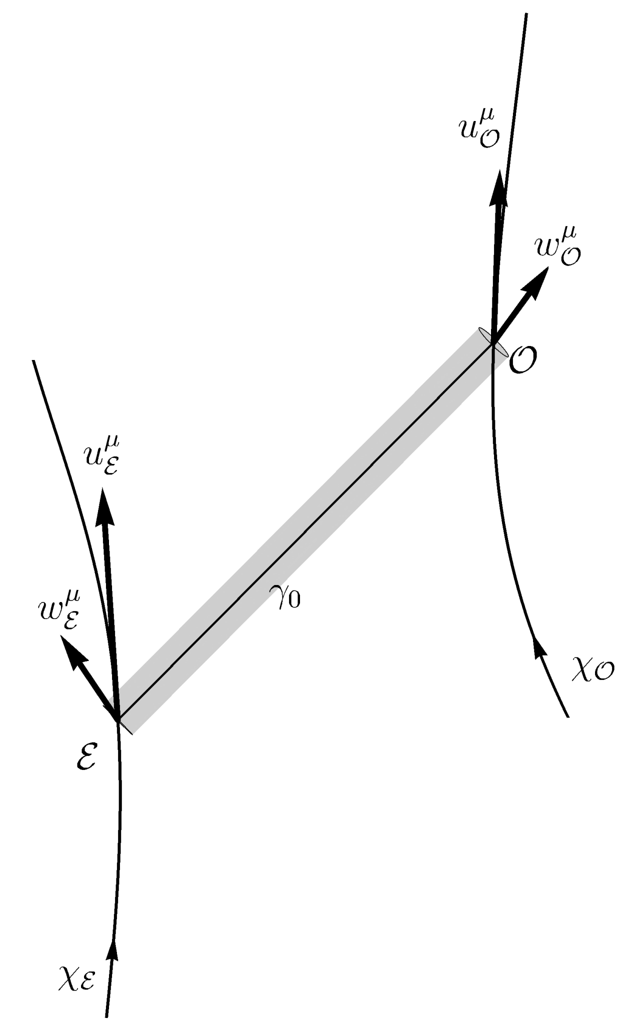
$\frac{1}{1+z}$ large (strong blueshift)

transverse velocity difference

$\Rightarrow$ $\delta_{\mathcal{O}} r^A$ large

Position drift (proper motion)

$$\delta_{\mathcal{O}} r^A = w_{\mathcal{O}}^A + \frac{1}{l_{\mathcal{O}\,\sigma} u_{\mathcal{O}}^{\sigma}} \mathcal{D}^{-1\,A}{}_B \left(\left(\frac{1}{1+z} u_{\mathcal{E}} - \hat{u}_{\mathcal{O}} \right)^B - m^B{}_\mu u_{\mathcal{O}}^\mu \right)$$



Position drift (proper motion)

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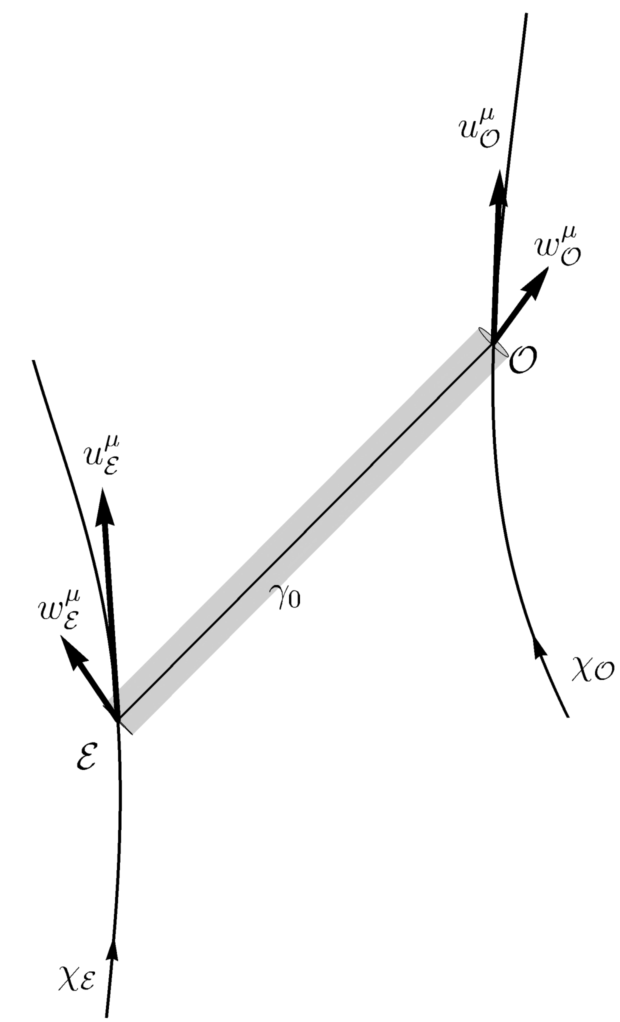

- Relation between lensing and position drift

Position drift (proper motion)

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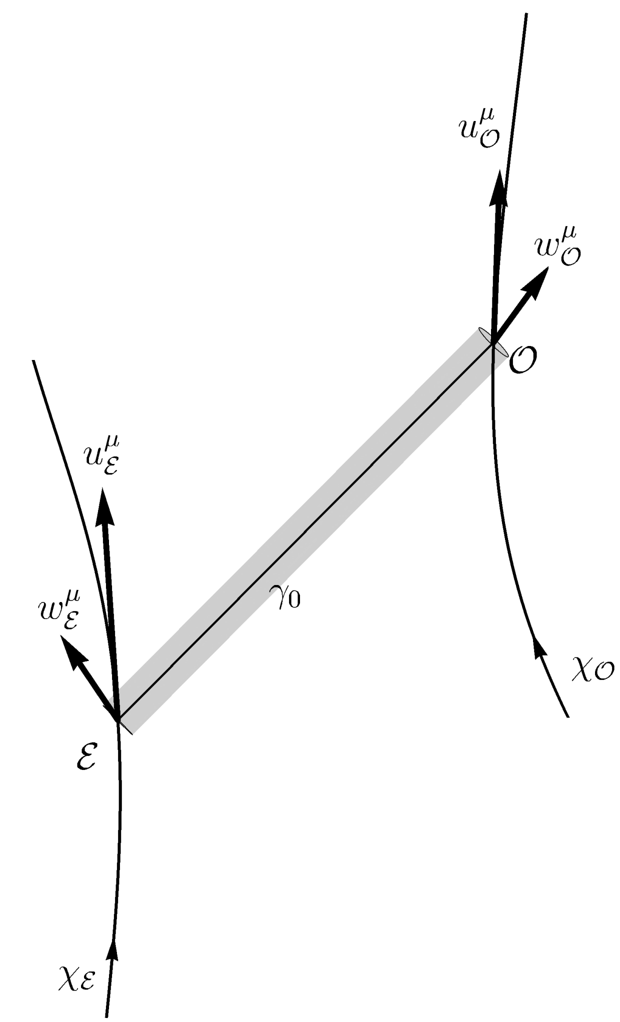

- Relation between lensing and position drift

  strong magnification = faster position drift rates

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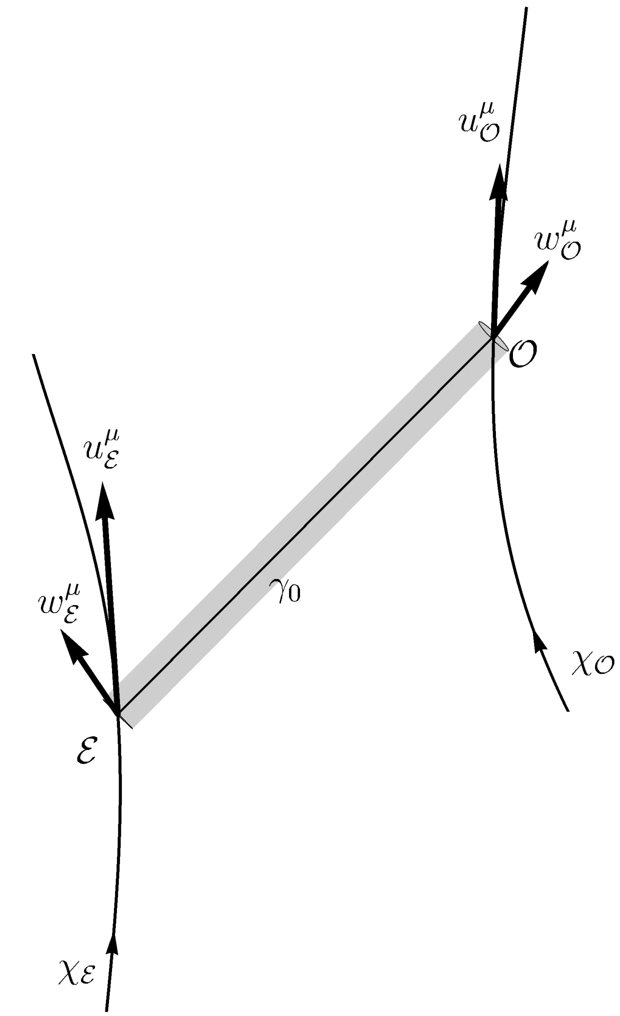
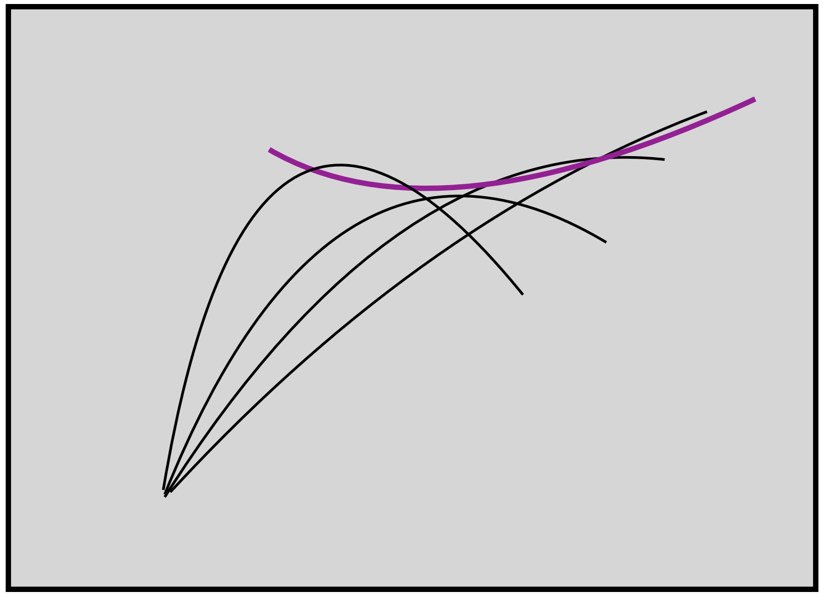


- Relation between lensing and position drift

  strong magnification = faster position drift rates

- Close to a caustic



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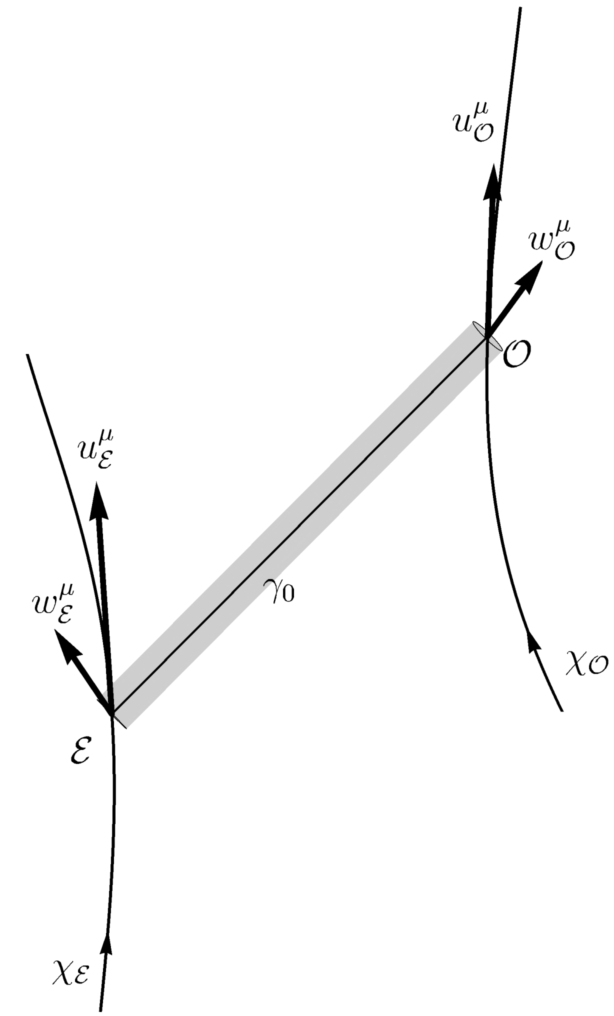

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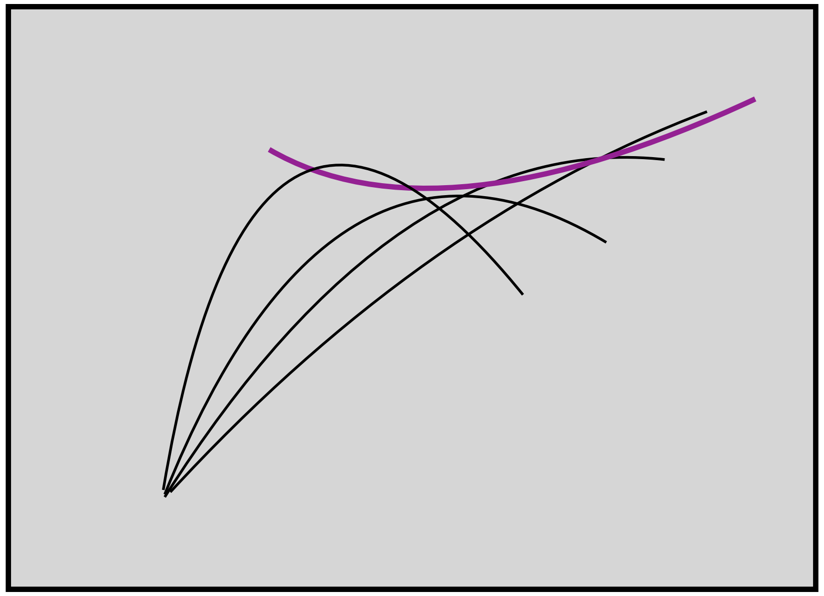

  $\mathcal{D}^A{}_B$ becomes degenerate

Position drift (proper motion)

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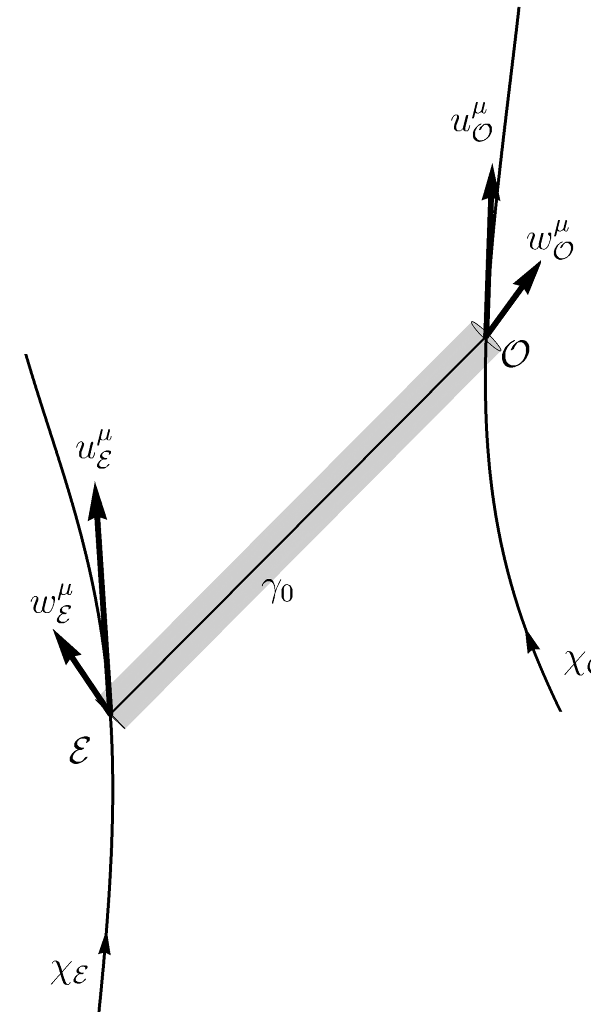

- Relation between lensing and position drift

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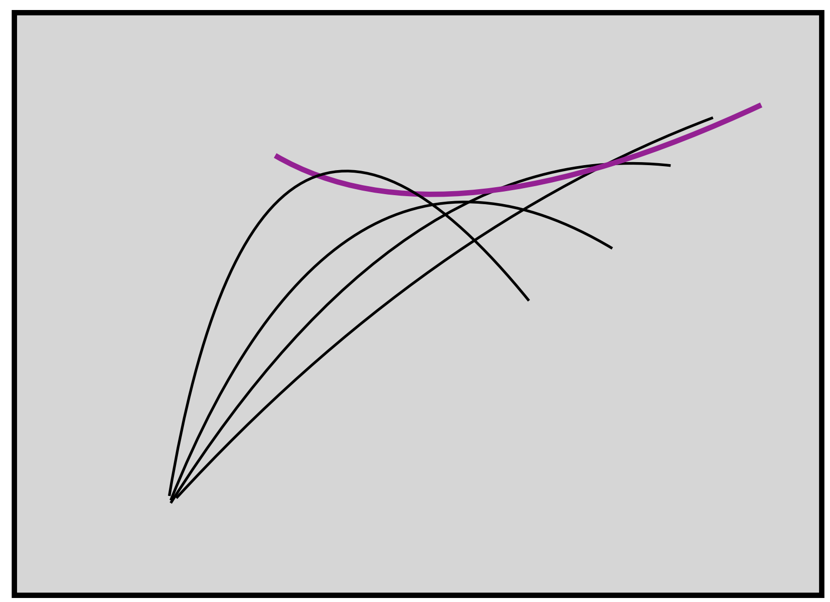
- Close to a caustic

  $\mathscr{D}^A{}_B$ becomes degenerate

  $M^A{}_B$ diverges (infinite magnification)



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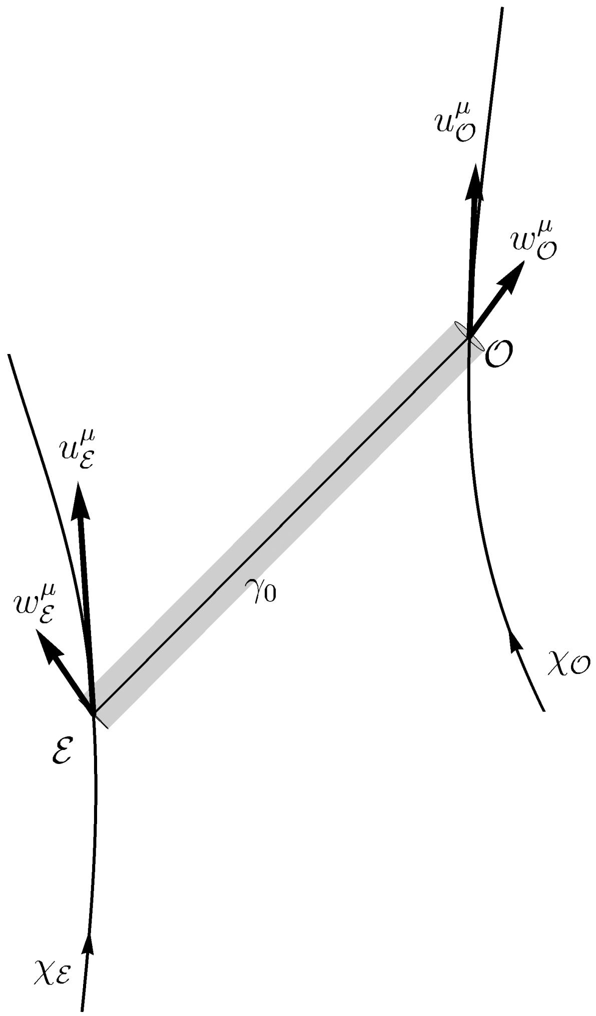

- Relation between lensing and position drift

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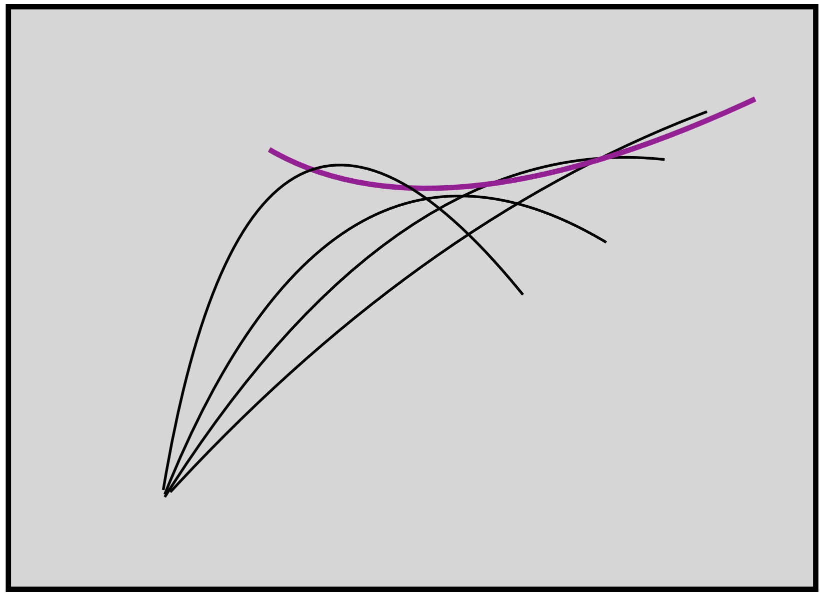
  $M^A{}_B$ diverges (infinite magnification)

  $\boldsymbol{\Pi}^A{}_B$ diverges (infinite parallax)



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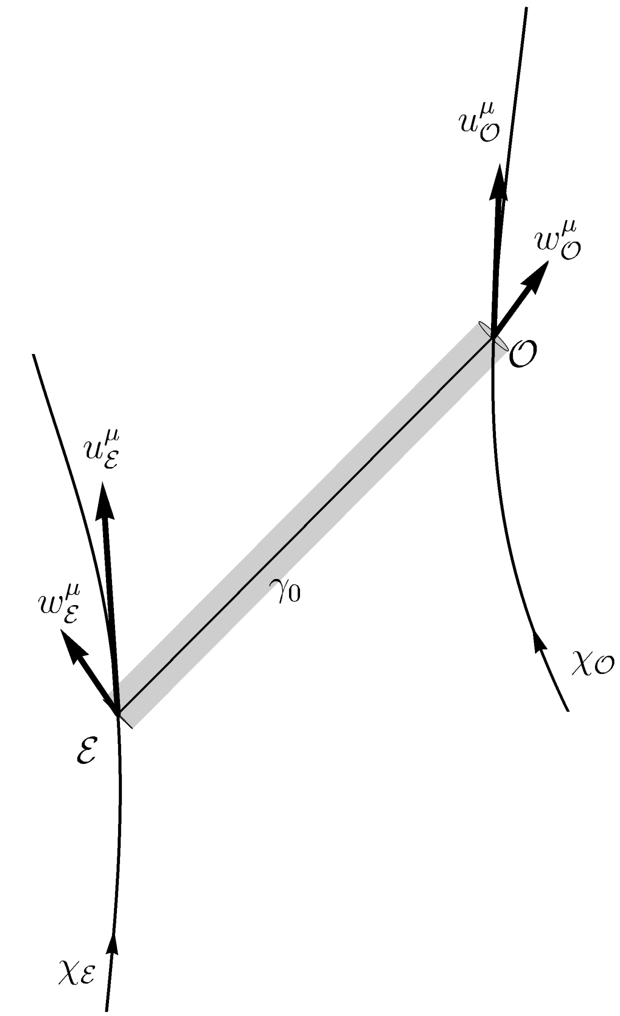

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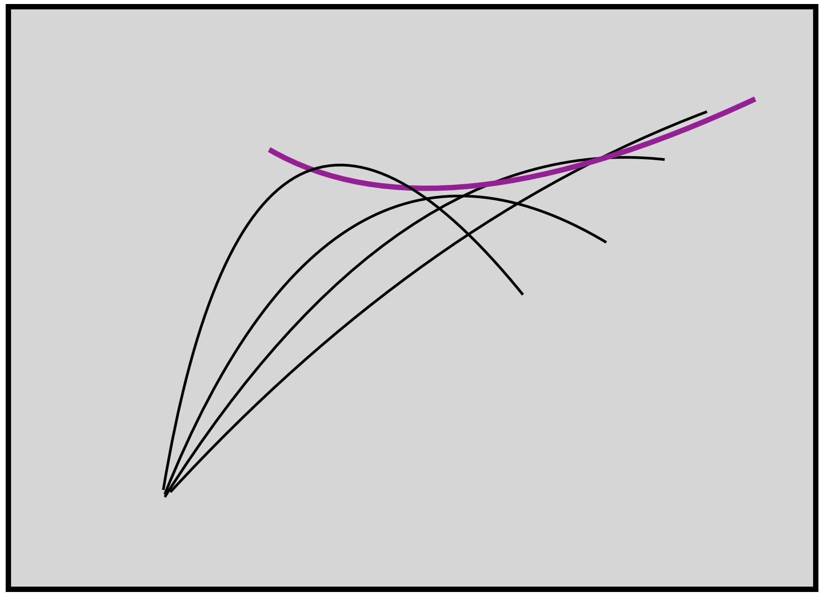
  $\boldsymbol{\Pi}^A{}_B$ diverges (infinite parallax)

  $\boldsymbol{\delta}_{\mathcal{O}} r^A$ diverges (infinite drift rate)



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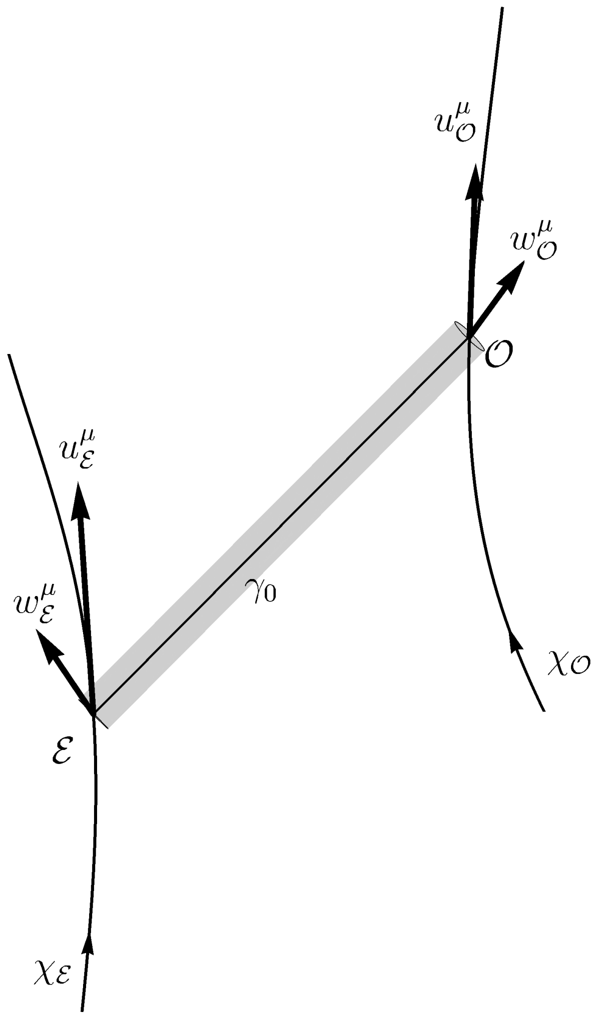

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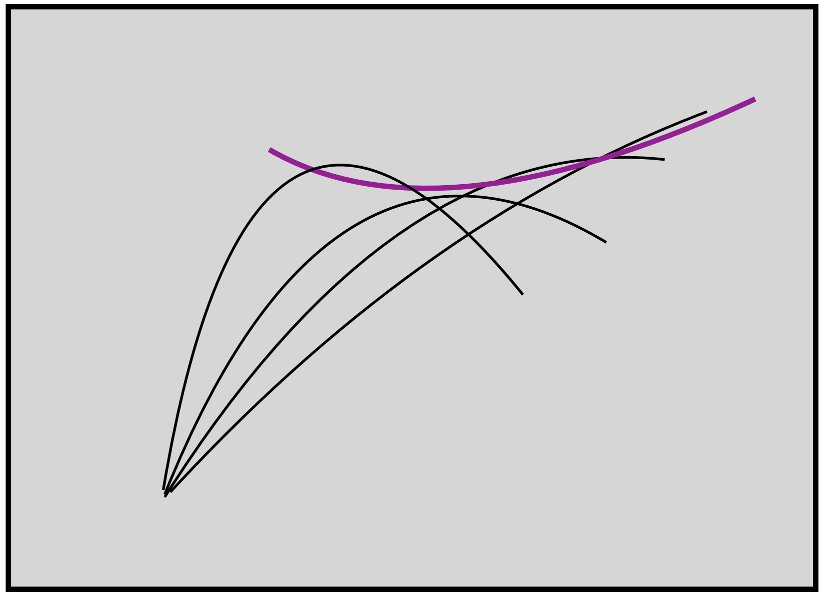

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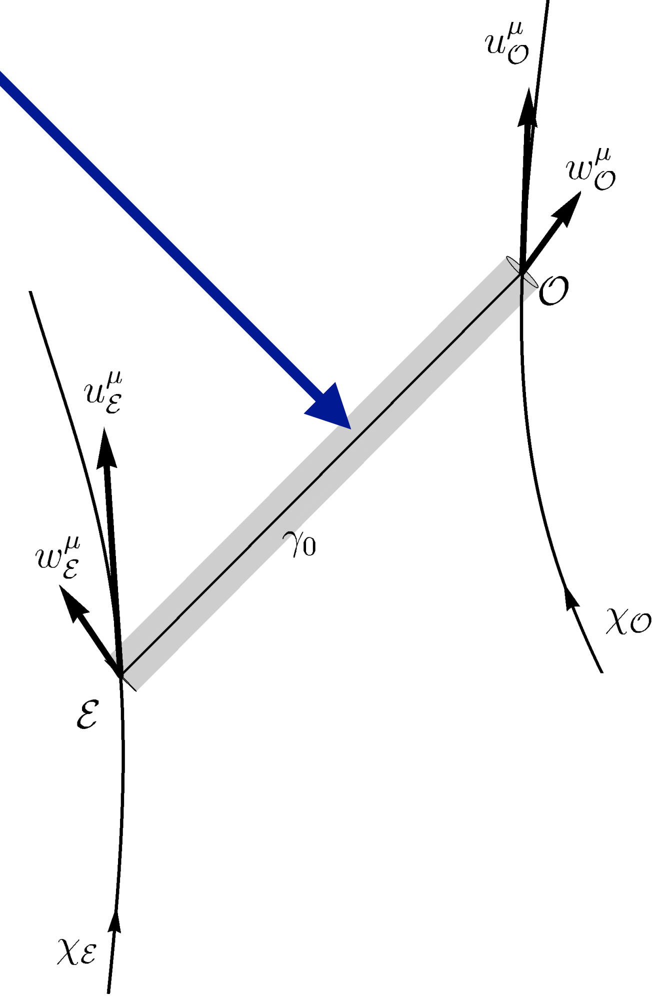
  $\boldsymbol{\delta}_{\mathscr{O}} r^A$ diverges (infinite drift rate)

caustics = position drift and parallax magnifiers

Observables

$$g_{\mu\nu} = \textbf{[flat]} + \textbf{[curvature corrections]} \left(R^{\mu}{}_{\nu\alpha\beta}\Big|_{\gamma_0} \right) + h.o.t.$$



Observables

spacetime geometry $g_{\mu\nu}(x), \Gamma^{\mu}{}_{\alpha\beta}(x)$

observation and emission points along a null geodesic $\gamma_0^{\mu}(\lambda), \mathcal{O}, \mathcal{E}$

curvature along the line of sight $R^{\mu}{}_{\nu\alpha\beta}\big|_{\gamma_0}$

(null) geodesic deviation equation

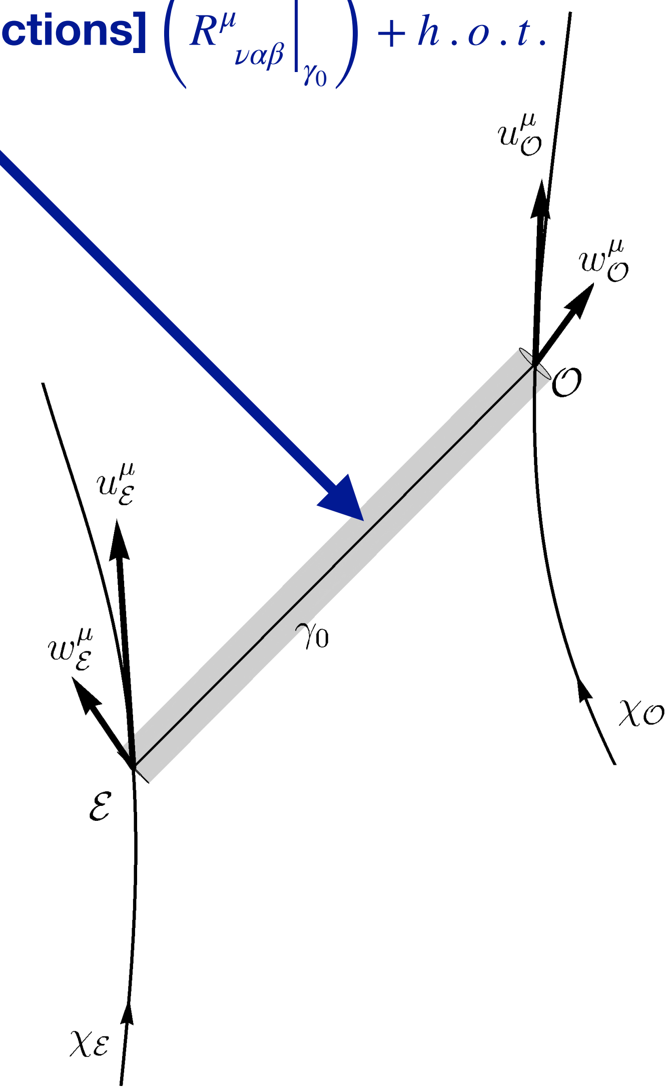
bilocal geodesic operators $W_{XX}, W_{XL}, W_{LL}, W_{LX}, \mathscr{D}, m$

covariant expressions for observables

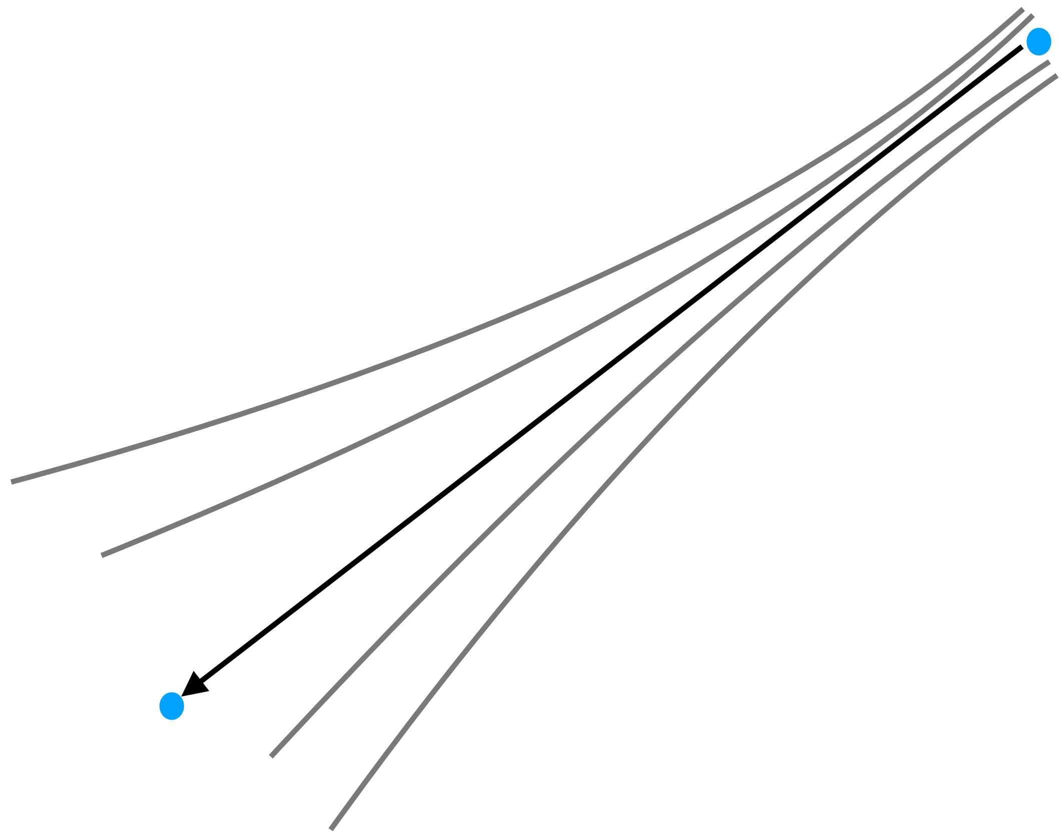
momentary positions and motions of the observer and emitter $\delta x^{\mu}_{\mathcal{O}}, \delta x^{\mu}_{\mathcal{E}}, u^{\mu}_{\mathcal{O}}, u^{\mu}_{\mathcal{E}}, w^{\mu}_{\mathcal{O}}, w^{\mu}_{\mathcal{E}}$

observables $r^{\mu}, z, \delta_{\mathcal{O}} r^{A}, \delta_{\mathcal{O}} z, \ldots$

$g_{\mu\nu}$ = **[flat]** + **[curvature corrections]** $\left(R^{\mu}{}_{\nu\alpha\beta}\big|_{\gamma_0} \right) + h.o.t.$

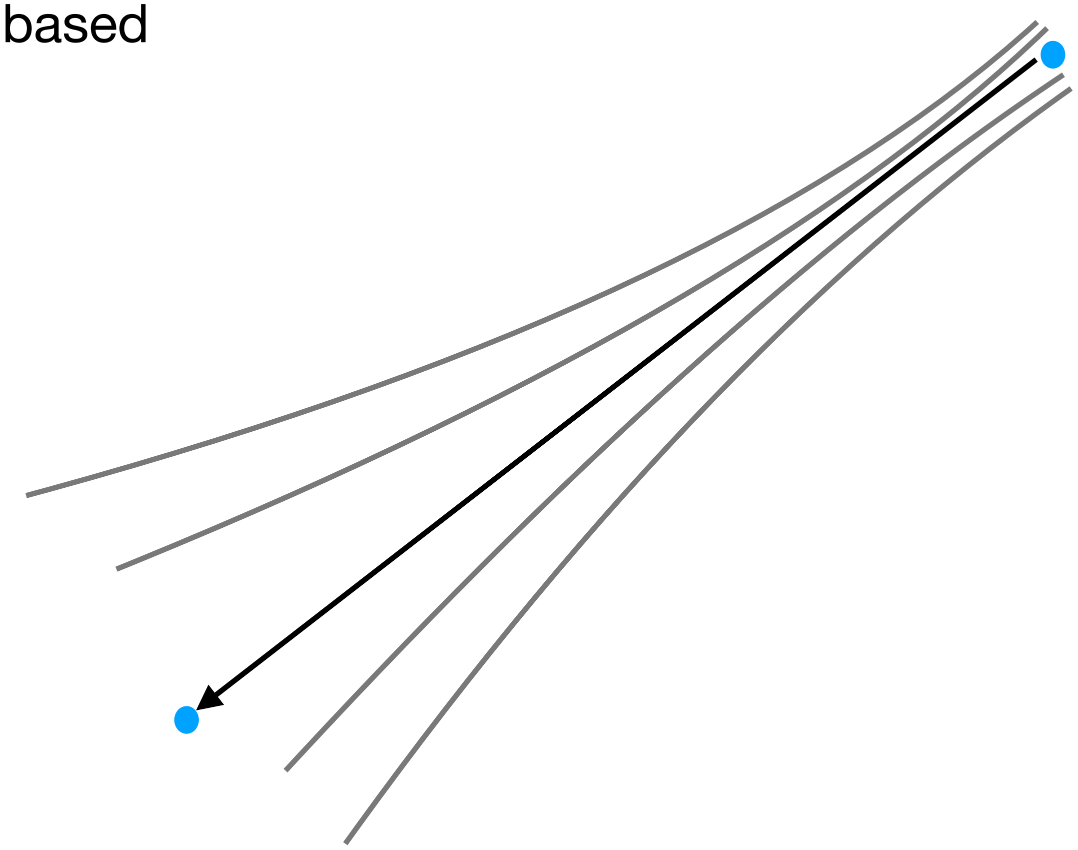


Relation to the Sachs formalism



Relation to the Sachs formalism

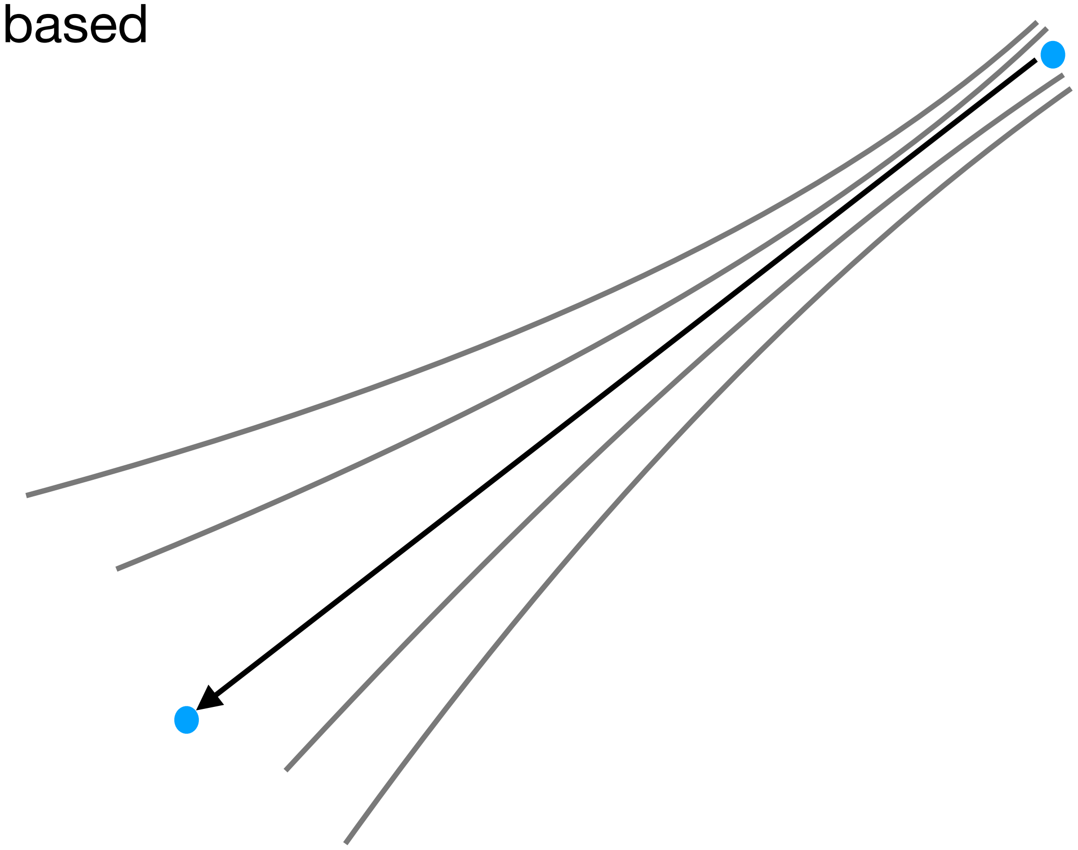
Sachs 1961, Ehlers, Jordan, Sachs 1961 - formalism based on congruences of null geodesics



Relation to the Sachs formalism

Sachs 1961, Ehlers, Jordan, Sachs 1961 - formalism based on congruences of null geodesics

congruence = 1 geodesic through each point = 3-parameter family



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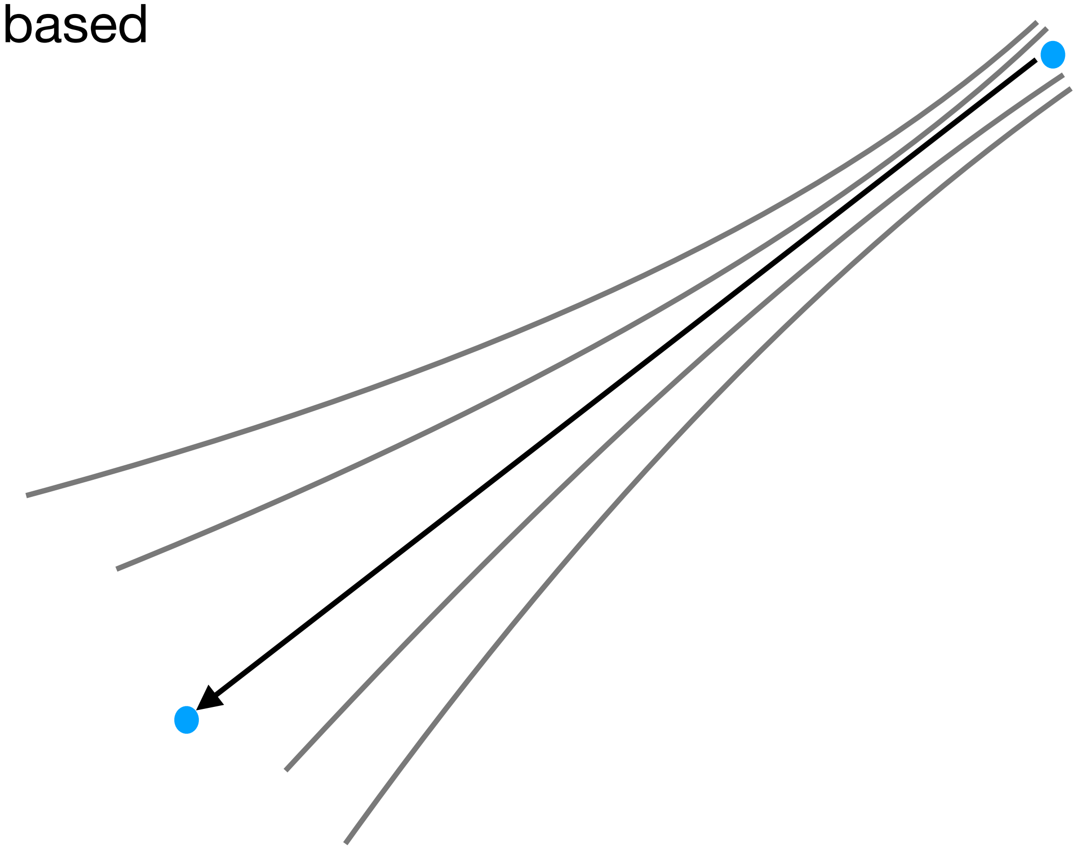
local tensorial quantities $\theta, \sigma_{AB}, \omega_{AB}$



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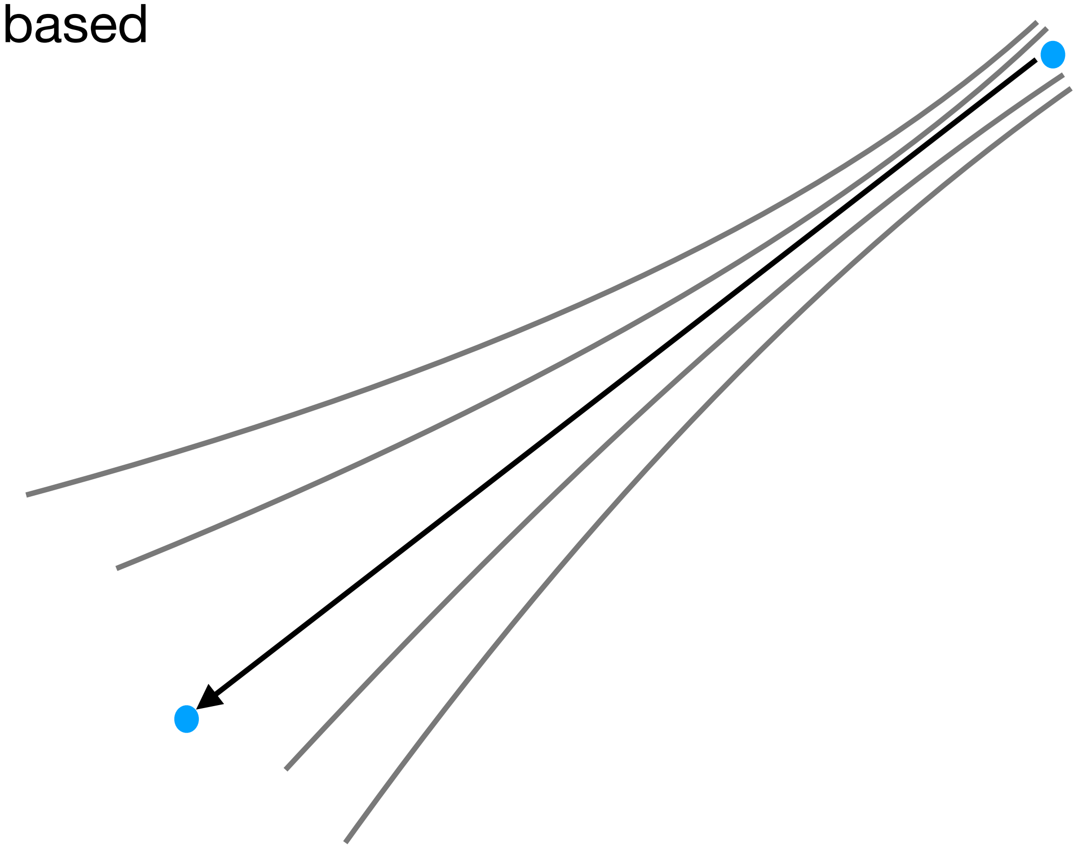
nonlinear, 1st order ODE's for them



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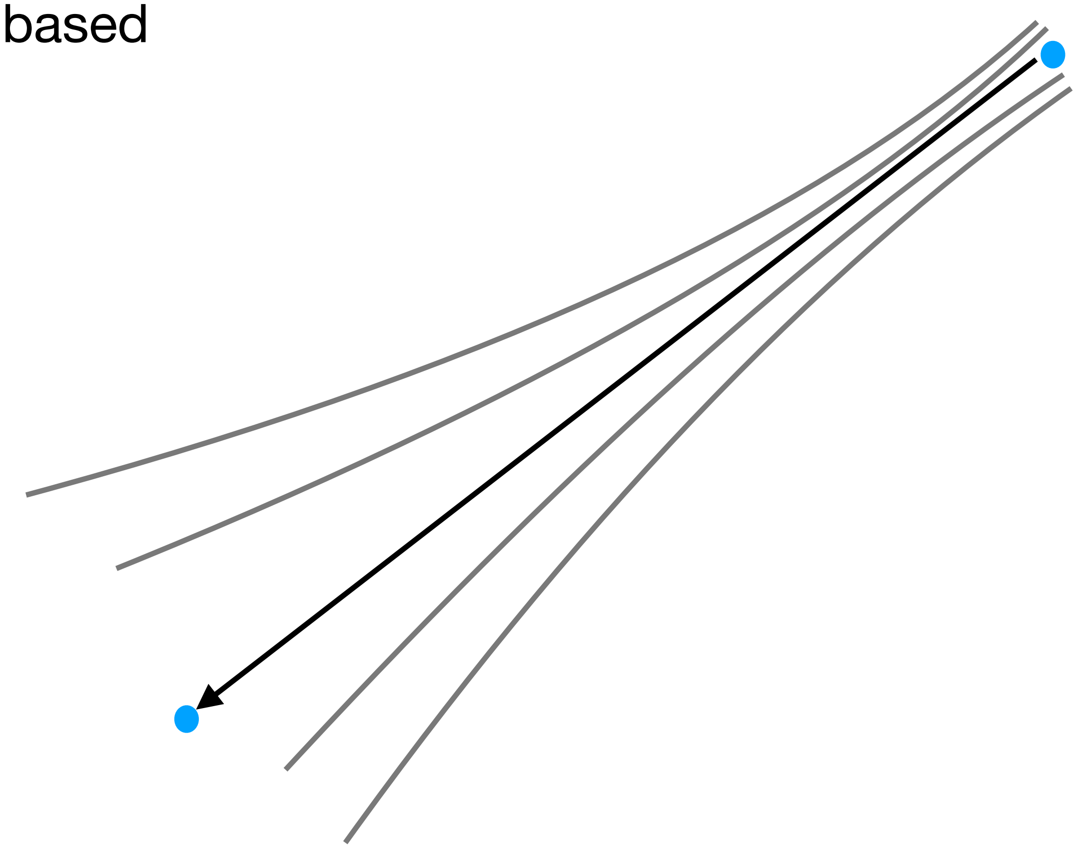

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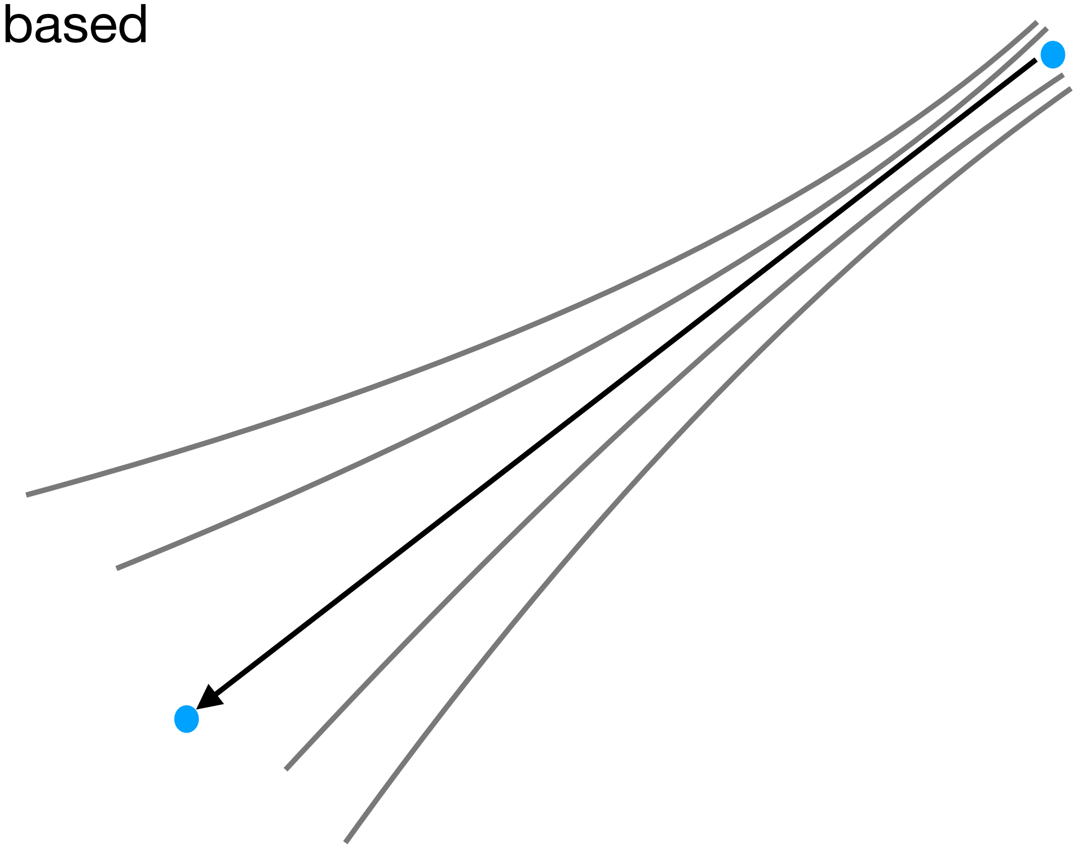

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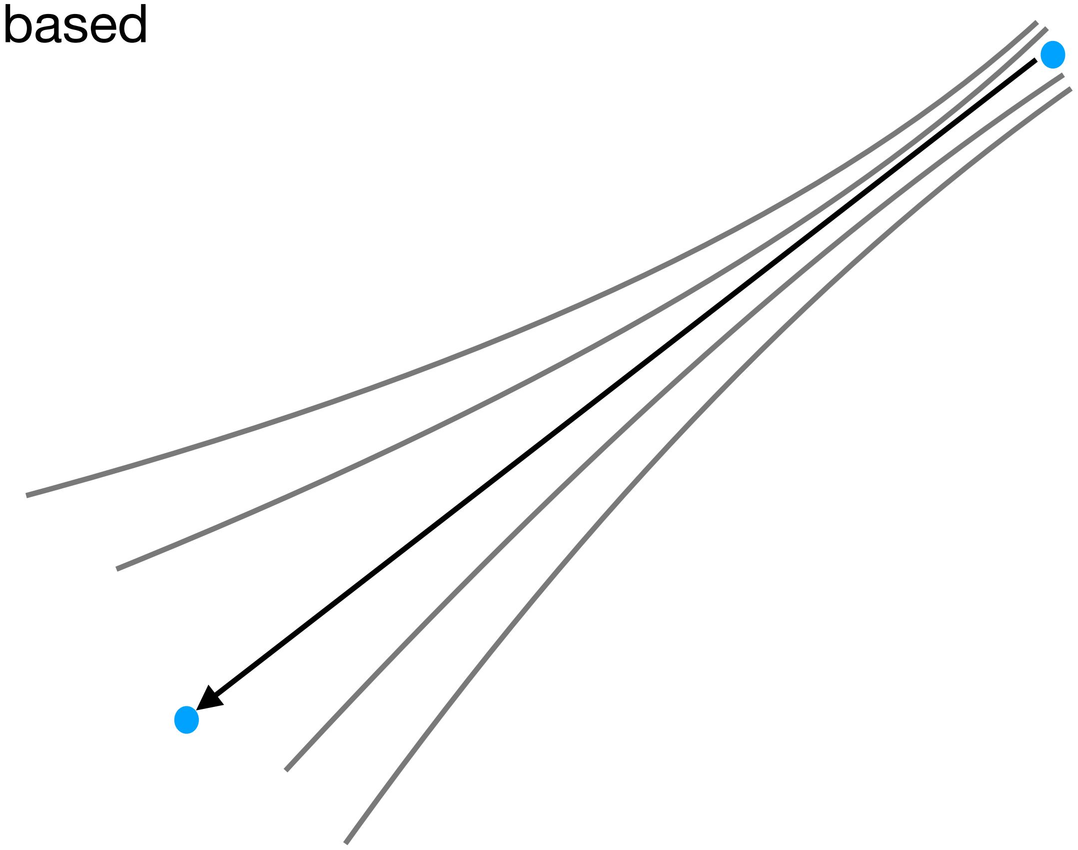

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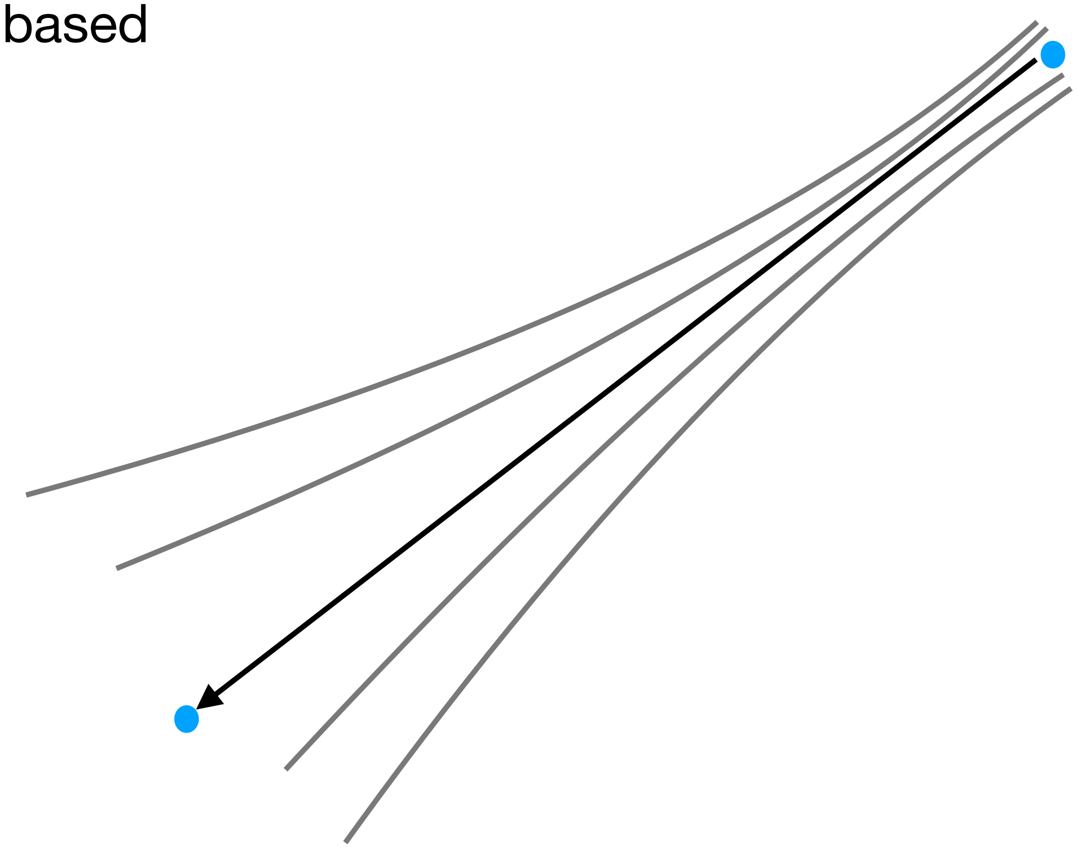

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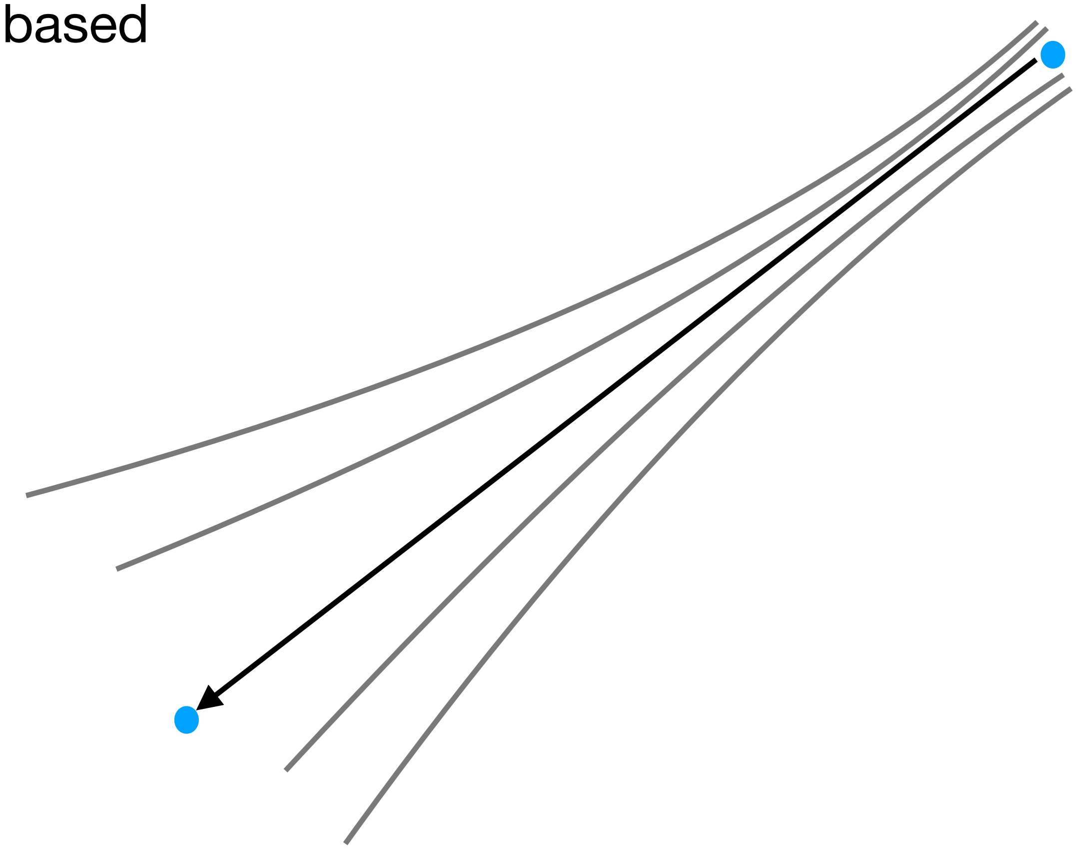

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equivalent to the BGO formalism

avoids bitensors, BUT…

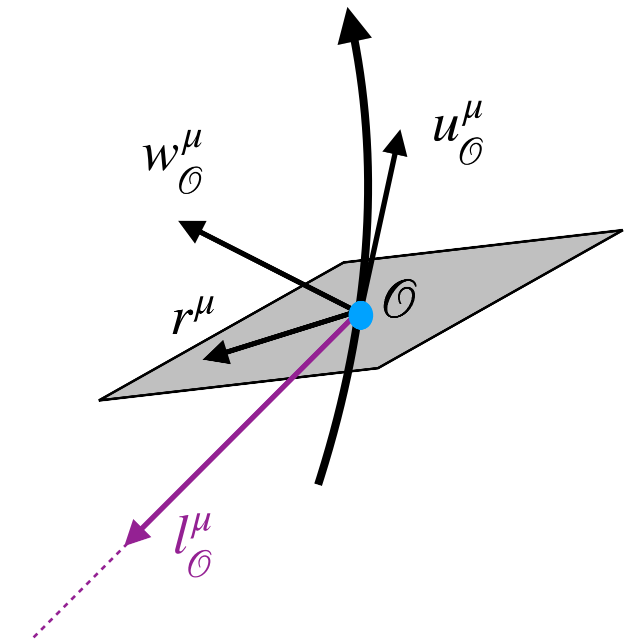
less flexible (need to fix the initial condition for the congruence, cannot change later)

cannot simultaneously displace both endpoints of geodesics

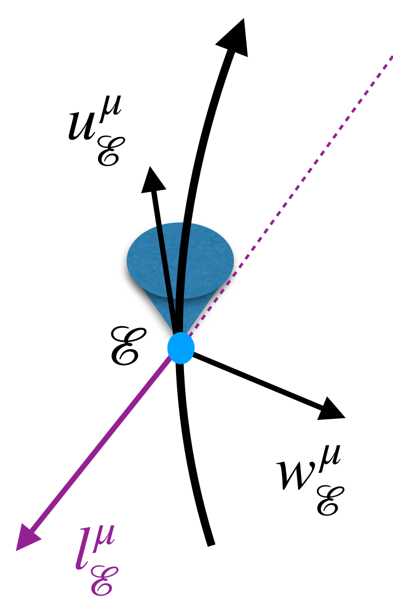
less useful to study drifts and parallax (although Räsänen 2014, Rosquist 1988…)

Problem

Light propagation in the geometric optics approximation



spacetime



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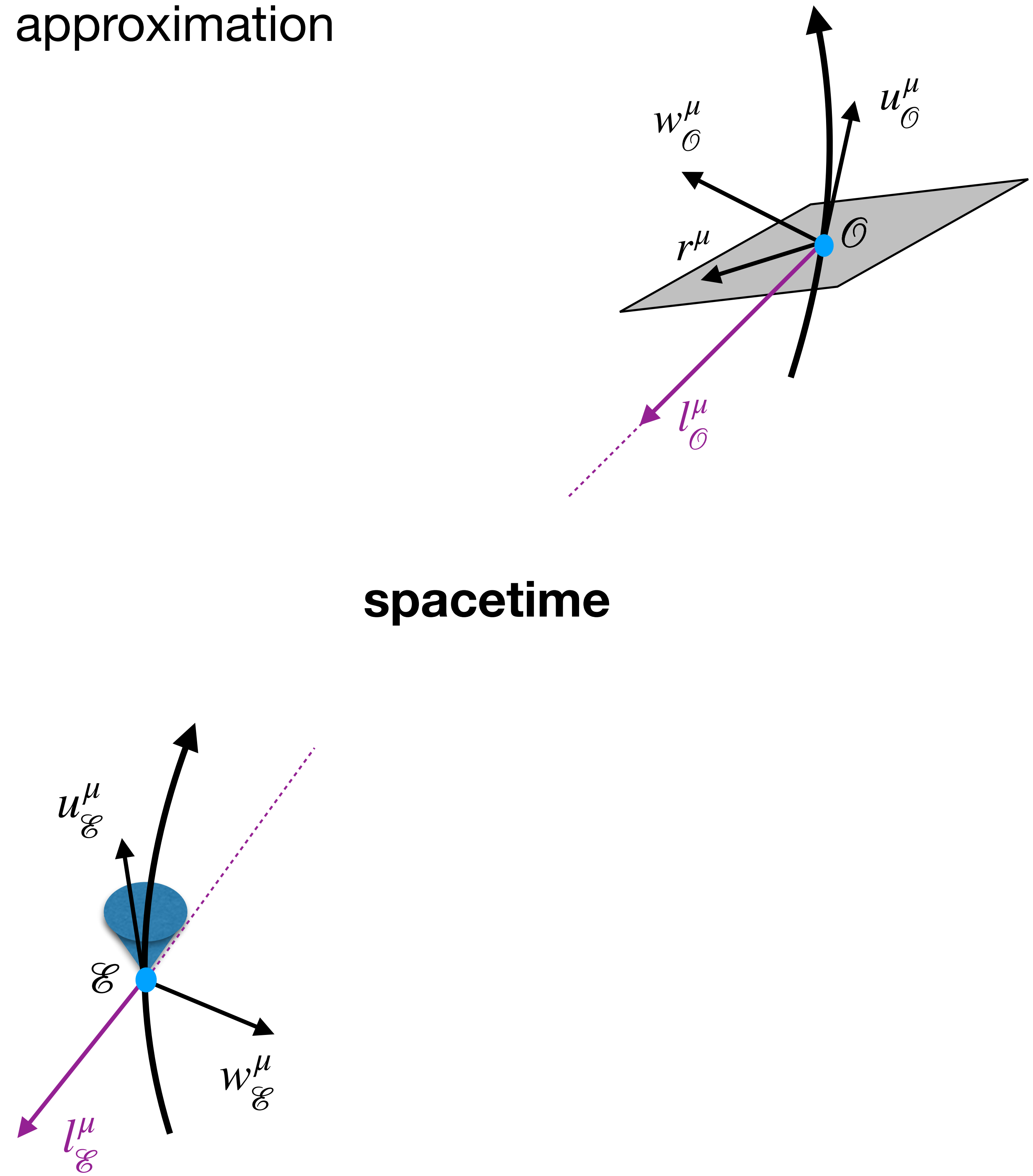
Observables:



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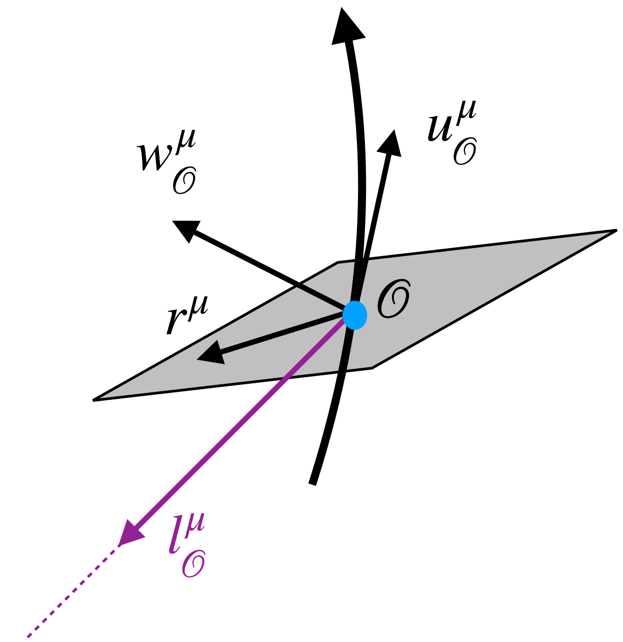
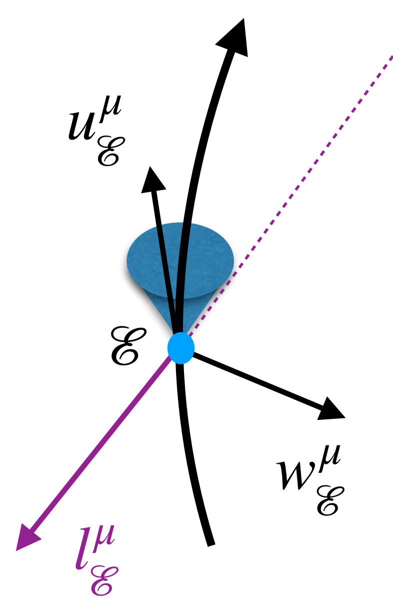


Observables:

- **time of arrival** $\tau_\mathcal{O}(\tau_\mathcal{E})$

spacetime



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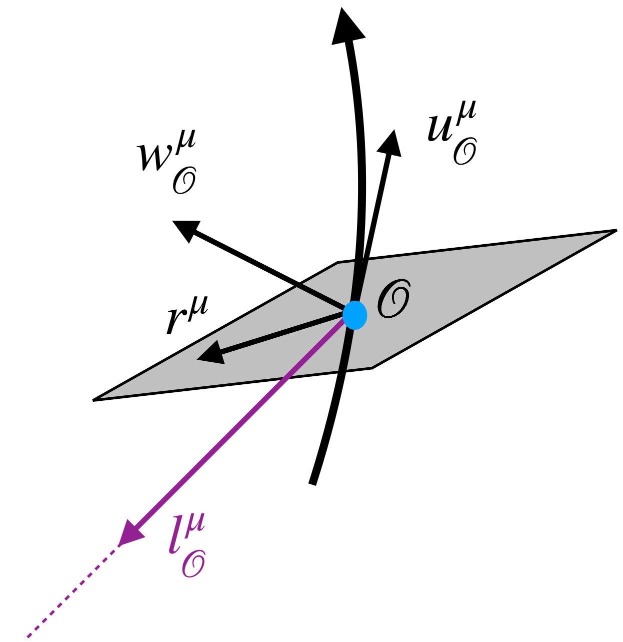
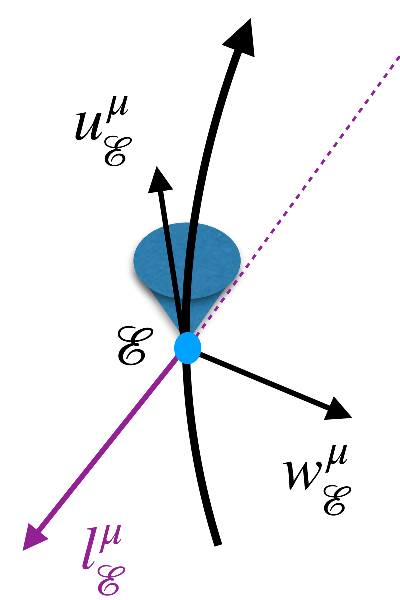


Observables:

- **time of arrival** $\tau_{\mathcal{O}}(\tau_{\mathcal{E}})$
- **position the sky** $r^\mu = \dfrac{l^\mu_{\mathcal{O}}}{l^\sigma_{\mathcal{O}}\, u_{\mathcal{O}_\sigma}} + u^\mu_{\mathcal{O}}$

spacetime

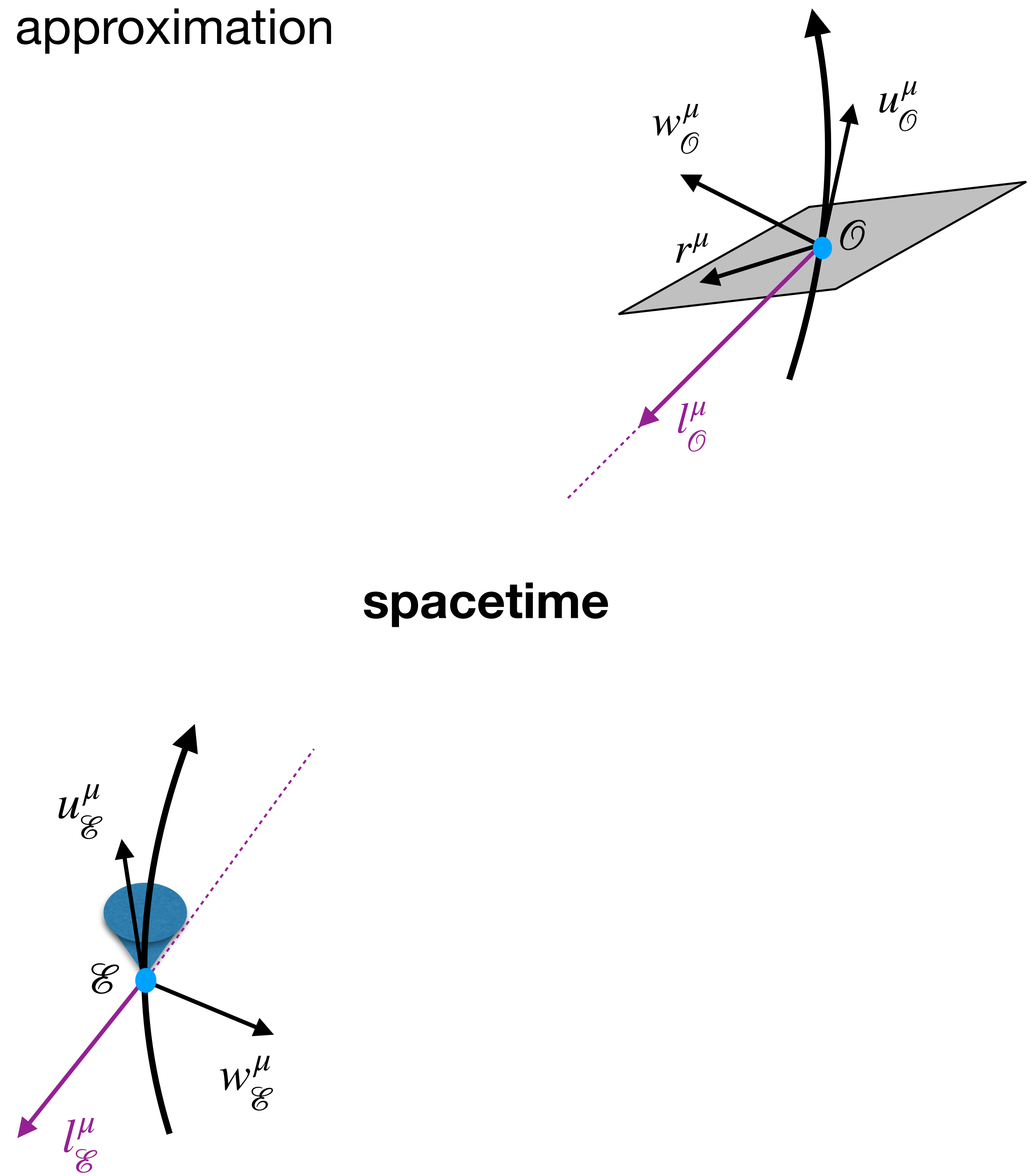


Problem

Light propagation in the geometric optics approximation

Observables:

- **time of arrival** $\tau_{\mathcal{O}}(\tau_{\mathcal{E}})$
- **position the sky** $r^\mu = \dfrac{l^\mu_{\mathcal{O}}}{l^\sigma_{\mathcal{O}}\, u_{\mathcal{O}_\sigma}} + u^\mu_{\mathcal{O}}$
- **image distortion** $\delta\theta^A \approx \delta r^A$



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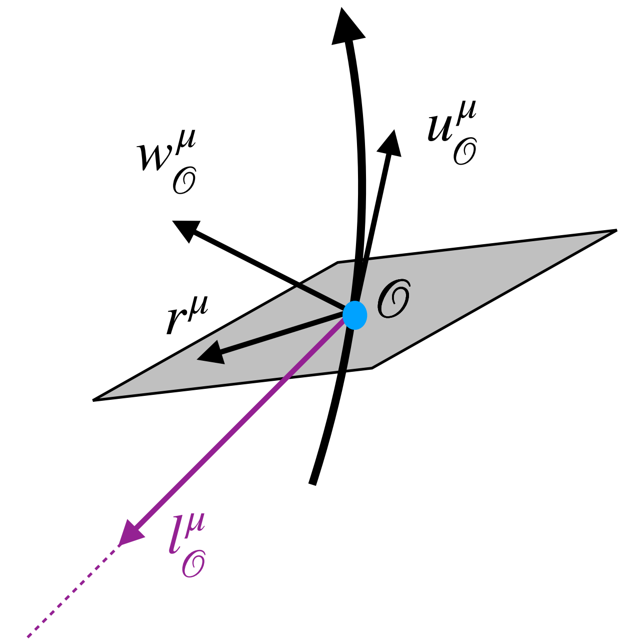

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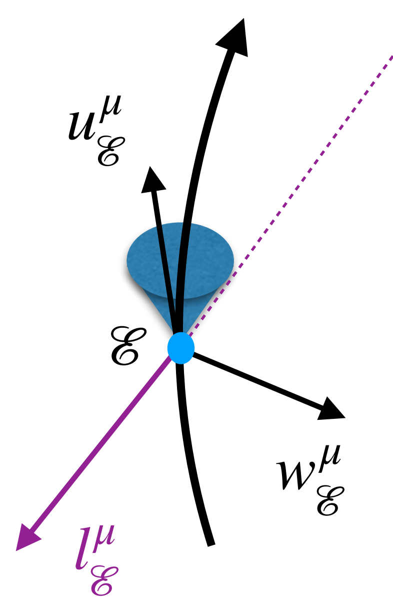
- **image distortion** $\delta\theta^A \approx \delta r^A$
- **parallax** $\delta\theta^A \approx \delta r^A$

spacetime



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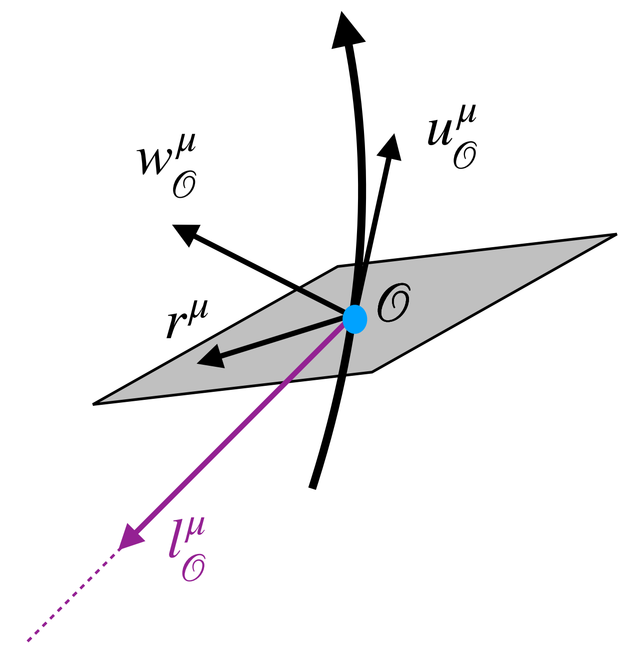

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- **position the sky** $r^\mu = \dfrac{l^\mu_{\mathcal{O}}}{l^\sigma_{\mathcal{O}}\, u_{\mathcal{O}_\sigma}} + u^\mu_{\mathcal{O}}$
- **image distortion** $\delta\theta^A \approx \delta r^A$
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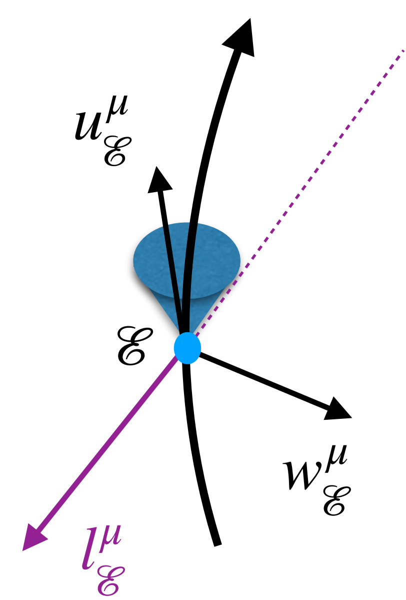
- **position drift** $\nabla_{u_{\mathcal{O}}} r^\mu$

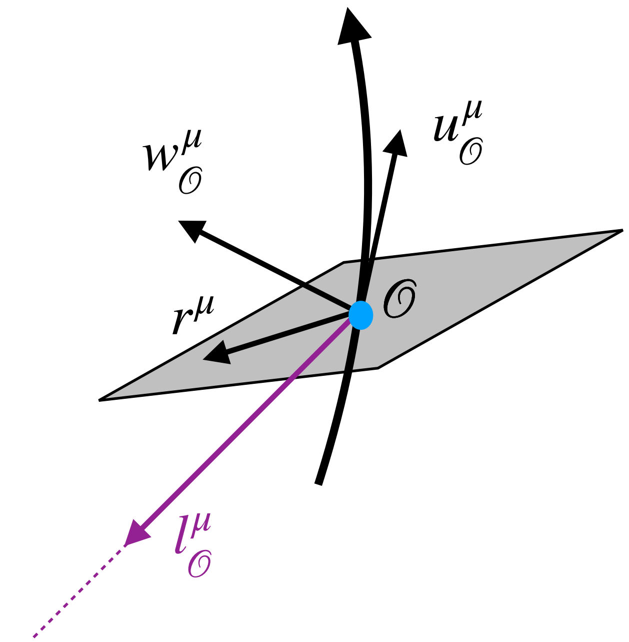
  or $\delta_{\mathcal{O}} r^A = \delta_{FW} r^A$

spacetime



Problem

Light propagation in the geometric optics approximation



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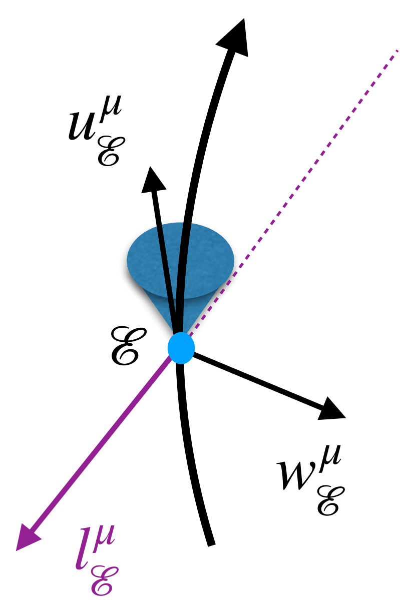

Fermi-Walker derivative

$$\delta_{FW} X^\mu = \nabla_{u_{\mathcal{O}}} X^\mu + \left(-w_{\mathcal{O}\,\nu}\, u^\mu_{\mathcal{O}} + u_{\mathcal{O}\,\nu}\, w^\mu_{\mathcal{O}}\right) X^\nu$$

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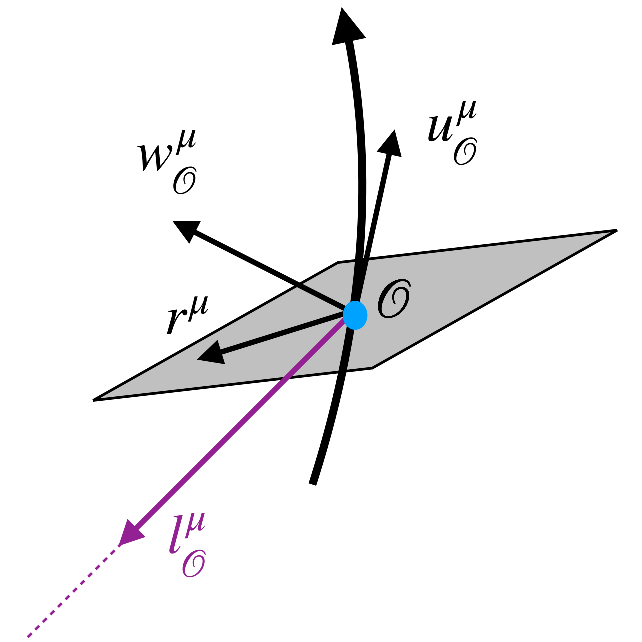

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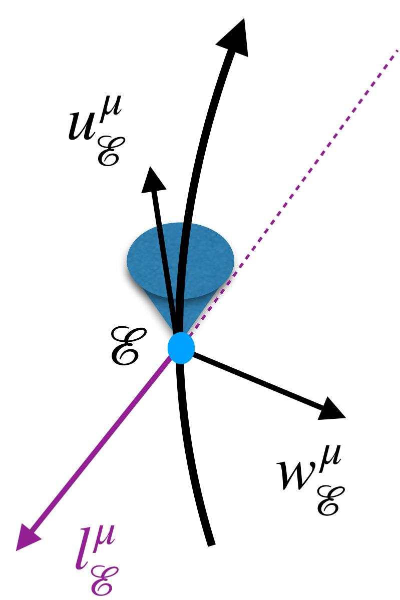

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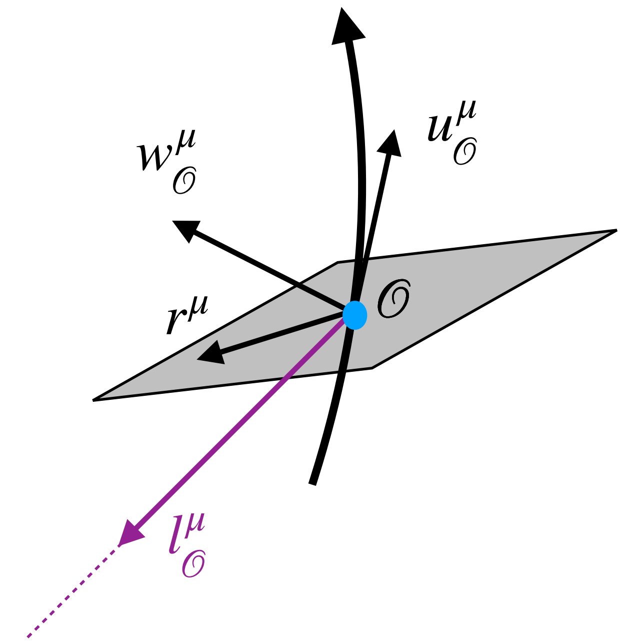

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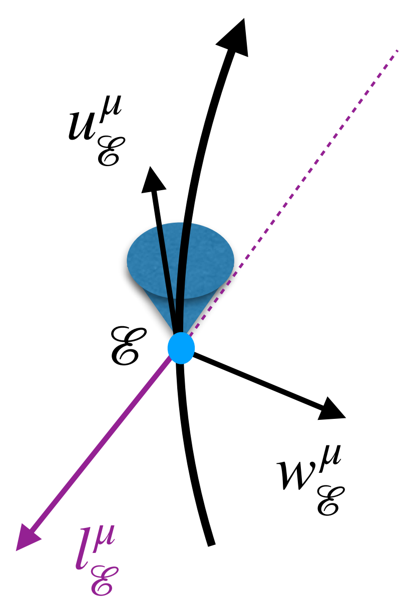
spacetime



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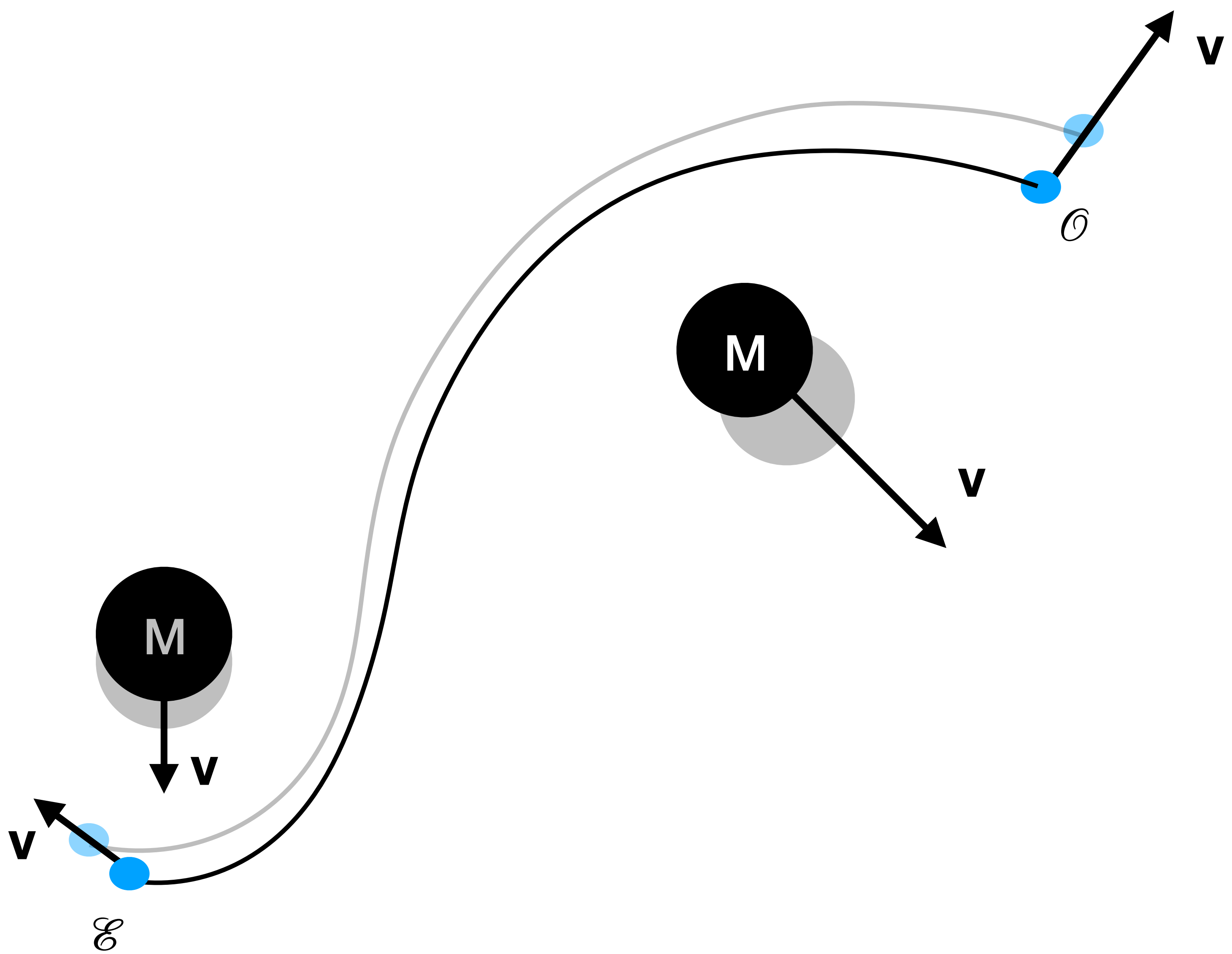
”Naive” approach

Redshift and position drift



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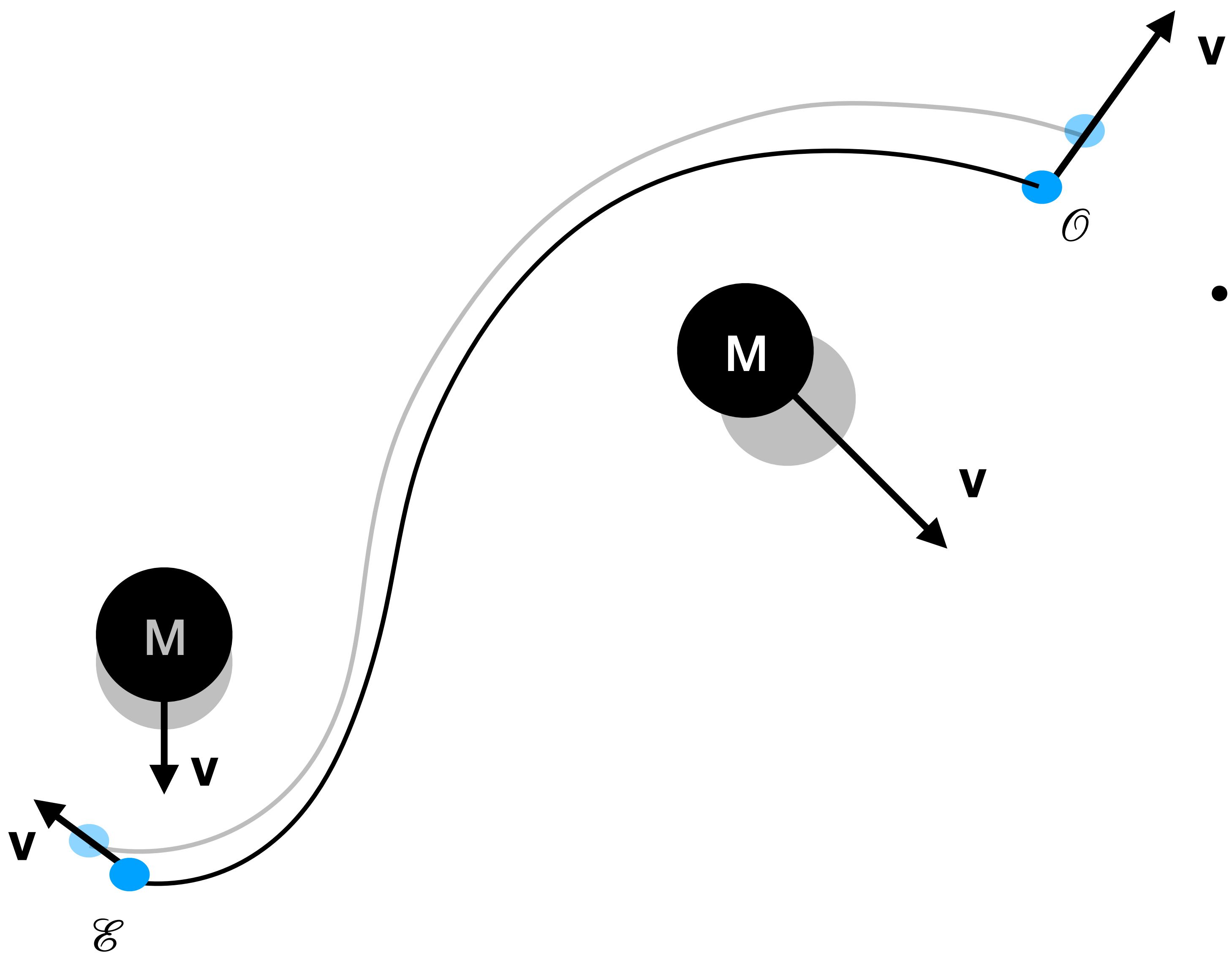
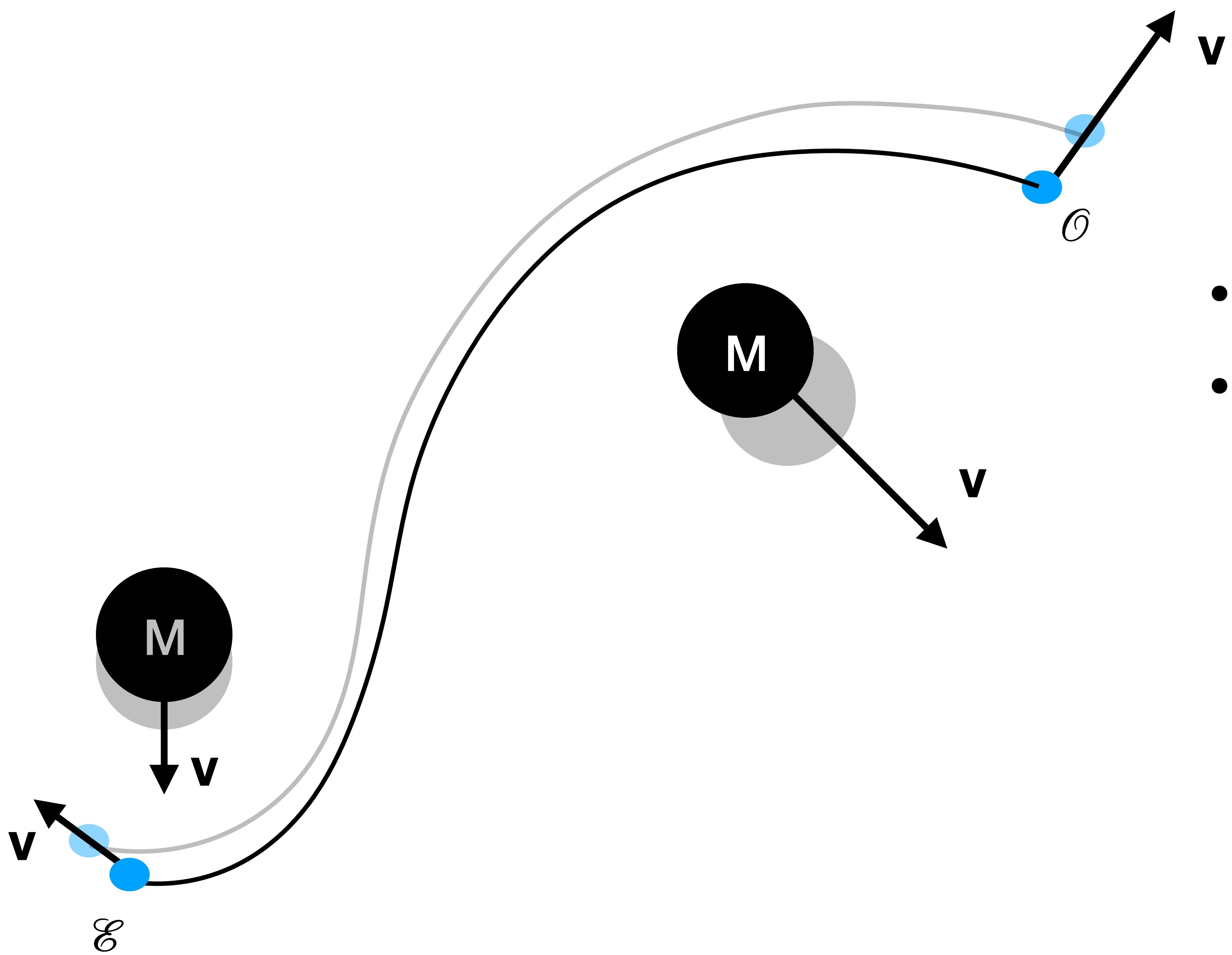


- Gravitational interactions between bodies

”Naive” approach

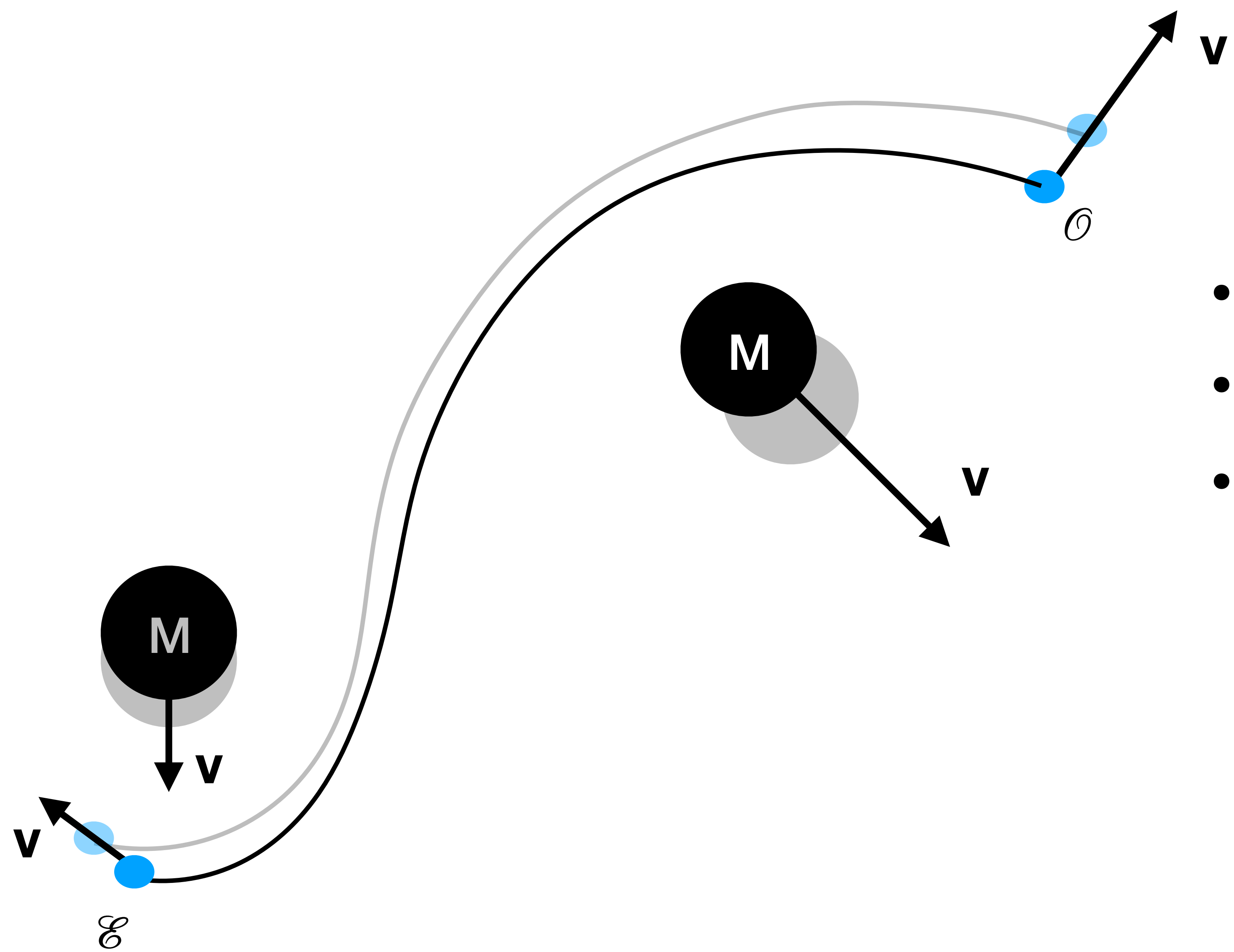
Redshift and position drift



- Gravitational interactions between bodies
- (Time-dependent) light ray bending

”Naive” approach

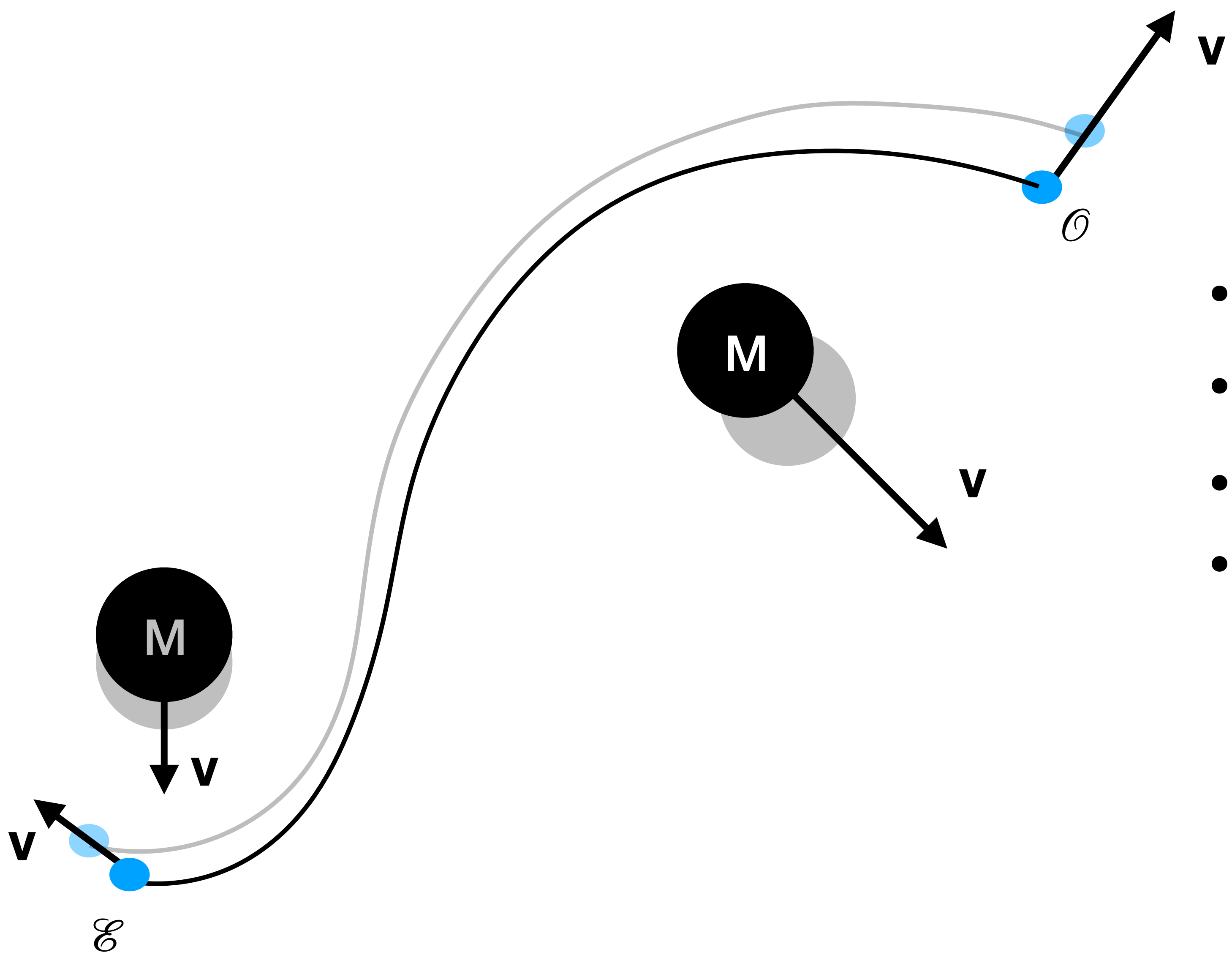
Redshift and position drift



- Gravitational interactions between bodies
- (Time-dependent) light ray bending
- (Time-dependent) ISW effect for z

”Naive” approach

Redshift and position drift



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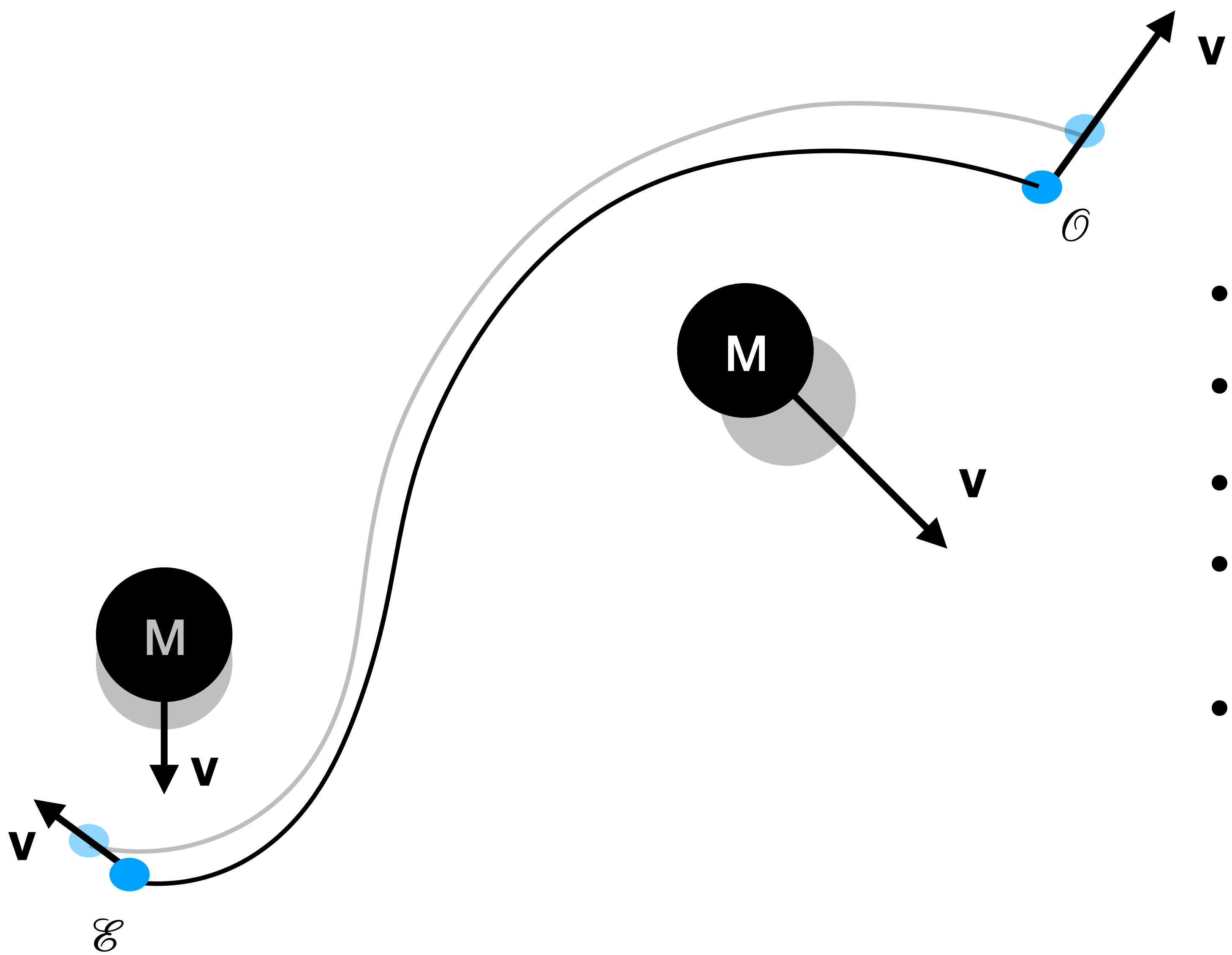
- (Time-dependent) Shapiro and geometric delays

"Naive" approach

Redshift and position drift



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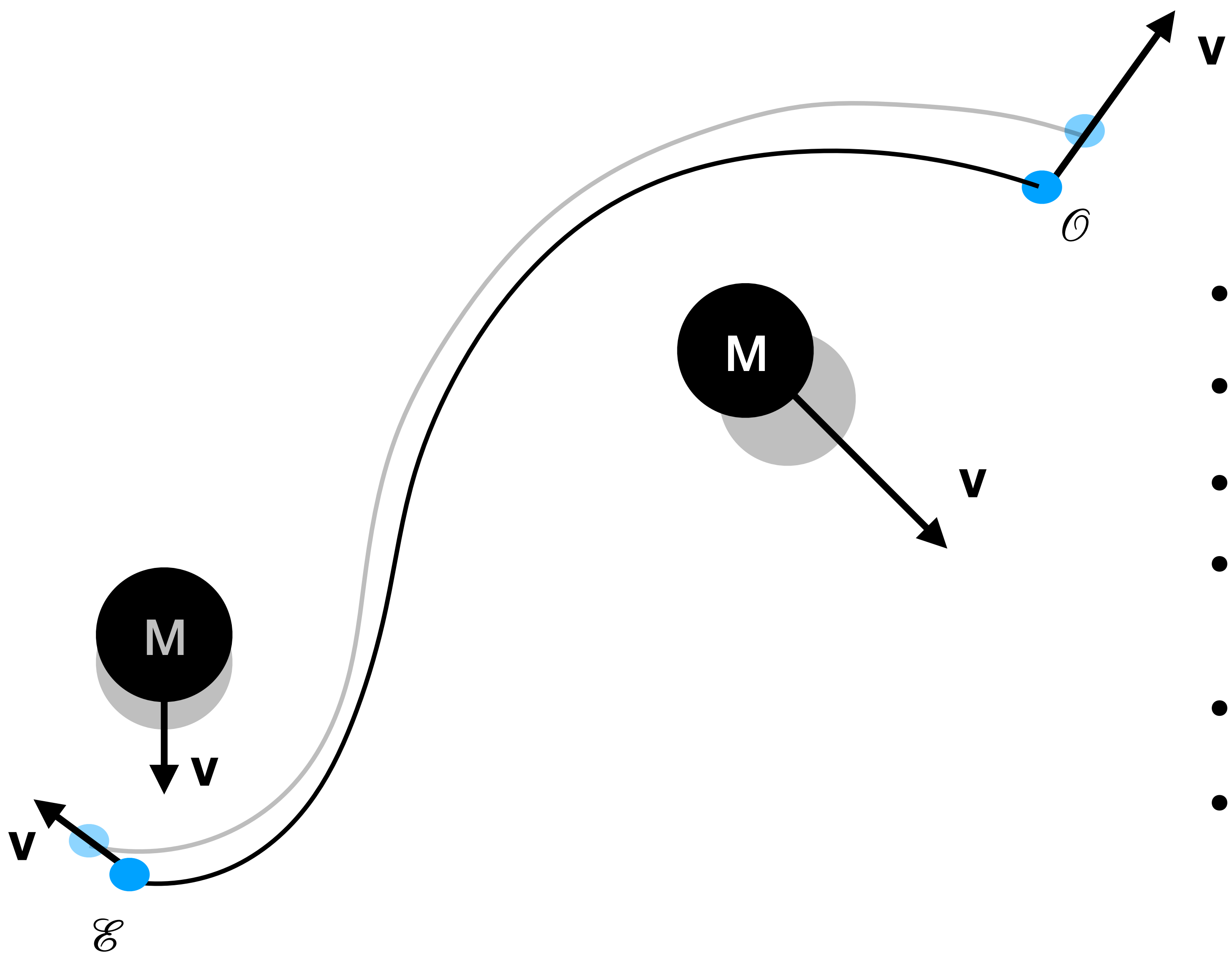
- (Time-dependent) motions of E and O

"Naive" approach

Redshift and position drift



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”Naive” approach

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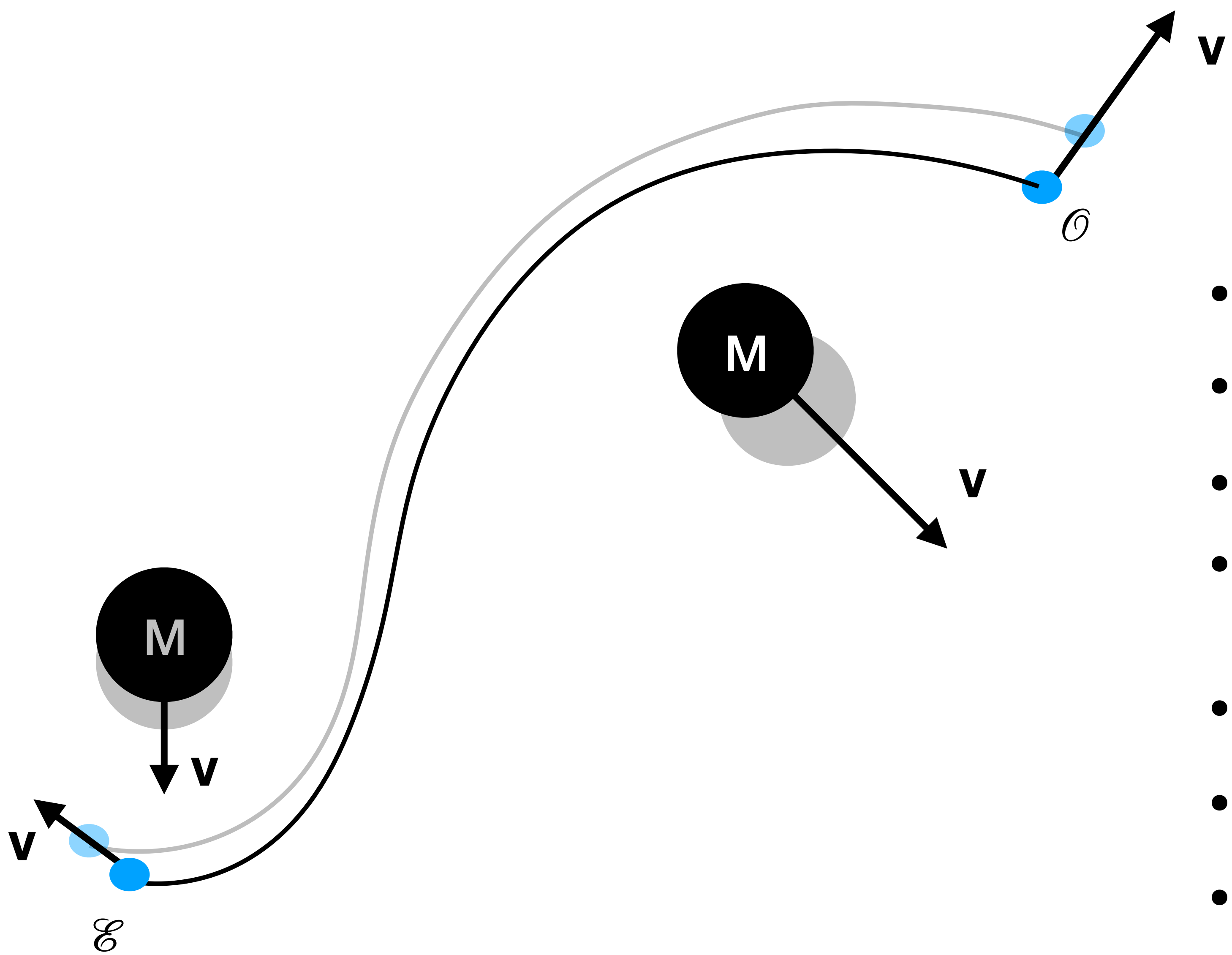

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”Naive” approach

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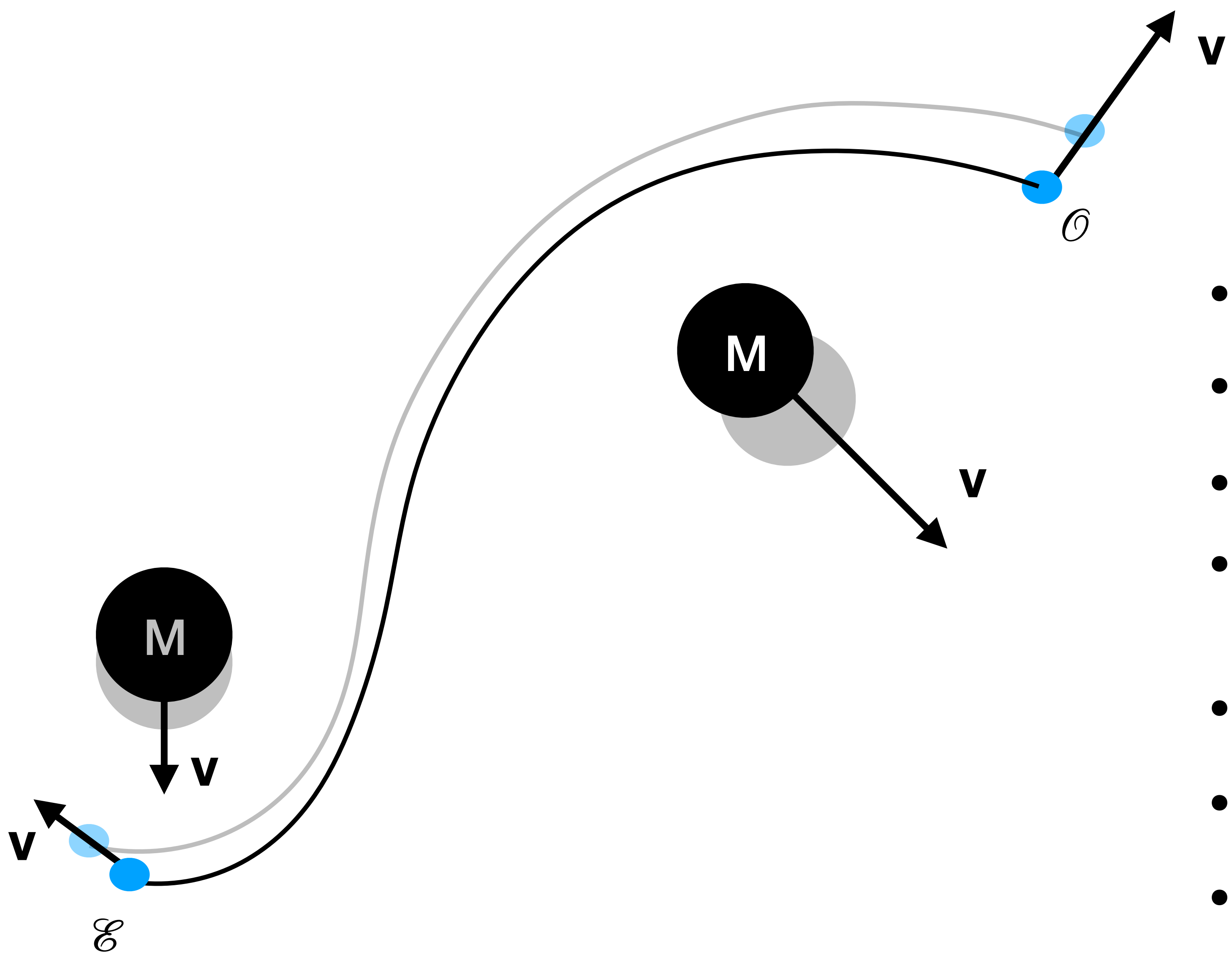

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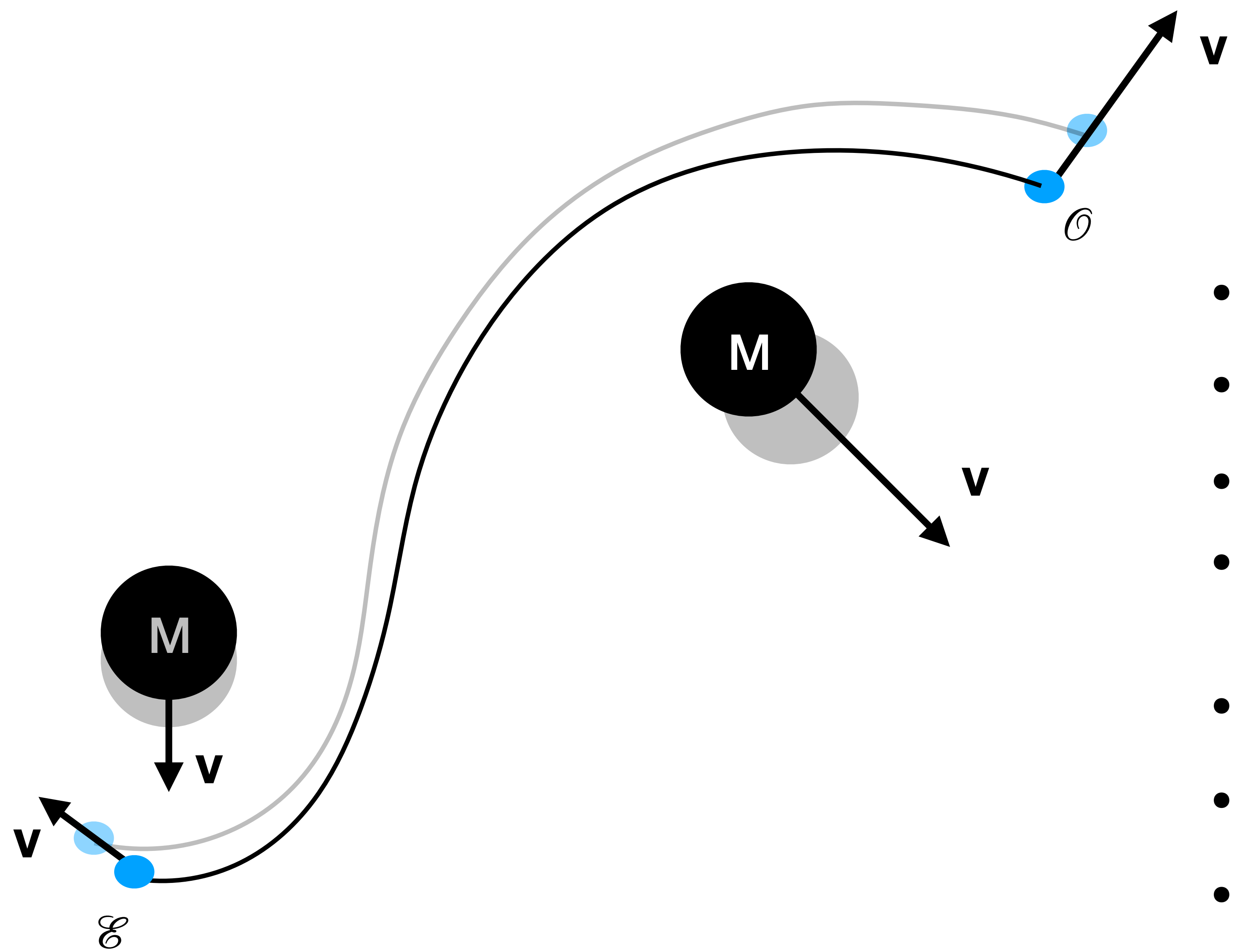
”Naive” approach

Redshift and position drift



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"Naive" approach

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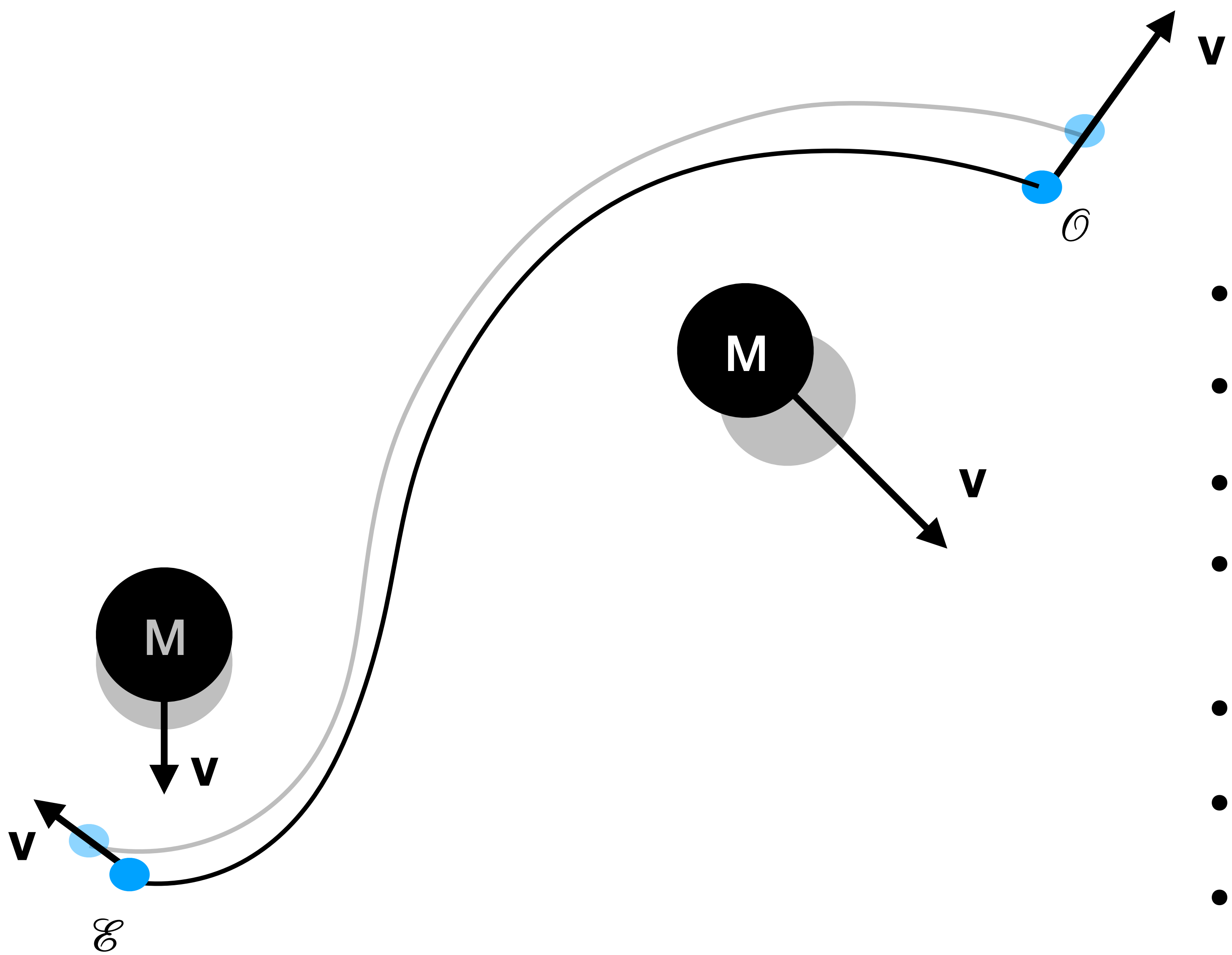

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- …

Very complicated problem!

"Naive" approach

spacetime geometry $g_{\mu\nu}(x), \Gamma^{\mu}{}_{\alpha\beta}(x)$

↓ *geodesic equation*

null geodesics $x^{\mu}(\lambda)$

↓ *nearby null geodesics*

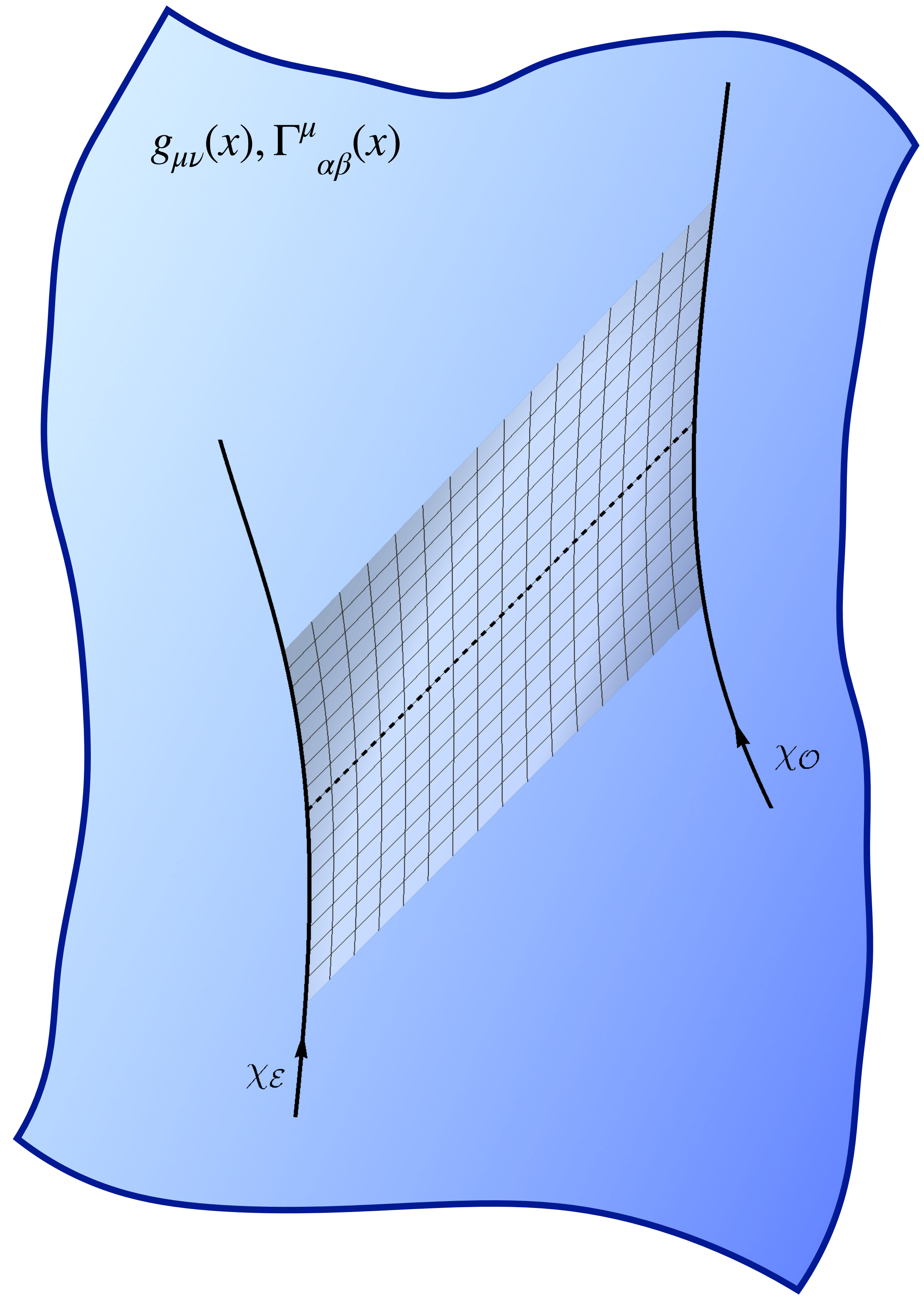
O's and E's worldlines $\chi^{\mu}_{\mathcal{O}}(\tau_{\mathcal{O}}), \chi^{\mu}_{\mathcal{E}}(\tau_{\mathcal{E}})$

↓

time of observation $\tau_{\mathcal{O}}$

↓

observables $r^{\mu}, z, \delta_{\mathcal{O}} r^{A}, \delta_{\mathcal{O}} z, \ldots$



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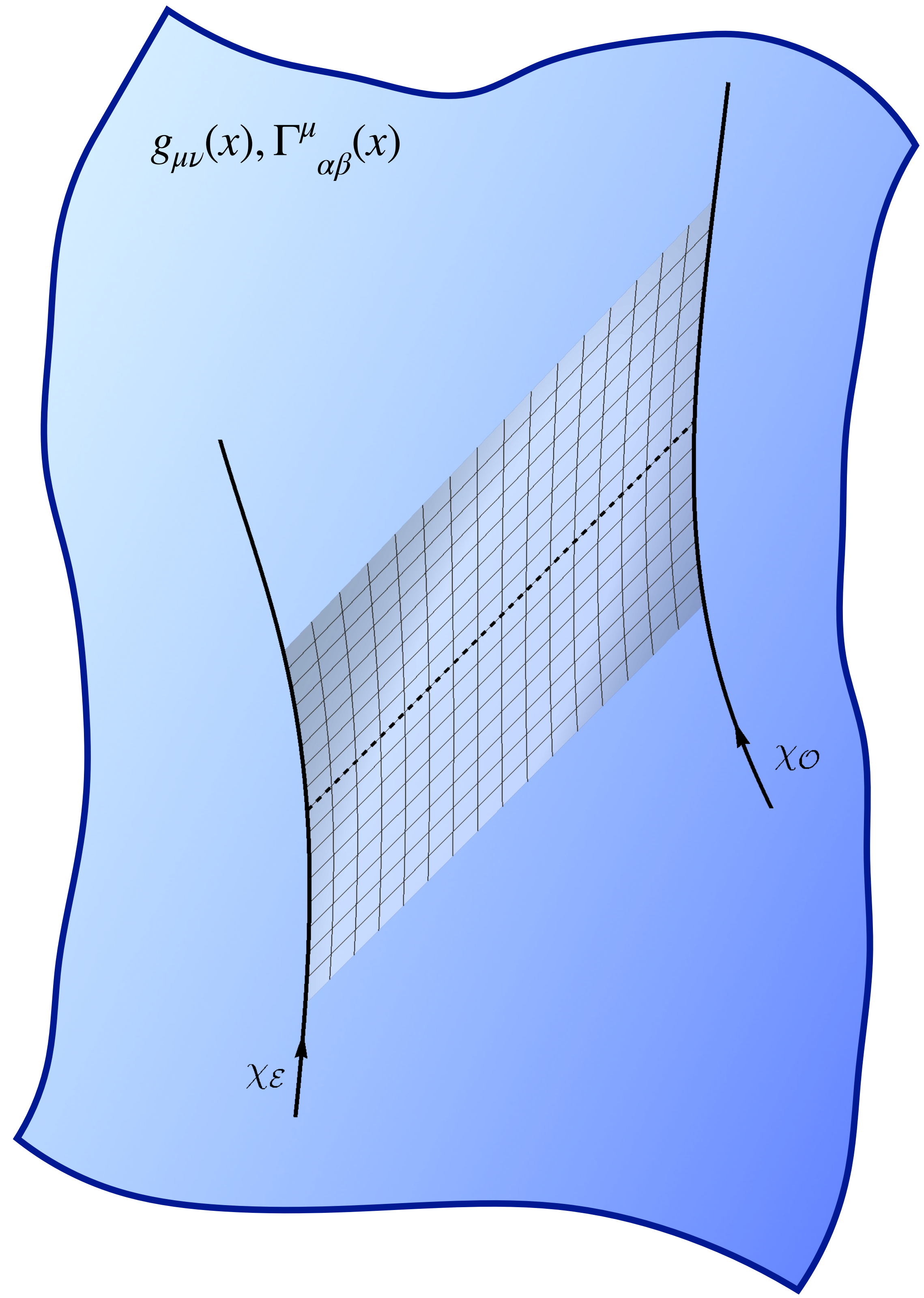
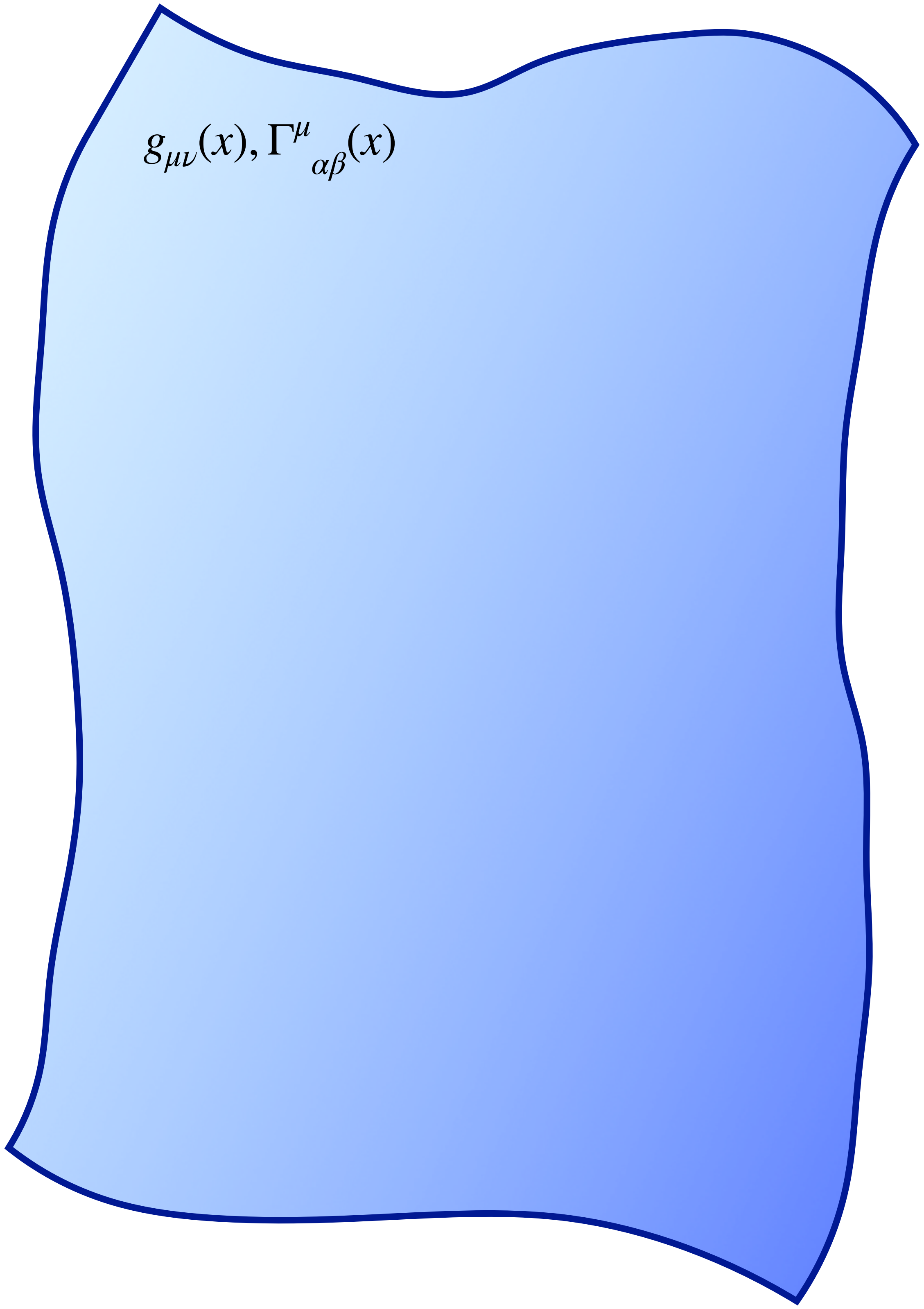


Everything depends on g in a complicated way

No general formulas or relations

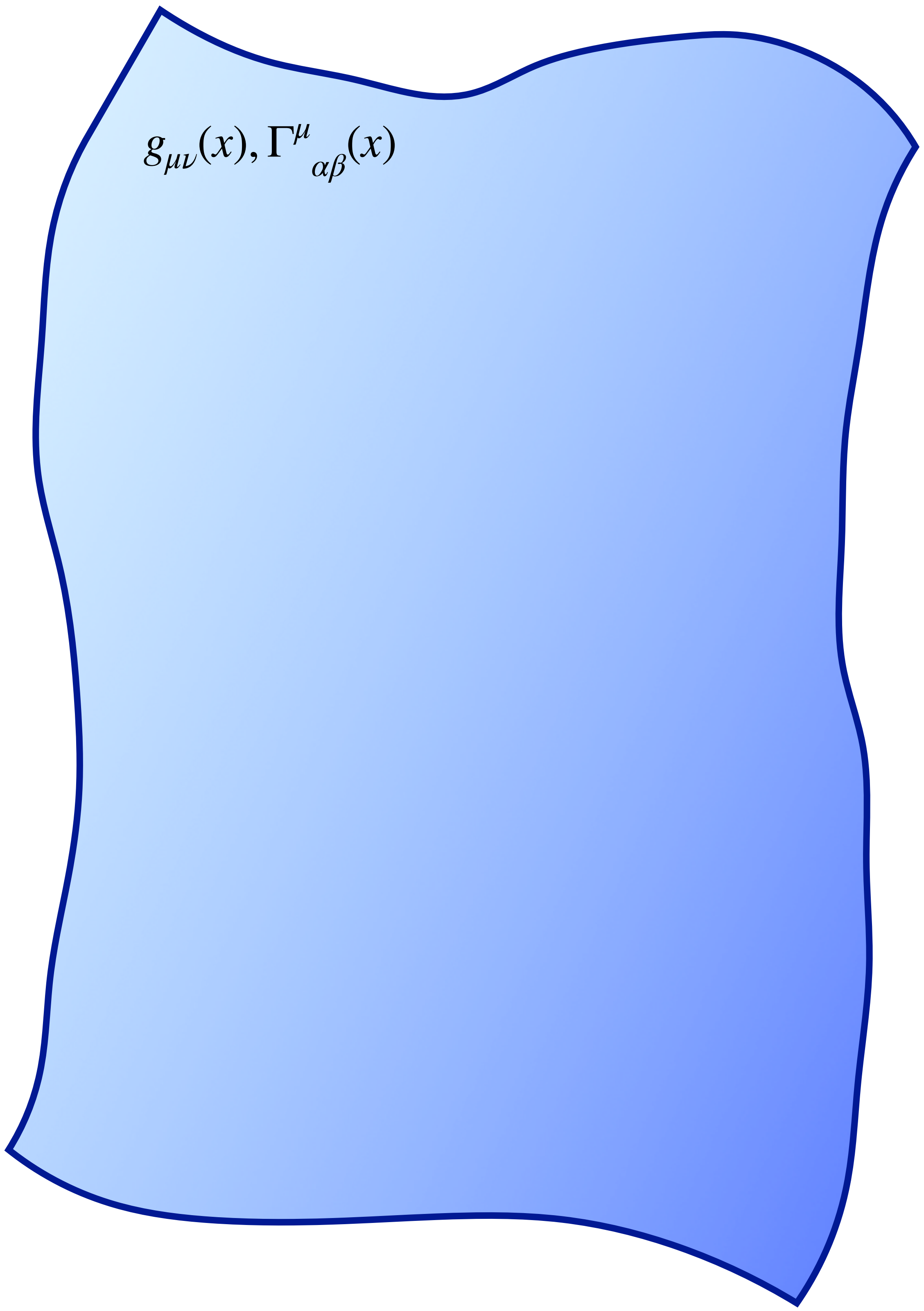
Geometric approach



Geometric approach

Normal coordinates around a point

$$g_{\mu\nu} = \eta_{\mu\nu} - \frac{1}{3} R_{\mu\alpha\nu\beta}\, y^\alpha\, y^\beta + O(y^3)$$



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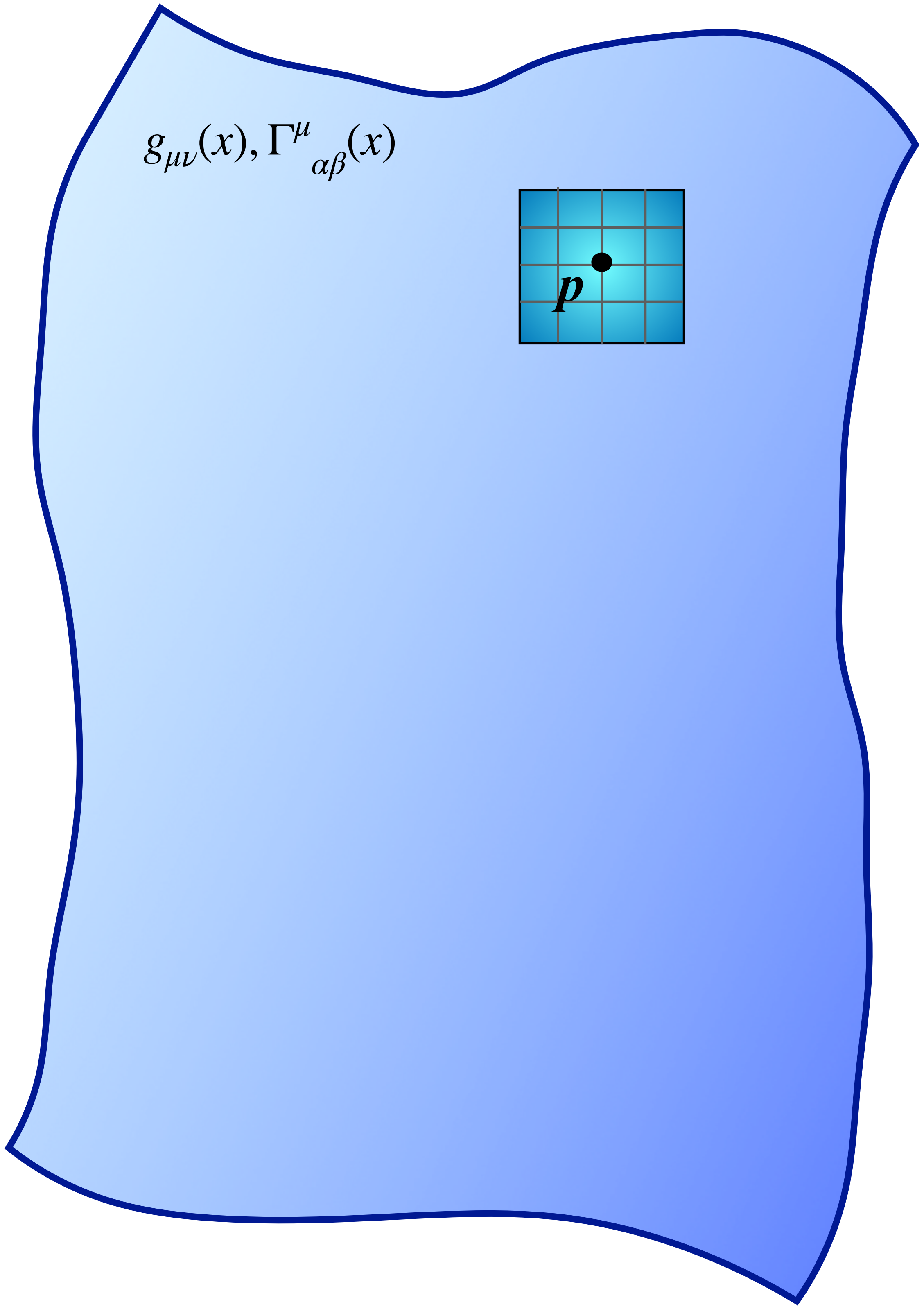

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Fermi normal coordinates around a geodesic (λ, y^i)

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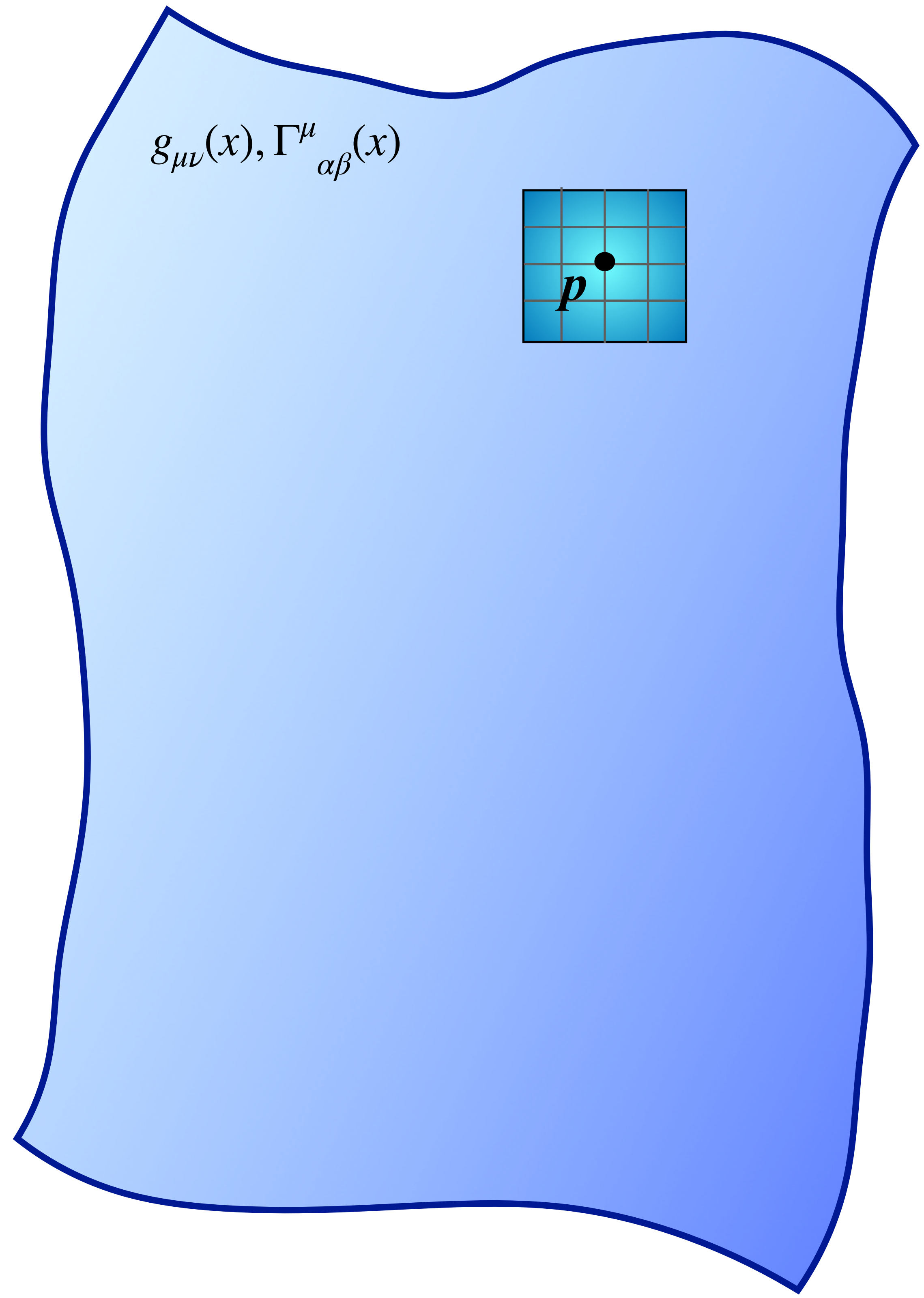

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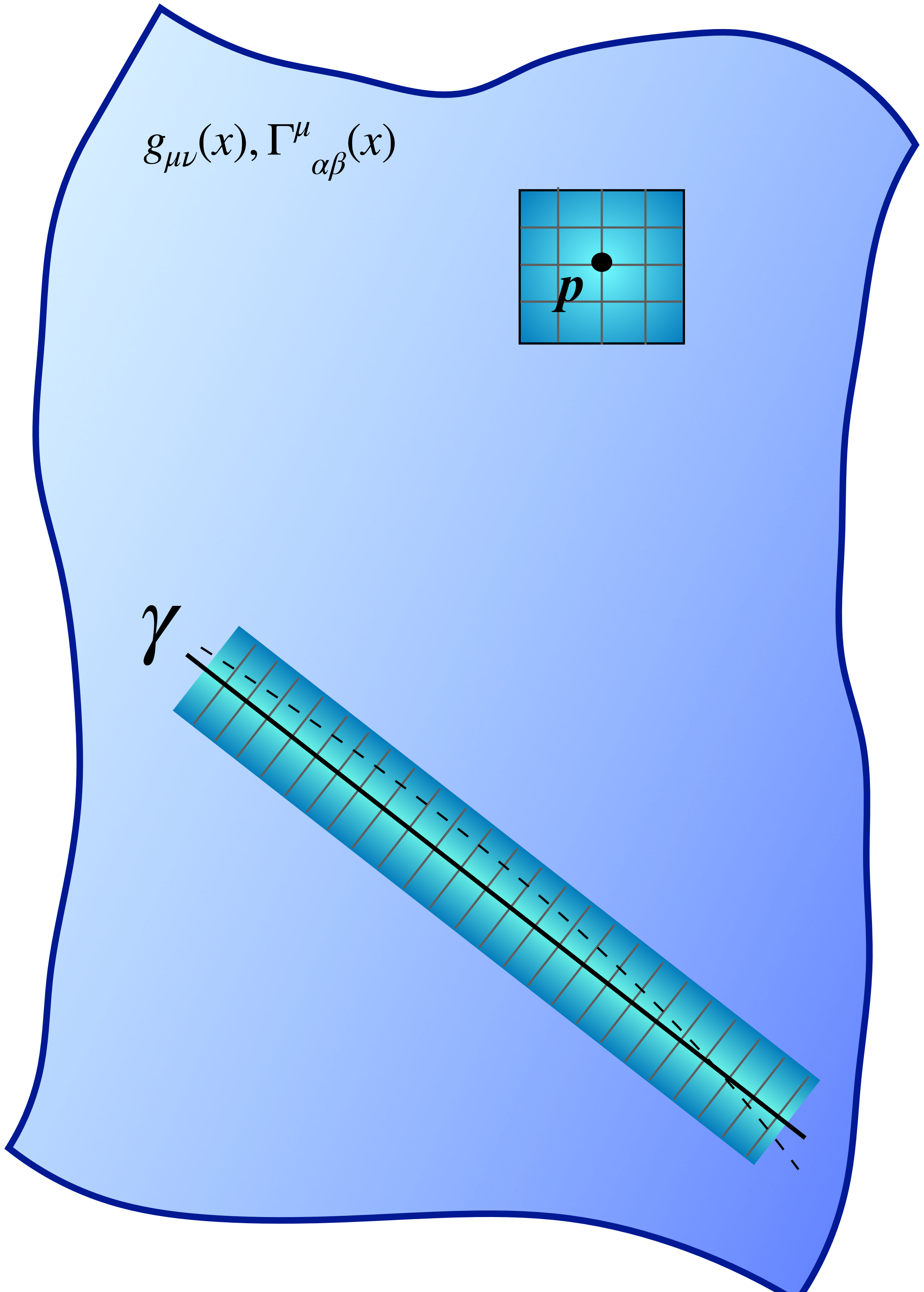

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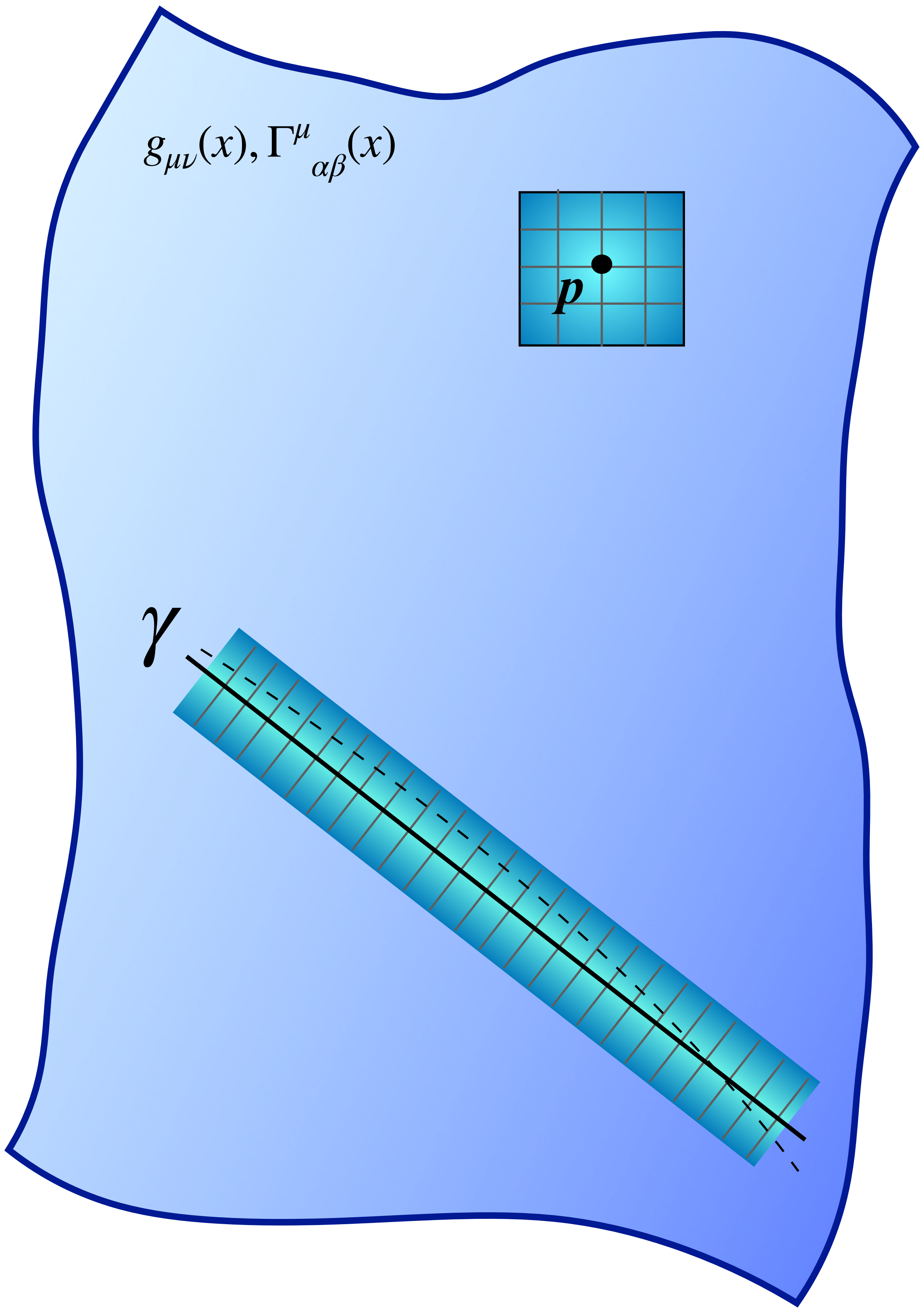
Geodesics nearby given by the GDE

$$\nabla_l \nabla_l \xi^\mu - R^\mu{}_{\nu\alpha\beta}(\lambda)\, l^\nu\, l^\alpha\, \xi^\beta = 0$$



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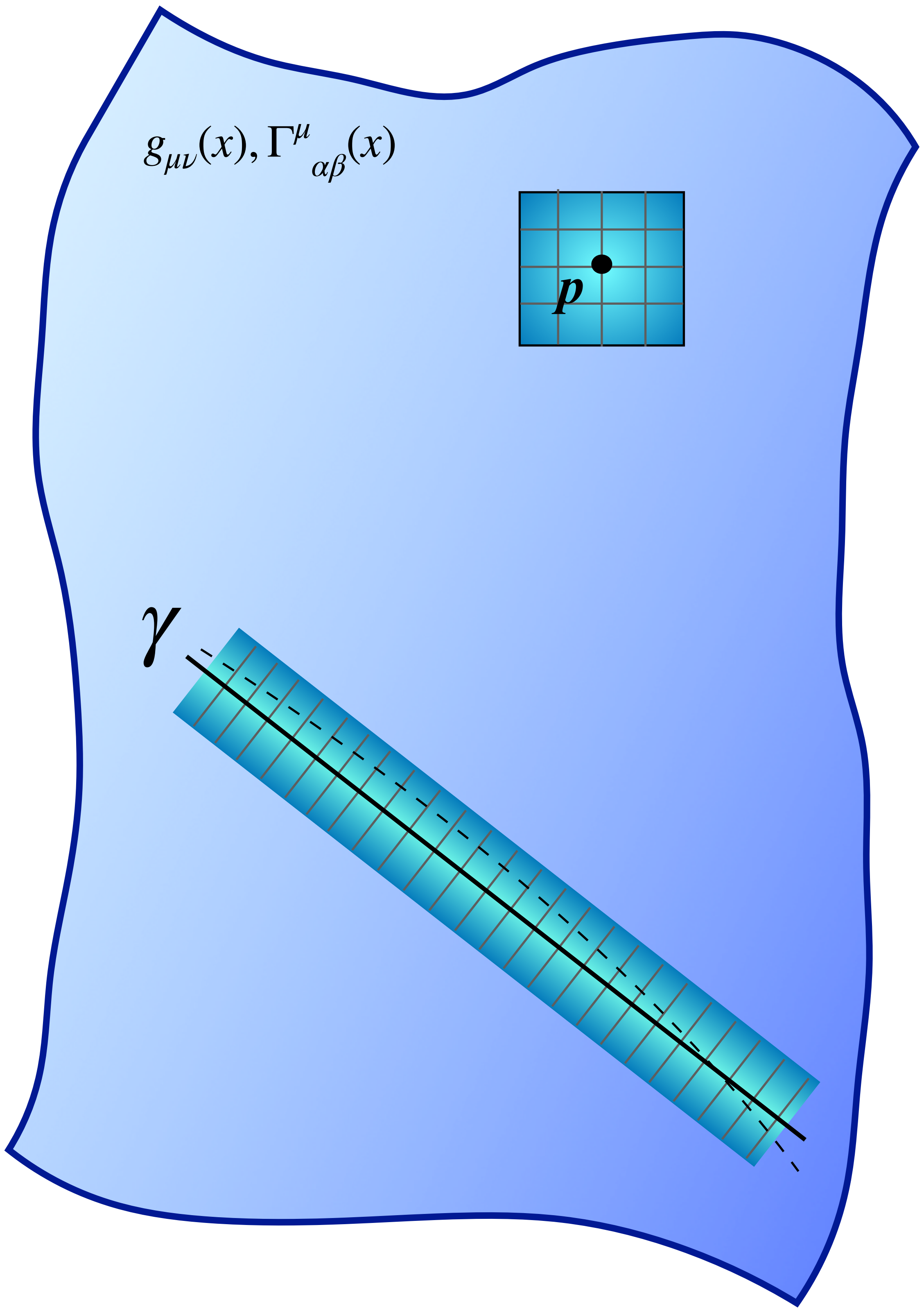
impact of curved geometry



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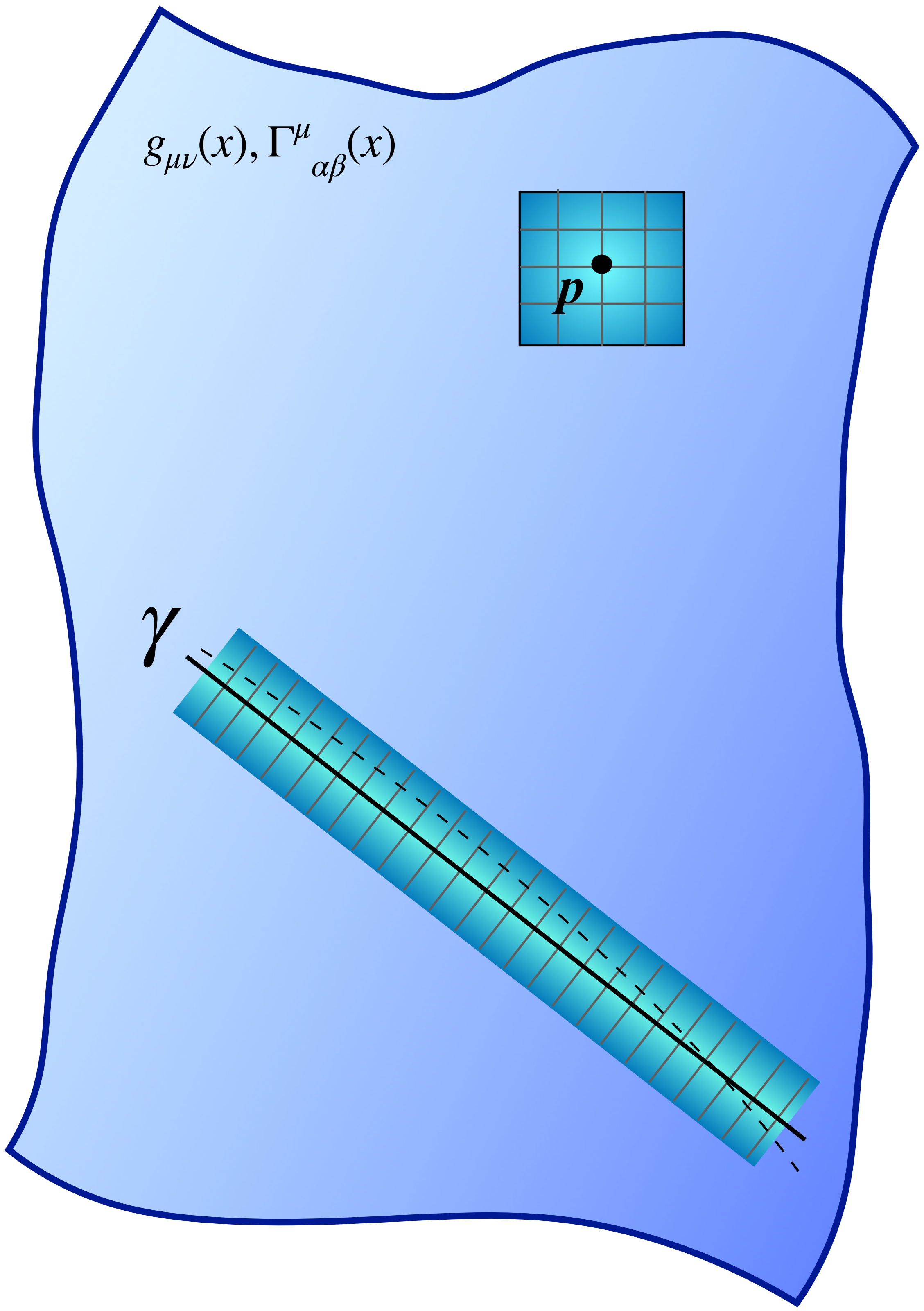
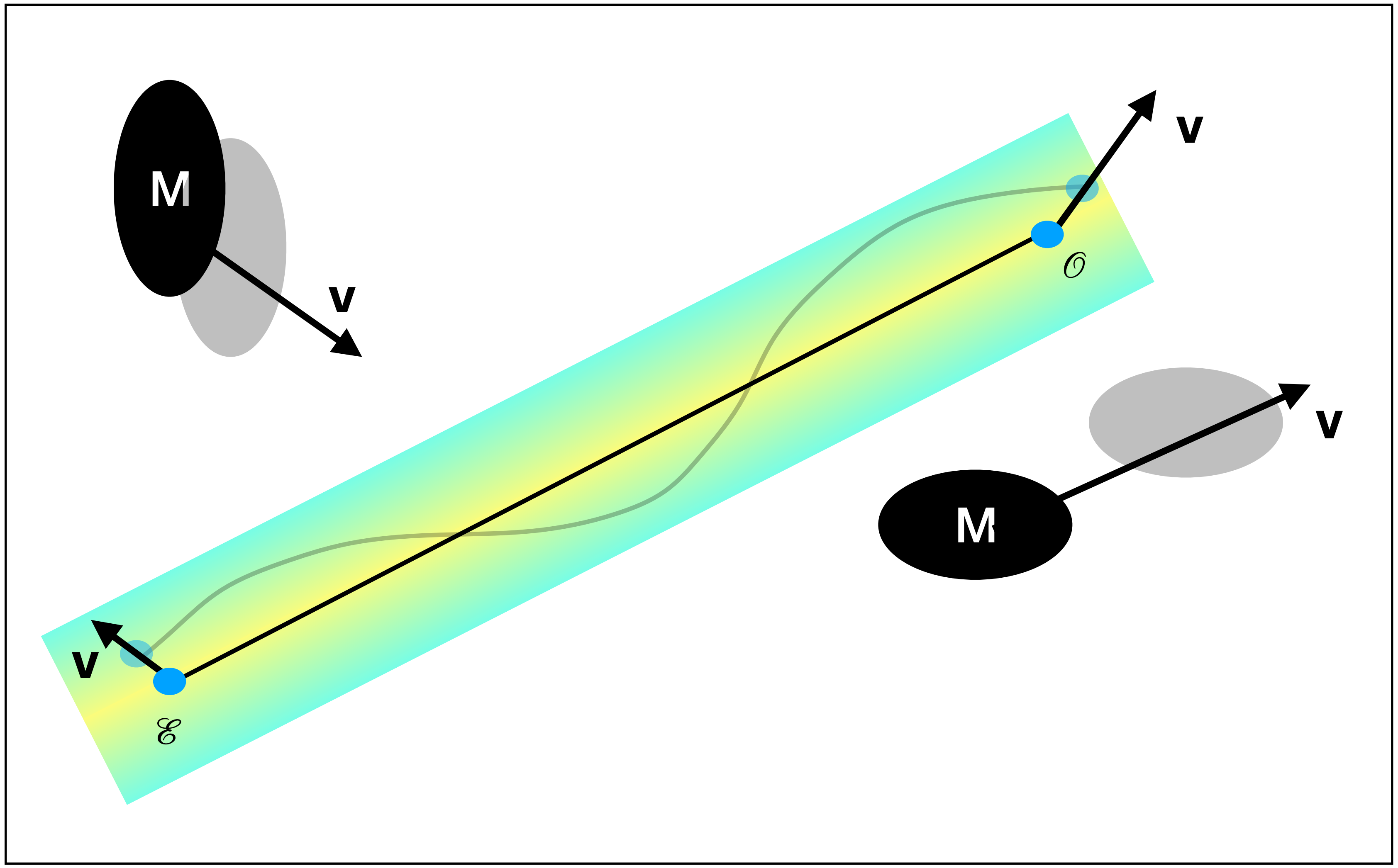
impact of curved geometry

Synge 1960, Bażański 1977, Alexandrov&Piragas 1979
Vines 2015, Puetzfeld&Obukhov 2016, Uzun 2018 …



Geometric approach

Redshift and position drift



Geometric approach

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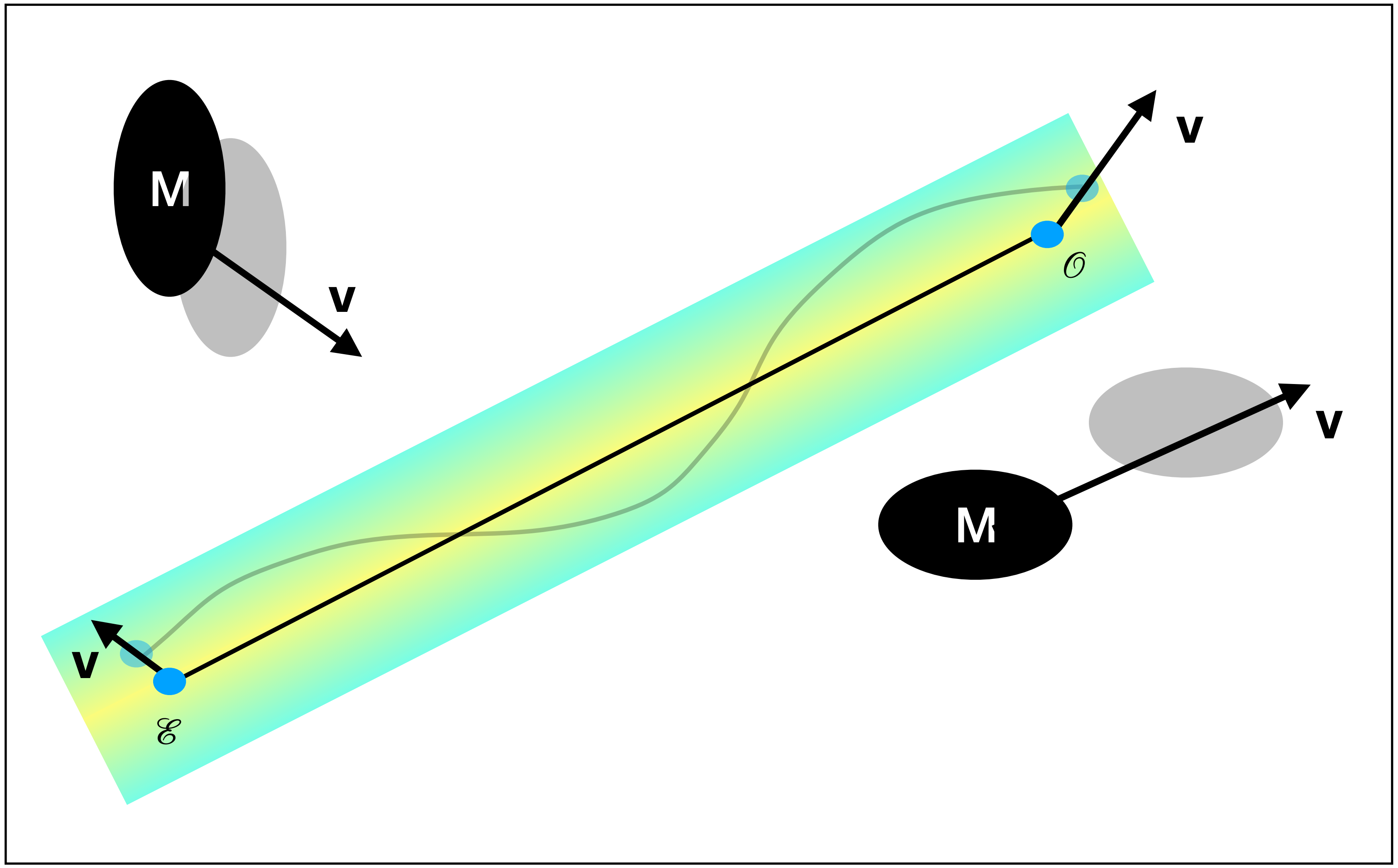

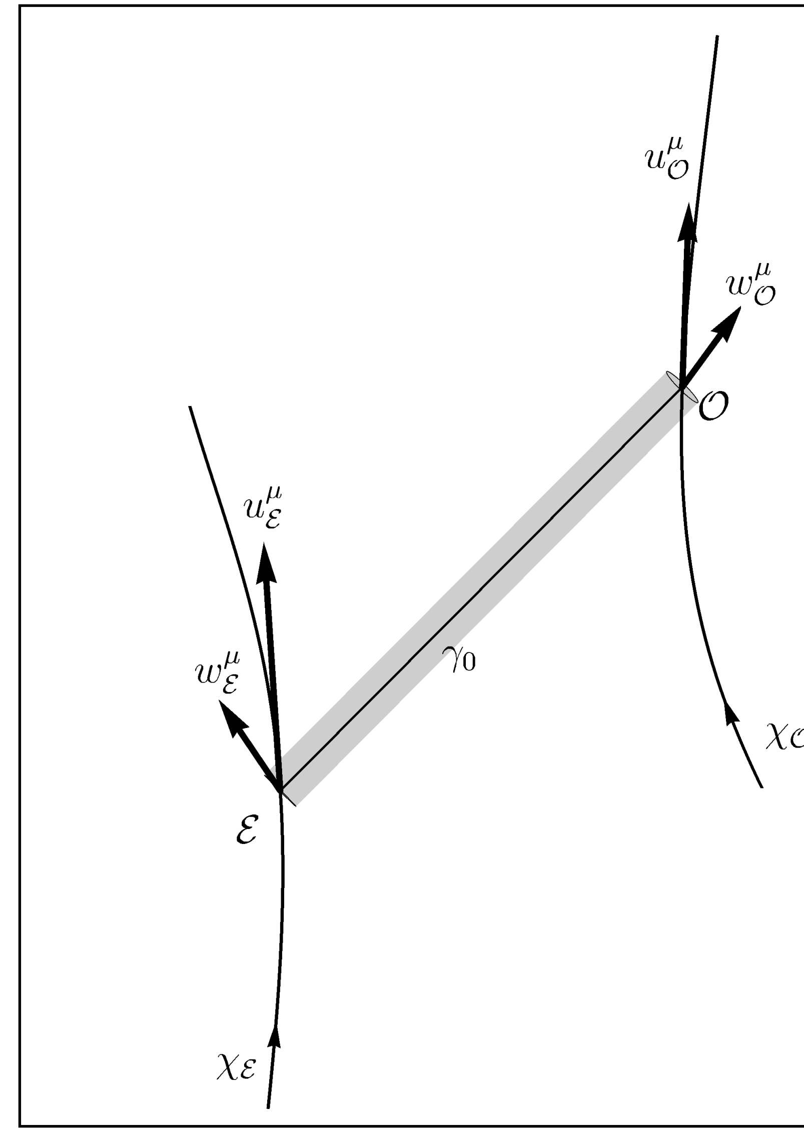

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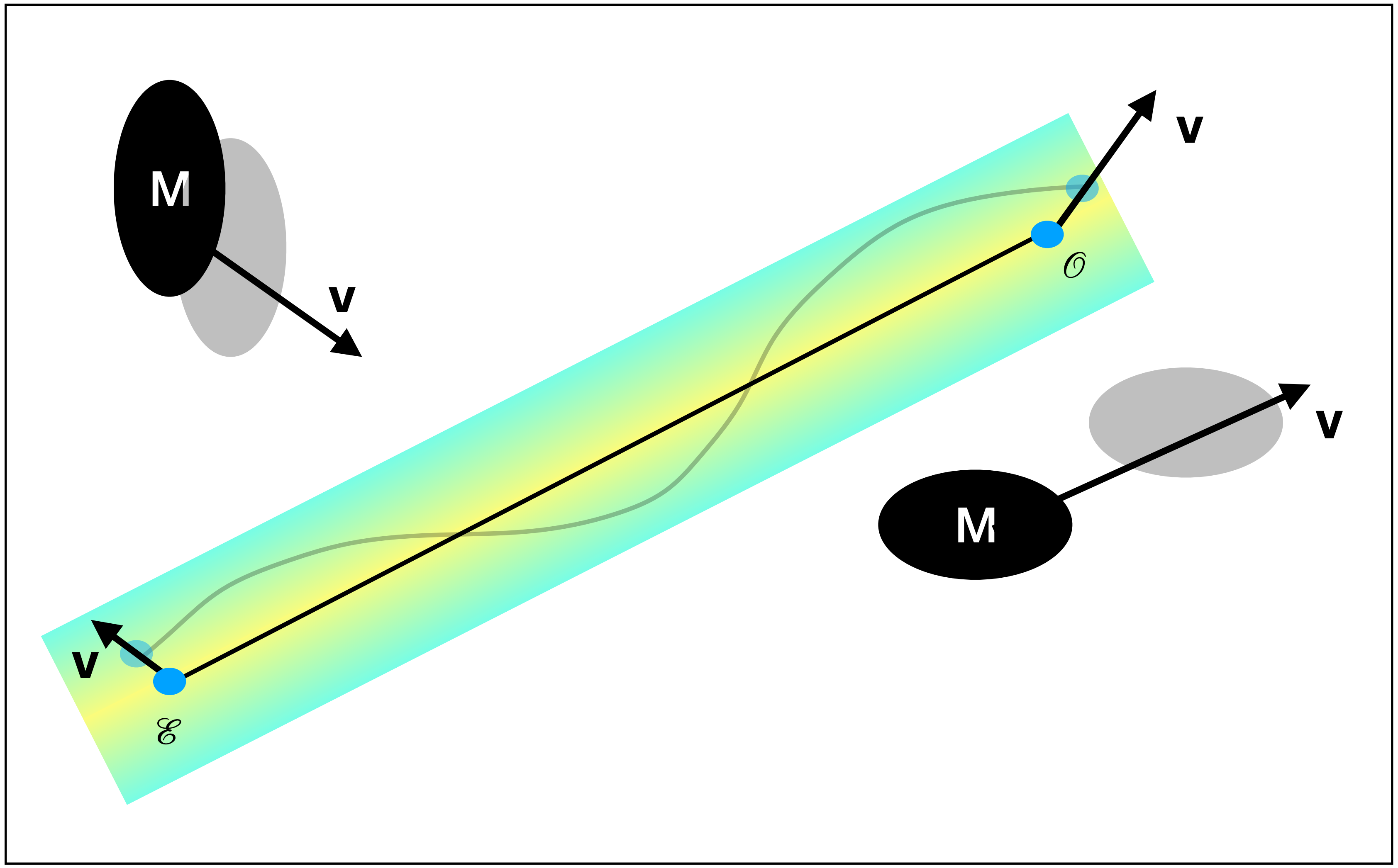

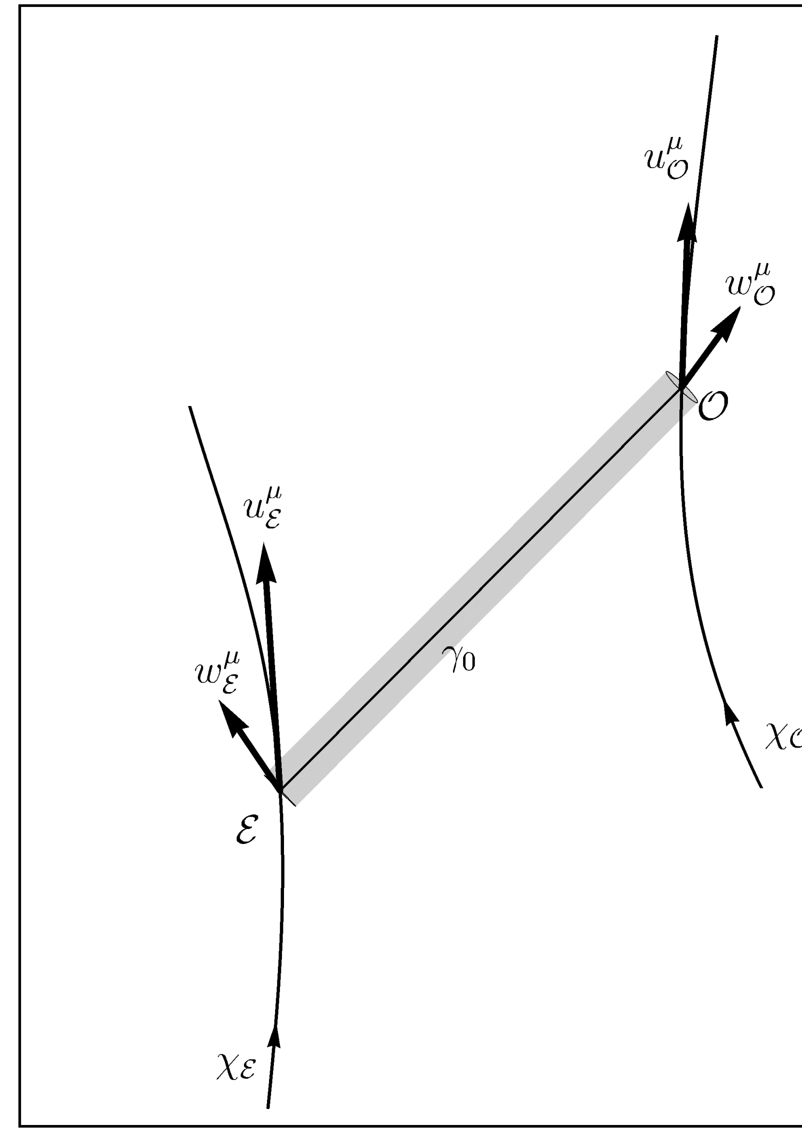

Impact of the spacetime geometry
only via curvature along the line of sight