The Real Quantum Gravity

CQ leads to FREE models

1. A Toy Model

The role of 'free' and 'pseudofree' models

2. An Ultralocal Model

A model with no spatial continuity

AQ leads to NON-FREE models

3. A Covariant Scalar

Adding spatial continuity to the previous model

4. Affine Quantum Gravity

The mother of all problems!

SOLVED !?! 🙂

NR: = NONRENORMALIZABLE

NR: A Toy Model

$$A_{g_0} = \int_0^T \{\tfrac{1}{2}[\dot{y}(t)^2 - y(t)^2] - g_0 y(t)^{-4}\}\, dt$$

$$\mathfrak{D}(A_{g_0=0}) \neq \mathfrak{D}(A_{g_0>0})$$

$f \neq pf$ 🙂

Classical to Quantum

$$\langle y'', T | y', 0\rangle_f = \sum_{n=0,1,2,3,\cdots} h_n(y'')h_n(y')e^{-i(n+1/2)T/\hbar}$$

$$\langle y'', T | y', 0\rangle_{pf} = 2\,\theta(y''y') \sum_{n=1,3,5,7,\cdots} h_n(y'')h_n(y')e^{-i(n+1/2)T/\hbar}$$

Affine Quantization-1

affine classical position and dilation variables

$$\{q,p\} = 1\,,\;\; q\{q,p\} = q\,,\;\; \{q,qp\} \equiv \{q,d\} = q \lessgtr 0$$

affine quantum position and dilation variables

$$[Q,P] = i\hbar I\,,\;\; Q[Q,P] = i\hbar Q\,,\;\; [Q,QP] = i\hbar Q$$
$$= [Q,(QP+PQ)+(QP-PQ)]/2 = i\hbar Q$$
$$= [Q,(QP+PQ)/2] \equiv [Q,D] = i\hbar Q\,,\quad Q \lessgtr 0$$

the Lie algebra for the affine group

Affine Quantization-2

action for Schrödinger's equation

$$A_Q = \int \langle \psi(t) | [i\hbar(\partial/\partial t) - \mathcal{H}(P,Q)] | \psi(t) \rangle \, dt$$

$$A'_Q = \int \langle \psi(t) | [i\hbar(\partial/\partial t) - \mathcal{H}'(D,Q)] | \psi(t) \rangle \, dt$$

canonical coherent states

$$|p,q\rangle = e^{-iqP/\hbar} \, e^{ipQ/\hbar} \, |\omega\rangle$$

affine coherent states

$$|p,q\rangle = e^{ipQ/\hbar} e^{-i \ln(q) D/\hbar} \, |b\rangle$$

action for enhanced classical equations

$$A_C = \int \langle p(t), q(t) | [i\hbar(\partial/\partial t) - \mathcal{H}(P,Q)] | p(t), q(t) \rangle \, dt$$

$$= \int \{ p(t)\dot{q}(t) - H(p(t), q(t)) \} \, dt$$

$$A'_C = \int \langle p(t), q(t) | [i\hbar(\partial/\partial t) - \mathcal{H}'(D,Q)] | p(t), q(t) \rangle \, dt$$

$$= \int \{ -q(t)\dot{p}(t) - H'(p(t), q(t)) \} \, dt,$$

Both operator pairs lead to similar classical stories, and with hbar > 0.

Favored Coordinates-1

Dirac: "Cartesian coordinates should lead to H(p,q)—>H(P,Q)"

canonical quantization

$$H(p,q) = \langle p,q|\mathcal{H}(P,Q)|p,q\rangle\ , \qquad (\omega Q + iP)|\omega\rangle = 0 \qquad 0$$

$$= \langle\omega|\mathcal{H}(P+p,Q+q)|\omega\rangle = \mathcal{H}(p,q) + \mathcal{O}(\hbar;p,q)$$

$$2\hbar[\,\|d|p,q\rangle\|^2 - |\langle p,q|d|p,q\rangle|^2] = \omega^{-1}dp^2 + \omega\,dq^2$$ 🙂

affine quantization

$$H'(pq,q) = \langle p,q|\mathcal{H}'(D,Q)|p,q\rangle\ , \quad [(Q-1)+iD/b]|b\rangle = 0 \qquad -2/b$$

$$= \langle b|\mathcal{H}'(D+pqQ,qQ)|b\rangle = \mathcal{H}'(pq,q) + \mathcal{O}'(\hbar;p,q)$$

$$2\hbar[\,d|p,q\rangle\|^2 - |\langle p,q|d|p,q\rangle|^2] = b^{-1}q^2dp^2 + bq^{-2}dq^2$$ 🙂

NR: Ultralocal Model-1

$$H(\pi,\varphi) = \int \{ \tfrac{1}{2}[\pi(x)^2 + m_0^2\varphi(x)^2] + g_0\varphi(x)^4 \}\, d^s\!x\,, \quad s \geq 1$$

$$f \neq pf$$

affine variables

$$\kappa(x) \equiv \pi(x)\varphi(x), \quad \varphi(x) \lessgtr 0$$

$$\{\varphi(x), \kappa(x')\} = \delta^s(x - x')\,\varphi(x), \quad \varphi(x) \lessgtr 0$$

$$H'(\kappa,\varphi) = \int \{ \tfrac{1}{2}[\kappa(x)\varphi(x)^{-2}\kappa(x) + m_0^2\varphi(x)^2] + g_0\varphi(x)^4 \}\, d^s\!x$$

Classical to Quantum

$$[\hat{\varphi}(x), \hat{\kappa}(x')] = i\hbar\,\delta^s(x - x')\,\hat{\varphi}(x)\,, \quad \hat{\varphi}(x) \lessgtr 0$$

🙂 🙂

$$\hat{\varphi}(x) = \int B(x,\lambda)^\dagger \lambda B(x,\lambda)\, d\lambda\,, \qquad B(x,\lambda) = A(x,\lambda) + c(\lambda)I\,, \quad A(x,\lambda)\,|0\rangle = 0$$

$$\hat{\kappa}(x) = -\,i\hbar\tfrac{1}{2}\int B(x,\lambda)^\dagger[\lambda(\partial/\partial\lambda) + (\partial/\partial\lambda)\lambda]B(x,\lambda)\; d\lambda$$

NR: Ultralocal Model-2

$$\hat{\varphi}(x) = \varphi(x) \quad , \quad \hat{\kappa}(x) = -\, i\hbar(1/2)[\varphi(x)\,(\delta/\delta\varphi(x)) + (\delta/\delta\varphi(x))\,\varphi(x)]$$

Schrödinger‘s equation

$$i\hbar\,\partial\Psi(\varphi,t)/\partial t = \left[\int\{(1/2)[\hat{\kappa}(x)\varphi(x)^{-2}\hat{\kappa}(x) + m_0^2\varphi(x)^2] + g_0\varphi(x)^4\}\;d^sx\right]\,\Psi(\varphi,t)$$

$$\hat{\kappa}(x)\,\varphi(x)^{-1/2} = 0\;, \qquad \Psi(\varphi) = e^{-W(\varphi)}\,\Pi_x\,|\,\varphi(x)\,|^{-1/2}$$ **formal**

regularization

$$\varphi(x) \to \varphi_{\mathbf{k}} \equiv \varphi(\mathbf{k}a)\,, \quad a > 0\,,\;\mathbf{k} \in \{0, \pm 1, \pm 2, \pm 3, \cdots\}^s$$

$$\Psi_r(\varphi) = e^{-W_r(\varphi)}\,\Pi_{\mathbf{k}}(ba^s)^{1/2}\,|\,\varphi_{\mathbf{k}}\,|^{-(1-2ba^s)/2}$$ **regularized**

$$\int|\,\Psi_r(\varphi)\,|^2\,\Pi_{\mathbf{k}}d\varphi_{\mathbf{k}} = 1$$

NR: Ultralocal Model-3

$$\Psi_r(\varphi) = \Pi_{\mathbf{k}} e^{-W_r(\varphi_{\mathbf{k}})} (ba^s)^{1/2} |\varphi_{\mathbf{k}}|^{-(1-2ba^s)/2}$$ 🙂

characteristic functional

$$C(f) = \lim_{a\to 0} \Pi_{\mathbf{k}} \int e^{if_{\mathbf{k}}\varphi_{\mathbf{k}}/\hbar} (ba^s)\, e^{-2W_r(\varphi_{\mathbf{k}})} |\varphi_{\mathbf{k}}|^{-(1-2ba^s)}\, d\varphi_{\mathbf{k}}$$

$$C(f) = \lim_{a\to 0} \Pi_{\mathbf{k}} \int \{1 - (ba^s) \int [1 - e^{if_{\mathbf{k}}\varphi_{\mathbf{k}}/\hbar}]\, e^{-2W_r(\varphi_{\mathbf{k}})} |\varphi_{\mathbf{k}}|^{-(1-2ba^s)}\, d\varphi_{\mathbf{k}}\}$$

$$C(f) = \exp\{-b \int d^s x \int [1 - e^{if(x)\lambda/\hbar}]\, e^{-2w(\lambda)}\, d\lambda/|\lambda|\}$$ **Poisson** 🙂

pseudofree model: equal-spacing spectrum, NO zero-point energy

NR: Covariant Model-1

$$s \geq 4 \quad , \qquad f \neq pf$$

$$H(\pi, \varphi) = \int \{(1/2)[\pi(x)^2 + (\nabla \varphi)(x)^2 + m_0^2 \varphi(x)^2] + g_0 \varphi(x)^4\} \, d^s x$$

affine variables

$$\kappa(x) \equiv \pi(x)\varphi(x) \,, \quad \varphi(x) \lessgtr 0$$

$$H'(\kappa, \varphi) = \int \{(1/2)[\kappa(x)\varphi(x)^{-2}\kappa(x) + (\nabla \varphi)(x)^2 + m_0^2 \varphi(x)^2] + g_0 \varphi(x)^4\} \, d^s x$$

Classical to Quantum

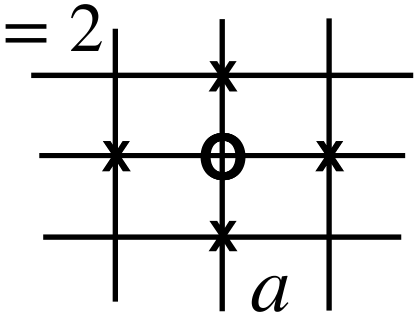
$$\hat{\varphi}(x) = \varphi(x) \quad , \qquad \hat{\kappa}(x) = -i\hbar(1/2)[\varphi(x)\,(\delta/\delta\varphi(x)) + (\delta/\delta\varphi(x))\,\varphi(x)]$$

Schrödinger's equation

$$i\hbar\,\partial\Psi(\varphi, t)/\partial t = \left[\int \{(1/2)[\hat{\kappa}(x)\varphi(x)^{-2}\hat{\kappa}(x) + (\nabla \varphi)(x)^2 + m_0^2 \varphi(x)^2] + g_0 \varphi(x)^4\} \; d^s x\right] \Psi(\varphi, t)$$

NR: Covariant Model-2

$$\hat{\kappa}(x)\,\varphi(x)^{-1/2} = 0\ , \quad \Psi(\varphi) = e^{-Y(\varphi)}\,\Pi_x\,|\,\varphi(x)\,|^{-1/2}$$ **formal**



a DIFFERENT regularization

$$\varphi(x) \to \varphi_{\mathbf{k}} \equiv \varphi(\mathbf{k}a)\,, \quad a > 0\,,\ \mathbf{k} \in \{0, \pm 1, \pm 2, \pm 3, \cdots\}^s$$

$J_{\mathbf{k},\mathbf{l}} \equiv 1/(1+2s)$, $\mathbf{l} = \mathbf{k}$ and 2s nearest spatial neighbors to $\mathbf{k}$; otherwise, $J_{\mathbf{k},\mathbf{l}} \equiv 0$

$$\Psi_r(\varphi) = e^{-Y_r(\varphi)}\,\Pi_{\mathbf{k}}[\Sigma_{\mathbf{l}} J_{\mathbf{k},\mathbf{l}}\,\varphi_{\mathbf{l}}^2]^{-(1-2ba^s)/4}$$ **regularized** 🙂

$$1 = \int |\,\Psi_r(\varphi)\,|^2\,\Pi_{\mathbf{k}} d\varphi_{\mathbf{k}}\ , \quad \varphi_{\mathbf{k}} = \rho\eta_{\mathbf{k}}\ , \quad R = 2ba^s N'$$ **‘divergence free’**

$$= \int e^{-2Y_r(\rho\eta)}\,\delta(1-\Sigma_{\mathbf{k}}\eta_{\mathbf{k}}^2)\,\Pi_{\mathbf{k}}[\Sigma_{\mathbf{l}} J_{\mathbf{k},\mathbf{l}}\,\eta_{\mathbf{l}}^2]^{-(1-2ba^s)/2}\,d\eta_{\mathbf{k}}]\,\rho^{R-1}\,d\rho$$ 🙂 🙂

There are indications that affine quantization of φ^4_4 models is non-trivial 🙂 🙂

NR: Quantum Gravity-1

$g_{ab}(x)\,dx^a\,dx^b > 0$, $g_{ac}(x)\,g^{bc}(x) \equiv \delta^b_a$, $\pi^{ac}(x)\,g_{bc}(x) \equiv \pi^a_b(x)$, $g(x) \equiv \det[g_{ab}(x)] > 0$

$$H'(\pi, g) = \int \{ g^{-1/2}[\pi^a_b\pi^b_a - (1/2)\pi^a_a\pi^b_b] + g^{1/2}\,{}^{(3)}\!R \}\, d^3x$$

Classical to Quantum

$f \neq pf$

$$[\hat{\pi}^a_b(x), \hat{\pi}^c_d(x')] = i\hbar(1/2)\,\delta^3(x,x')\,[\delta^a_d\hat{\pi}^c_b(x) - \delta^c_b\hat{\pi}^a_d(x)]$$
$$[\hat{g}_{ab}(x), \hat{\pi}^c_d(x')] = i\hbar(1/2)\,\delta^3(x,x')\,[\delta^c_a\hat{g}_{bd}(x) + \delta^c_b\hat{g}_{ad}(x)]$$
$$[\hat{g}_{ab}(x), \hat{g}_{cd}(x')] = 0\ ,$$

$\{\hat{g}_{ab}(x)\} > 0$ 🙂 🙂

$$\hat{g}_{ab}(x) = \int_+ B^\dagger(x,\gamma)\gamma_{ab}B(x,\gamma)d\gamma\,, \qquad \{\gamma_{ab}\} > 0\,,\ d\gamma \equiv \Pi_{a\le b}\,d\gamma_{ab}$$
$$\hat{\pi}^a_b(x) = -\,i\hbar(1/2)\int_+ \{B(x,\gamma)^\dagger[\gamma_{bc}(\partial/\partial\gamma_{ac}) + (\partial/\partial\gamma_{ac})\gamma_{bc}]B(x,\gamma)\}d\gamma$$

NR: Quantum Gravity-2

$$\hat{g}_{ab}(x) = g_{ab}(x)$$

N.B. The field $\pi^{ab}(x)$ is NOT made an operator.

$$\hat{\pi}^a_b(x) = -i\hbar(1/2)[g_{bc}(x)(\delta/\delta g_{ac}(x)) + (\delta/\delta g_{ac}(x))g_{bc}(x)]$$

Schrödinger's equation

$$i\hbar\,\partial\Psi(\{g\},t)/\partial t = \Big[\int[\hat{\pi}^a_b(x)g(x)^{-1/2}\hat{\pi}^b_a(x) - \tfrac{1}{2}\hat{\pi}^a_a(x)g(x)^{-1/2}\hat{\pi}^b_b(x) + g(x)^{1/2}\,{}^{(3)}R(x)]\,d^3x\Big]\;\Psi(\{g\},t)$$

🙂

$$\hat{\pi}^a_b(x)\,g(x)^{-1/2} = 0$$

$$\Psi(\{g\}) = Y(\{g\})\,\Pi_x\,g(x)^{-1/2}$$ **formal**

$$\Psi_r(\{g\}) = Y_r(\{g\})\Pi_{\mathbf{k}}\,[\Sigma_{\mathbf{l}}J_{\mathbf{k},\mathbf{l}}\,g_{\mathbf{l}}]^{-(1-ba^3)/2}$$ **regularized**

🙂 🙂 **SMOOTH METRICS & CONSTRAINTS** 🙂 🙂

$$\{[\hat{\pi}^a_b(x)\hat{\pi}^b_a(x) - \tfrac{1}{2}\hat{\pi}^a_a(x)\hat{\pi}^b_b(x)] + g(x)\,{}^{(3)}R(x)\,\}\Phi(\{g\}) = 0 \quad , \quad \{g(\cdot)\} \in C^2$$

NR: Quantum Gravity-3

affine gravity coherent states

$$\eta(x) = \{\eta^a_b(x)\}$$

$$|\pi,\eta\rangle = e^{(i/\hbar)\int \pi^{ab}(x)\,\hat{g}_{ab}(x)\,d^3x}\, e^{-(i/\hbar)\int \eta^a_b(x)\,\hat{\pi}^b_a(x)\,d^3x}\,|\beta\rangle\ \ [=|\pi,g\rangle\,]$$

$$\langle\pi,\eta|\,\hat{g}_{ab}(x)\,|\pi,\eta\rangle = [e^{\eta(x)/2}]^c_a\,\langle\beta|\,\hat{g}_{cd}(x)\,|\beta\rangle\,[e^{\eta(x)/2}]^d_b \equiv g_{ab}(x)$$ 🙂

$$\langle\pi,\eta|\,\hat{\pi}^a_b(x)\,|\pi,\eta\rangle = \pi^{ac}(x)\,g_{cb}(x) \equiv \pi^a_b(x)$$ 🙂 🙂

$$\langle\pi'',g''|\pi',g'\rangle = \exp\Big\{-2\int\beta(x)\,d^3x \times \ln\Big\{\frac{\det\{\frac{1}{2}[g''^{ab}(x)+g'^{ab}(x)] + i\frac{1}{2\hbar}\beta(x)^{-1}[\pi''^{ab}(x)-\pi'^{ab}(x)]\}}{\det[g''^{ab}(x)]^{1/2}\ \det[g'^{ab}(x)]^{1/2}}\Big\}\Big\}$$

ultralocal : NO constraints imposed

Favored Coordinates-2

enhanced classical Hamiltonian density

$$H'(\pi^a_b(x), g_{cd}(x)) = \langle \pi, g \,|\, \mathscr{H}'(\hat{\pi}^a_b(x), \hat{g}_{cd}(x)) \,|\, \pi, g \rangle$$

$$= \langle \beta \,|\, \mathscr{H}'(\hat{\pi}^a_b(x) + \pi^{aj}(x)[e^{\eta(x)/2}]^e_j \hat{g}_{ef}(x)[e^{\eta(x)/2}]^f_b, [e^{\eta(x)/2}]^e_c \hat{g}_{ef}(x)[e^{\eta(x)/2}]^f_d) \,|\, \beta \rangle$$

$$\langle \beta \,|\, [e^{\eta(x)/2}]^e_c \hat{g}_{ef}(x)[e^{\eta(x)/2}]^f_d \,|\, \beta \rangle = [e^{\eta(x)/2}]^e_c \langle \beta \,|\, \hat{g}_{ef}(x) \,|\, \beta \rangle [e^{\eta(x)/2}]^f_d \equiv g_{cd}(x)$$

$$= \mathscr{H}'(\pi^{aj}(x) g_{jb}(x), g_{cd}(x)) + \mathscr{O}'(\hbar; \pi, g)$$

$$= \mathscr{H}'(\pi^a_b(x), g_{cd}(x)) + \mathscr{O}'(\hbar; \pi, g)$$ 🙂 🙂

these are favored affine gravity coordinates

Favored Coordinates-3

$$|p,q\rangle = e^{-iqP/\hbar}\, e^{ipQ/\hbar}\,|\omega\rangle\ , \qquad (\omega Q + iP)\,|\omega\rangle = 0$$

$$2\hbar[\,\|\,d\,|p,q\rangle\,\|^2 - |\langle p,q\,|\,d|p,q\rangle\,|^2] = \omega^{-1}dp^2 + \omega\,dq^2$$

$$|p,q\rangle = e^{ipQ/\hbar} e^{-i\ln(q)D/\hbar}\,|b\rangle\ , \qquad [(Q-1) + iD/b]\,|b\rangle = 0$$

$$2\hbar[\,\|\,d\,|p,q\rangle\,\|^2 - |\langle p,q\,|\,d|p,q\rangle\,|^2] = b^{-1}q^2dp^2 + bq^{-2}dq^2$$

$$|\pi,g\rangle = e^{(i/\hbar)\int \pi^{ab}(x)\,\hat{g}_{ab}(x)\,d^3x}\; e^{-(i/\hbar)\int \eta^a_b(x)\,\hat{\pi}^b_a(x)\,d^3x}\,|\beta\rangle$$

$$C\hbar[\,\|\,d\,|\pi,g\rangle\,\|^2 - |\langle \pi,g\,|\,d|\pi,g\rangle\,|^2]$$

$$= \int [(\beta(x)\hbar)^{-1} g_{ab}g_{cd}\,d\pi^{bc}d\pi^{da} + (\beta(x)\hbar)\,g^{ab}g^{cd}\,dg_{bc}dg_{da}]\;d^3x$$

Fubini-Study metrics

NR: Quantum Gravity-4

$$|\pi, g\rangle = e^{(i/\hbar)\int \pi^{ab}(x)\,\hat{g}_{ab}(x)\,d^3x}\, e^{-(i/\hbar)\int \eta^a_b(x)\,\hat{\pi}^b_a(x)\,d^3x}\,|\beta\rangle$$

$$\langle \pi, g\,|\,\pi', g'\rangle = \exp\Big\{-2\int \beta(x)\,\ln\Big\{\frac{\det\{\frac{1}{2}[g^{ab}(x)+g'^{ab}(x)] + i\frac{1}{2\hbar}\beta(x)^{-1}[\pi^{ab}(x)-\pi'^{ab}(x)]\}}{\det[g^{ab}(x)]^{1/2}\;\det[g'^{ab}(x)]^{1/2}}\Big\}\,d^3x\Big\}$$

complex polarization

$$C^r_s(x)\,\langle \pi, g\,|\,\Psi\rangle \equiv \Big[-i\hbar g^{rt}(x)\frac{\delta}{\delta\pi^{st}(x)} + \delta^r_s + \beta(x)^{-1}g_{st}(x)\frac{\delta}{\delta g_{rt}(x)}\Big]\langle \pi, g\,|\,\Psi\rangle = 0$$

$$A \equiv \frac{1}{2\hbar}\int C^s_r(x)^\dagger C^r_s(x)\,\beta(x)\,d^3x \quad , \quad \lim_{\nu\to\infty} \mathcal{N}_\nu e^{-\nu\,TA}\,\delta\{\pi-\pi'\}\delta\{g-g'\} = \langle \pi, g\,|\,\pi', g'\rangle$$

an analog to Wiener measure

NR: Quantum Gravity-5

$$\pi'' = \pi^{ab}(\,\cdot\,,T)\ ,\ g'' = g_{ab}(\,\cdot\,,T) \quad ; \quad \pi' = \pi^{ab}(\cdot,0)\ ,\ g' = g_{ab}(\cdot,0) \quad ; \quad T > 0$$

kinematic and physical functional integrals

$$\langle \pi'', g'' | \pi', g' \rangle = \lim_{\nu \to \infty} \mathcal{N}_\nu \int \exp[-(i/\hbar) \textstyle\int g_{ab}\, \dot{\pi}^{ab}\ d^3x\, dt]$$

$$\times \exp\{-(1/2\nu\hbar) \textstyle\int [(\beta(x)\hbar)^{-1} g_{ab} g_{cd} \dot{\pi}^{bc} \dot{\pi}^{da} + (\beta(x)\hbar)\, g^{ab} g^{cd} \dot{g}_{bc} \dot{g}_{da}]\ d^3x\, dt\}$$

$$\times\, \Pi_{x,t} \Pi_{[a \le b]}\, d\pi^{ab}(x,t)\, dg_{ab}(x,t)$$ 🙂

projection operator IE enforces the constraints

$$\langle \pi'', g'' | \mathbb{E} | \pi', g' \rangle = \lim_{\nu \to \infty} \mathcal{N}'_\nu \int \exp\{-(i/\hbar) \textstyle\int [g_{ab}\, \dot{\pi}^{ab} + N^a H_a + NH]\, d^3x\, dt\}$$

$$\times \exp\{-(1/2\nu\hbar) \textstyle\int [(\beta(x)\hbar)^{-1} g_{ab} g_{cd} \dot{\pi}^{bc} \dot{\pi}^{da} + (\beta(x)\hbar)\, g^{ab} g^{cd} \dot{g}_{bc} \dot{g}_{da}]\ d^3x\, dt\}$$

$$\times\, [\Pi_{x,t} \Pi_{[a \le b]}\, d\pi^{ab}(x,t)\, dg_{ab}(x,t)\,]\, \mathscr{D}R\{N^a, N\}$$ 🙂 🙂

first and second class constraints

Comparison List

The Canonical Story

$$\{g_{ab}(x)\} \geqslant 0$$

complex variables

part Euclidean

discrete metric

$$\{\tilde{E}^a_i(x)\ ,A_b{}^j(y)\} = -\,i\delta^a_b\delta^j_i\ \delta^3(x,y)$$

$$\hat{\tilde{E}}^a_i(x) = -\,i\,\delta/\delta A_a{}^i(x)$$

FAVORED CANONICAL VARIABLES ???

NON-Cartesian coordinates yield a FALSE quantum theory

The Affine Story

$$\{g_{ab}(x)\} > 0$$

real variables

all Lorentzian

continuous metric* **C.R. *lives***

$$[\hat{\pi}^a_b(x),\hat{\pi}^c_d(y)] = i\hbar(1/2)\,\delta^3(x,y)\,[\delta^a_d\hat{\pi}^c_b(x) - \delta^c_b\hat{\pi}^a_d(x)]$$

$$[\hat{g}_{ab}(x),\hat{\pi}^c_d(y)] = i\hbar(1/2)\,\delta^3(x,y)\,[\delta^c_a\hat{g}_{bd}(x) + \delta^c_b\hat{g}_{ad}(x)]$$

$$[\hat{g}_{ab}(x),\hat{g}_{cd}(y)] = 0$$

FAVORED AFFINE VARIABLES !!! 🙂

proper affine coordinates yield a VALID quantum theory 🙂

Today's Message

1. Nonrenormalizable models are NOT continuously connected to their own free model.

2. If Q&P then Q&D, where D=(PQ+QP)/2. Note that [Q,P]=i hbar, and then [Q,D]=i hbar Q, which is the Lie algebra of the affine group.

3. The favored pair q&p to promote are "Cartesian coordinates" which make a flat plane. The favored pair q&pq to promote have an affine form on a Lobachevsky plane of constant negative curvature.

4. Canonical quantization of nonrenormalizable models fails, but affine quantization is successful.

5. Affine quantization of gravity offers self-adjoint momentric and metric fields that respect positivity requirements.

THANK YOU

arXiv:1811.09582

arXiv:1903.11211 ⟵

Favored Coordinates-4

$$[S_2\,,S_3] = i\hbar\,S_1 \quad , \quad S_1^2 + S_2^2 + S_3^2 = \hbar^2\,s(s+1)I_{2s+1} \quad , \quad s \in (1/2)\{1,2,3,\cdots\}$$

$$S_3\,|\,s,m\rangle = m\hbar\,|\,s,m\rangle \;\; , \;\; m \in \{-s,\cdots,s-1,s\} \;\; , \qquad (S_1 + iS_2)\,|\,s,s\rangle = 0$$

spin coherent states

$$|\,\theta,\varphi\rangle \equiv e^{-i\varphi S_3/\hbar}e^{-i\theta S_2/\hbar}\,|\,s,s\rangle$$

$$|\,p,q\rangle \equiv e^{-i(q/(s\hbar)^{1/2})S_3/\hbar}\,e^{-i\cos^{-1}(p/(s\hbar)^{1/2})S_2/\hbar}\,|\,s,s\rangle$$

$$-\pi(s\hbar)^{1/2} < q \leq \pi(s\hbar)^{1/2} \quad , \quad -(s\hbar)^{1/2} \leq p \leq (s\hbar)^{1/2}$$

$$\begin{aligned} d\sigma^2 &= 2\hbar[\,\|\,d|\,\theta,\varphi\rangle\,\|^2 - |\,\langle\theta,\varphi\,|\,d|\,\theta,\varphi\rangle\,|^2\,] \\ &= (s\hbar)[d\theta^2 + \sin^2(\theta)^2\,d\varphi^2] \\ &= (1 - p^2/s\hbar)^{-1}dp^2 + (1 - p^2/s\hbar)\,dq^2 \end{aligned}$$

False Quantum Theories

classical harmonic oscillator

$$\{q, p\} = 1 \quad , \qquad \{\bar{q}, \bar{p}\} = 1$$

$$H(p, q) = (1/2)[p^2 + q^2] \quad , \qquad \mathbf{e\,g}\ , \quad p = \bar{p}/\bar{q}^2 \ , \quad q = \bar{q}^3/3$$

$$H(p, q) = \bar{H}(\bar{p}, \bar{q}) = (1/2)[\bar{p}^2/\bar{q}^4 + \bar{q}^6/9]$$

quantum harmonic oscillator

$$\mathscr{H}(P, Q) = (1/2)[P^2 + Q^2]$$ ✓

$$\bar{\mathscr{H}}(\bar{P}, \bar{Q}) = (1/2)[\bar{P}\bar{Q}^{-4}\bar{P} + \bar{Q}^6/9]$$ ✗

VERY DIFFERENT SPECTRUMS

Basic Path Integrals

a canonical quantization

$$|p,q\rangle = e^{-iqP/\hbar}\, e^{ipQ/\hbar}\, |\omega = 1\rangle\ , \quad \mathcal{H}(P,Q) = M\int h(p,q)\ |p,q\rangle\langle p,q|\ dp\, dq$$

$$\langle p'',q''|p',q'\rangle = \lim_{\nu->\infty} \mathcal{N}_\nu \int e^{(i/\hbar)\int\{p\dot{q}-h(p,q\}\,dt}\, e^{-(1/2\nu)\int\{\dot{p}^2+\dot{q}^2\}\,dt}\mathcal{D}p\,\mathcal{D}q$$

an affine quantization

$$|p,q\rangle = e^{ipQ/\hbar} e^{-i\ln(q)D/\hbar}\, |b = 1\rangle\ , \quad \mathcal{H}'(D,Q) = M'\int h'(p,q)\ |p,q\rangle\langle p,q|\ dp\, dq$$

$$\langle p'',q''|p',q'\rangle = \lim_{\nu->\infty} \mathcal{N}'_\nu \int e^{(i/\hbar)\int\{p\dot{q}-h'(p,q\}\,dt}\, e^{-(1/2\nu)\int\{q^2\dot{p}^2+q^{-2}\dot{q}^2\}\,dt}\mathcal{D}p\,\mathcal{D}q$$

Freedman, et al, 1982

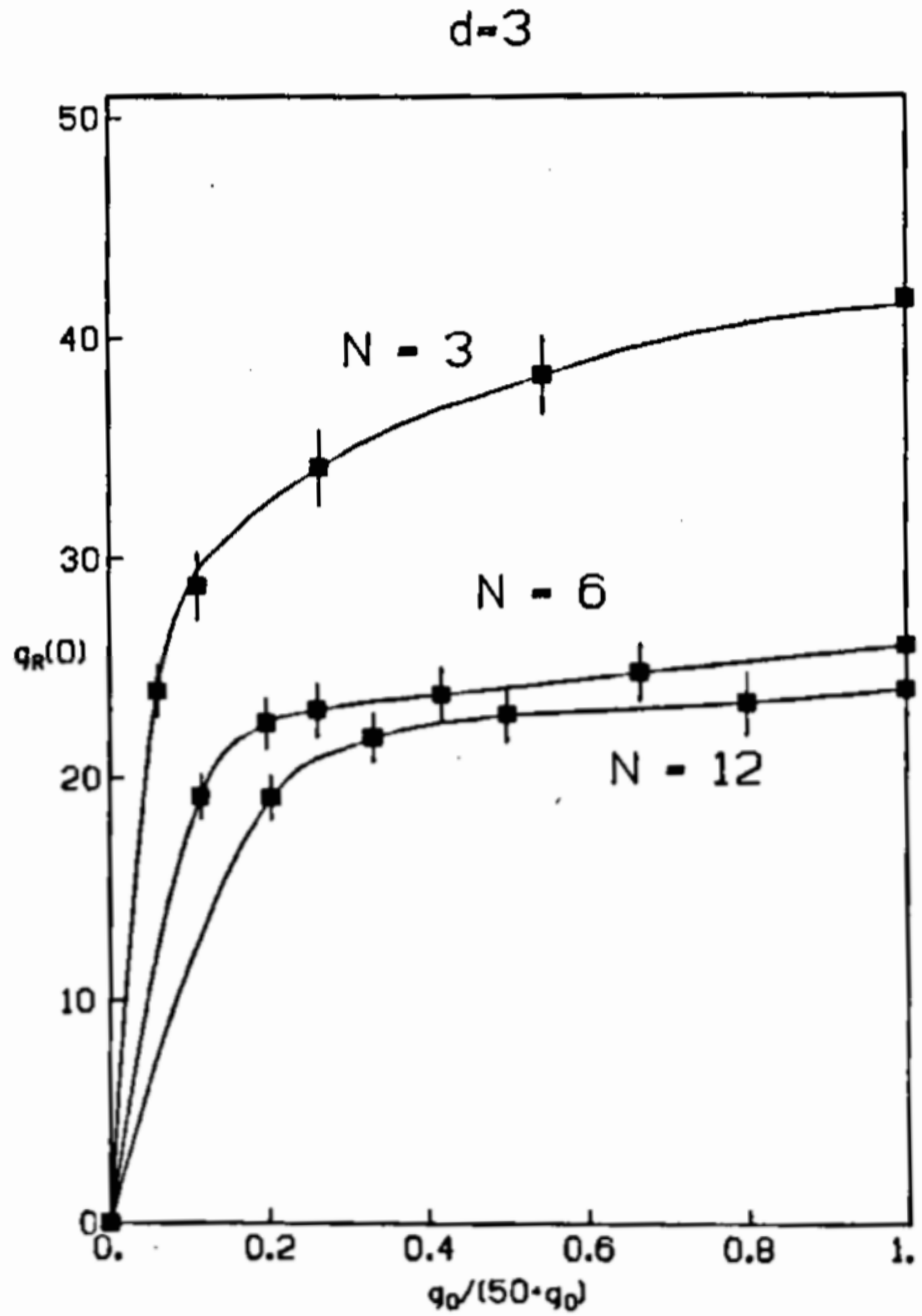


Fig. 3. The zero-momentum coupling constant $g_R(0)$ as a function of the bare coupling constant ratio $g_0/(50 + g_0)$ for $(\phi^4)_3$ on lattices of size $N = 3$, 6 and 12.

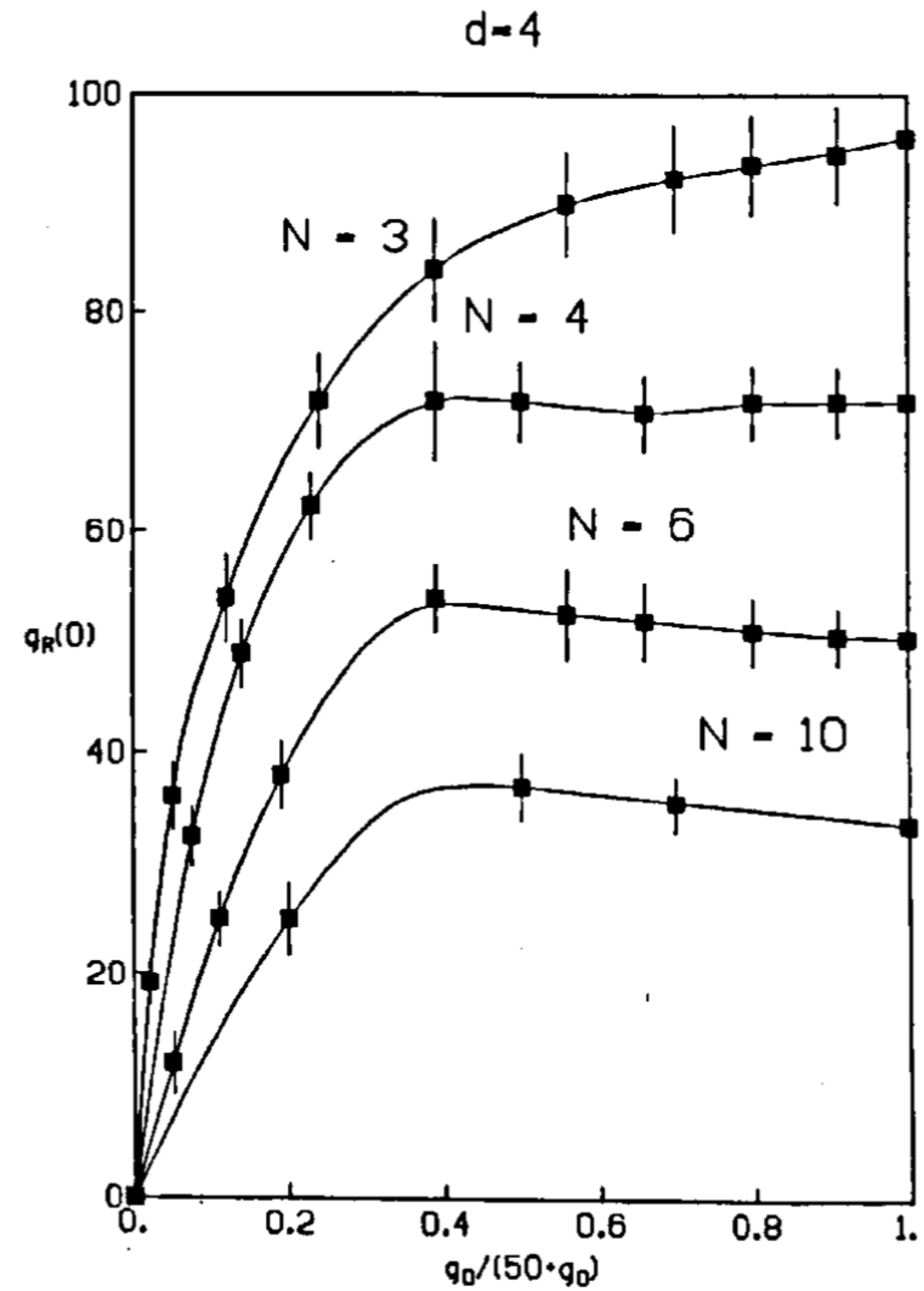
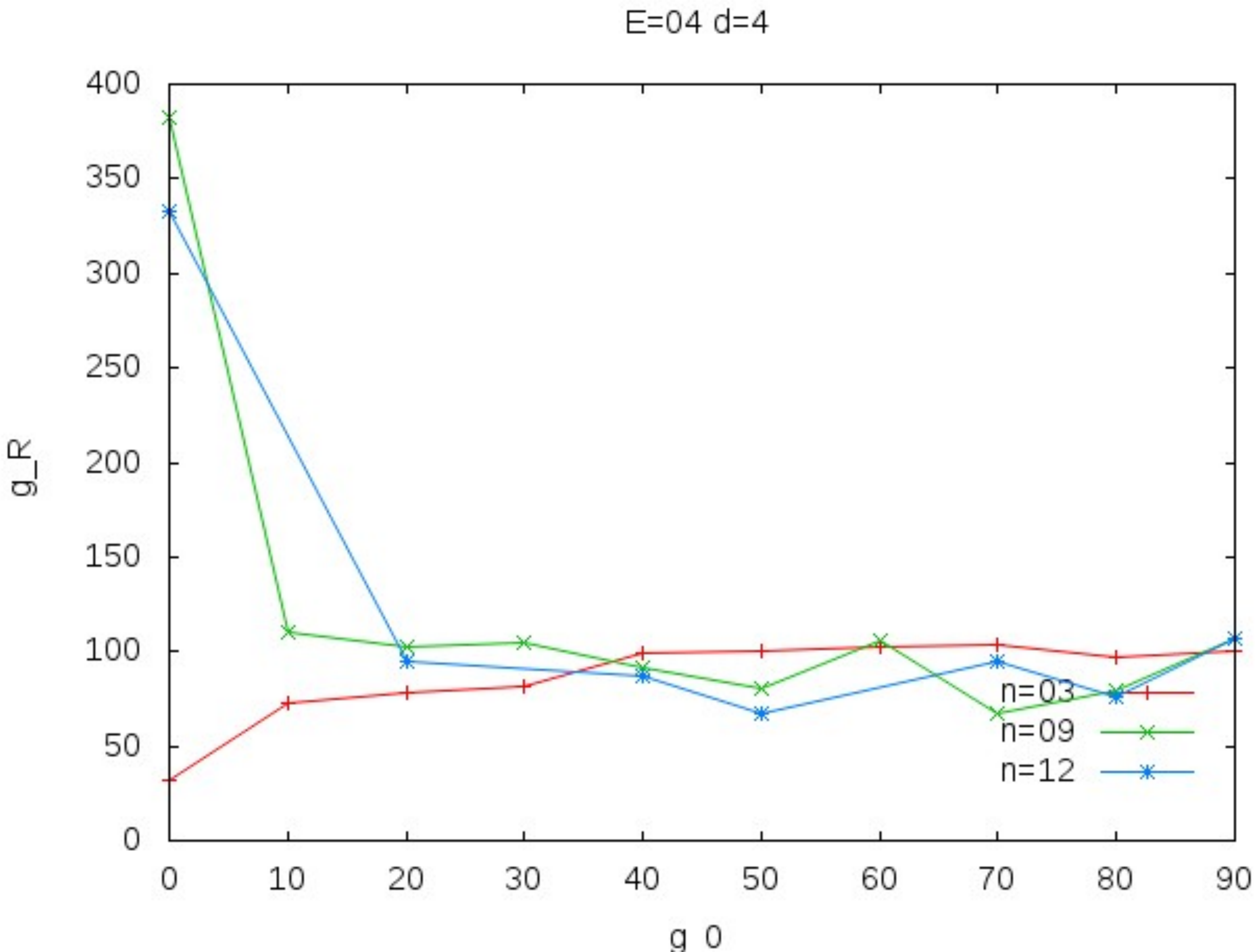


Fig. 1. The zero-momentum renormalized coupling constant $g_R(0)$ as a function of the bare coupling constant ratio $g_0/(50 + g_0)$ for $(\phi^4)_4$ on lattices of sizes $N = 3$, 4, 6 and 10.

Phi^4_4 With Counter Term



NR: Quantum Gravity-6

regularized equation argument

smooth $\{g_{ab\,\mathbf{k}}\}$, **lattice spacing** a

create related sets

$$g_{\mathbf{k}} = det(g_{ab\,\mathbf{l}}) \quad , \qquad g^{ab}_{\mathbf{k}} = invert(g_{ab\,\mathbf{l}}, g_{\mathbf{l}})$$

$$C^{a}_{bc\,\mathbf{k}} = Chris(a\,, g_{ab\,\mathbf{l}}\,, g^{ab}_{\mathbf{l}})$$

$$g^{1/2}_{\mathbf{k}} R_{\mathbf{k}} = scalar(a\,, g_{\mathbf{k}}\,, C^{a}_{bc\,\mathbf{l}}\,, g^{ab}_{\mathbf{l}})$$

pre-fixed required sets

ready to study the regularized Schrödinger equation

Affine Quantization-3

a classical example

$$H(p,q) = p^2 + q^2 \quad , \quad q > 0$$ 🙂

an affine quantization

$$\mathscr{H} = P^2 + Q^2 \,, \qquad Q > 0\,,\ \mathfrak{D}(P^\dagger) \supset \mathfrak{D}(P)$$ bad!

$$\mathscr{H}' = DQ^{-2}D + Q^2 \,, \quad Q > 0\,,\ \mathfrak{D}(D^\dagger) = \mathfrak{D}(D)$$ good!

$$[Q, D] = i\hbar\, Q$$

$$Q \rightarrow x\,,\ D \rightarrow -i\hbar(1/2)[x(d/dx) + (d/dx)x]\ ,\ x > 0$$ 🙂

$$-i\hbar[x(d/dx) + (1/2)]\ ,\ x > 0$$ 🙂

Affine Quantization-4

$$\mathscr{H}' = -\hbar^2[x(d/dx) + (1/2)]\,x^{-2}[x(d/dx) + (1/2)] + x^2$$

$$= -\hbar^2(d^2/dx^2) + \hbar^2(3/4x^2) + x^2$$

🙂

affine coherent states

$$|p,q\rangle = e^{ipQ/\hbar}\,e^{-i\ln(q)\,D/\hbar}\,|b\rangle\;, \quad q > 0$$

$$[(Q-1) + (i/b)D]\,|b\rangle = 0$$

🙂 $$\langle p,q\,|D,\;\; Q,\;\; Q^n\,|p,q\rangle = pq,\;\; q,\;\; q^n(1 + O_n(\hbar))$$ 🙂

Affine Quantization-5

enhanced classical Hamiltonian

$$H(p,q) = \langle p,q\,|\,DQ^{-2}D + Q^2\,|\,p,q\rangle$$

🙂 🙂

$$= \langle b\,|\,(D + pqQ)(qQ)^{-2}(D + pqQ) + (qQ)^2\,|\,b\rangle$$

$$= p^2 + q^2(1 + O_2(\hbar)) + C(\hbar)/q^2 \;\;,\;\; C(\hbar) = \langle b\,|\,DQ^{-2}D\,|\,b\rangle > 0$$

classical limit

$$H_c(p,q) = \lim_{\hbar\to 0} H(p,q) = p^2 + q^2, \quad q > 0$$

Affine Quantization-6

“canonical quantization”

$$p^2 + q^2 \;\text{–>}\; PP^\dagger + Q^2$$

$$\psi_n(x) = h_{n=0,\,2,\,4,\cdots}(x) \quad , \quad x > 0$$

$$E_n = \hbar(1,\,3,\,5\;,\cdots)$$

$$p^2 + q^2 \;\text{–>}\; P^\dagger P + Q^2$$

$$\psi_n(x) = h_{n=1,\,3,\,5,\cdots}(x) \quad , \quad x > 0$$

$$E_n = \hbar(2,\,4\;,6\;,\cdots)$$

mixed eigenvalues

$$E_n = \hbar(1,\,3,\,4,\,6,\,8,\,9,\cdots)$$

Cartesian-coordinates?

$$d\sigma^2 = A\,dp^2 + B\,dq^2$$

$$dp\,dq = [A\,B]^{1/2}\,dp\,dq$$ **phase space**

$$d\sigma^2 = A\,dp^2 + A^{-1}dq^2$$

$$dE_i^a\;dA_a^i$$ **phase space**

$$d\sigma^2 = C\,dE_i^{a\,2} + C^{-1}dA_a^{i\,2}$$???