On the relation between loop quantum gravity and spin-foam models

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Relation between the EPRL spin-foam model and Loop Quantum Gravity Spin-foam model derived from Loop Quantum Gravity coupled to massless scalar Summary and outlook

Outline



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Spin-foam model derived from Loop Quantum Gravity coupled to massless scalar field



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Section 1

Introduction

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Spin network

- A spin network is a triple $s = (\gamma, \rho, \iota)$:
- \bullet oriented graph γ



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- A spin network is a triple $s = (\gamma, \rho, \iota)$:
- \bullet oriented graph γ
- \bullet coloring of links with irreducible representations ρ_ℓ of a compact group G



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Spin network

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- \bullet oriented graph γ
- \bullet coloring of links with irreducible representations ρ_ℓ of a compact group G
- coloring of nodes with invariant tensors

$$\iota_n \in \operatorname{Inv}\left(\bigotimes_i \mathcal{H}_i^* \otimes \bigotimes_j \mathcal{H}_j\right)$$

where i / j labels the links incoming / outgoing in n.



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Loop Quantum Gravity

• The (kinematical) Hilbert space is spanned by spin-network states.



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Loop Quantum Gravity

- The (kinematical) Hilbert space is spanned by spin-network states.
- Quantum operators of geometry such as area or volume operators are defined in this space.



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Loop Quantum Gravity

- The (kinematical) Hilbert space is spanned by spin-network states.
- Quantum operators of geometry such as area or volume operators are defined in this space.
- Gravity is fully constrained system (the Hamiltonian vanishes on the constraint surface). In Loop Quantum Gravity the constrains become quantum constraint operators. The states annihilated by the constraint operators form the physical Hilbert space.



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Spin foams

Spin foams are histories of spin networks (Reisenberger-Rovelli [1997], Markopoulou [1997], Baez [1998]).



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Spin foams

A spin foam is a triple $s = (\kappa, \rho, \iota)$:

• oriented 2-complex κ with boundary $\partial \kappa$



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Spin foams

A spin foam is a triple $s = (\kappa, \rho, \iota)$:

- oriented 2-complex κ with boundary $\partial \kappa$
- coloring of faces with irreducible representations ρ_f of a compact group ${\rm G}$
- coloring of internal edges with invariant tensors ι_e ∈ Inv (H_e)



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The idea of the relation between spin foams and Loop Quantum Gravity

The physical scalar product between spin-network states is expressed as sum over spin foams:

$$ig\langle s'|sig
angle_{ ext{phys}}:=$$
 " $ig\langle s'|\delta(\hat{C})sig
angle$ " $=\sum_{F:s
ightarrow s'}\mathcal{A}_{F},$

where $\textit{F} = (\kappa, \rho, \iota)$, $\partial \textit{F} = \textit{s}^{\dagger} \cup \textit{s}'$,

$$\mathcal{A}_{\mathcal{F}} = \prod_{\ell \in \mathrm{Links}(\partial \kappa)} \mathcal{A}_{\ell} \prod_{f \in \mathrm{Faces}(\kappa)} \mathcal{A}_{f} \prod_{e \in \mathrm{Edges}(\mathrm{int}\kappa)} \mathcal{A}_{e} \prod_{\nu \in \mathrm{Vertices}(\mathrm{int}\kappa)} \mathcal{A}_{\nu}.$$

The relation between Loop Quantum Gravity and the spin-foam models has been studied for years but precise link between the theories is still missing (Reisenberger, Rovelli [1997]; Noui, Perez [2005]; Engle, Livine, Pereira, Rovelli [2008]; Kaminski, Kisielowski, Lewandowski [2010]; Ashtekar, Campiglia, Henderson [2010]; Henderson, Rovelli, Vidotto, Wilson-Ewing [2011]; Engle, Han, Thiemann [2010]; Dittrich, Hohn [2010]; Alesci, Thiemann, Zipfel [2012,2014]; Pranzetti [2014]; Kisielowski, Lewandowski [2019] ...)

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Section 2

Relation between the EPRL spin-foam model and Loop Quantum Gravity

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Holst-Palatini formulation of General Relativity

The theory is defined by an action

$$S_{\rm HP}[e,\omega] = rac{1}{4k} \int_{\mathcal{M}} \varepsilon_{IJKL} e^{l} \wedge e^{J} \wedge F^{KL} + rac{\sigma}{2k\beta} \int_{\mathcal{M}} e^{l} \wedge e^{J} \wedge F_{IJ},$$

where e_{μ}^{I} is a $V = \mathbb{R}^{4}$ valued 1-form e_{μ}^{I} on \mathcal{M} called a tetrad, ω is a connection one-form on \mathcal{M} . The space V is equipped with a fixed metric $\overline{\eta}_{IJ}$, σ is the sign of the determinant of the metric $\overline{\eta}_{IJ}$. The number β is called the Barbero-Immirzi parameter, $k = 8\pi G$.

General Relativity as a constrained BF theory

The gravity theory can be interpreted as given by the action

$$S[B,\omega] = \frac{1}{2k} \int_{\mathcal{M}} B^{IJ} \wedge F_{IJ}$$

with additional simplicity constraint

$$B^{IJ} = rac{1}{2} arepsilon^{IJ}_{KL} \mathbf{e}^K \wedge \mathbf{e}^L + rac{\sigma}{eta} \mathbf{e}^I \wedge \mathbf{e}^J.$$

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The Euclidean Engle-Pereira-Rovelli-Livine model

Let
$$k_{l} \in \frac{1}{2}\mathbb{N}$$
, $l \in \{1, \dots, N\}$ be such that $\forall_{l} \quad j_{l}^{\pm} := \frac{|1 \pm \beta|}{2}k_{l} \in \frac{1}{2}\mathbb{N}$. An EPRL map

$$\iota_{\mathrm{EPRL}}:\mathrm{Inv}\left(\mathcal{H}_{k_{1}}\otimes\cdots\otimes\mathcal{H}_{k_{N}}\right)\to\mathrm{Inv}\left(\mathcal{H}_{j_{1}^{+}j_{1}^{-}}\otimes\cdots\otimes\mathcal{H}_{j_{N}^{+}j_{N}^{-}}\right)$$

assigns to an SU(2) intertwiner \mathcal{I} a Spin(4)=SU(2)×SU(2) (double cover of SO(4)) intertwiner

$$\mathcal{I} \mapsto \iota_{\mathrm{EPRL}}(\mathcal{I}) = P\left(C_{k_1}^{j_1^+ j_1^-} \otimes ... \otimes C_{k_N}^{j_N^+ j_N^-}(\mathcal{I})\right).$$

The image of an EPRL map is called a space of EPRL intertwiners and denoted

$$\mathrm{Inv}_{\mathrm{EPRL}}\left(\mathcal{H}_{j_1^+ j_1^-} \otimes \ldots \otimes \mathcal{H}_{j_N^+ j_N^-}\right).$$

The elements of this space are called EPRL intertwiners.

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Generalization of the EPRL model

The EPRL maps were originally defined for N = 4 [Engle,Pereira,Rovelli,Livine 2008]. We noticed that they can be generalized to arbitrary N. This allowed us to extend the model from foams defined by triangulations to a broader class that describes histories of arbitrary spin networks [Kamiński,Kisielowski,Lewandowski 2010].

The generalized EPRL spin-foam model

The generalized EPRL spin foam (κ, ρ, ι) is a foam κ with a coloring such that

• $\rho_f = \rho_{j_f^+ j_f^-}$, where $j_f^+ = \frac{|\beta+1|}{|\beta-1|} j_f^-$.

• ι_e is an EPRL intertwiner, $\iota_e \in \operatorname{Inv}_{\operatorname{EPRL}}(\mathcal{H}_e) \subset \operatorname{Inv}(\mathcal{H}_e)$.

The amplitude of each spin foam $F = (\kappa, \rho, \iota)$ is defined by the formula:

$$\begin{aligned} \mathcal{A}_{\mathcal{F}} &= \prod_{\ell \in \mathrm{Links}(\partial \kappa)} \frac{1}{\sqrt{\dim \mathcal{H}_{f_{\ell}}}} \prod_{f \in \mathrm{Faces}(\kappa)} \dim \mathcal{H}_{f} \\ &\prod_{v \in \mathrm{Vertices}(\kappa)} \mathrm{Tr}_{v} \left(\bigotimes_{\mathrm{outgoing } e} \iota_{e}^{\dagger} \otimes \bigotimes_{\mathrm{incoming } e'} \iota_{e'} \right), \end{aligned}$$
where $\dim \mathcal{H}_{f} = (2j_{f}^{+} + 1)(2j_{f}^{-} + 1).$



Further developments

- The generalized model has a Lorentzian version (Bianchi, Rovelli, Regoli [2010]; Rovelli [2011]) but the Lorentzian group needs to be dealt with special care (Kaminski [2010]).
- The semiclassical limit of the model has been studied (Barrett, Dowdall, Fairbairn, Gomes, Hellmann, Pereira [2009,2010]; Bonzom [2009]; Perini[2012]; Han [2013,2014]; Hellmann, Kaminski [2013]; ...; Simone Speziale's talk).
- The model suffers from sum-over-spin divergences which need to be regularized (Riello [2013]; Fairbairn, Meusburger [2012]; Han [2011]; Haggard, Han, Kaminski, Riello [2015], ...).
- The generalization raises the problem of allowed foams (Hellmann [2011]; Kisielowski, Lewandowski, Puchta [2012]; Oriti, Ryan, Thürigen [2015]; Sarno, Speziale, Stagno [2018]).
- The generalized model has been applied to cosmology and black-hole physics (Bianchi, Rovelli, Vidotto [2010]; Rennert, Sloan [2013, 2014]; Christodoulou, Rovelli, Speziale, Vilensky [2016]; Vilensky [2017]; Sarno, Speziale, Stagno [2018]; ...). Technical tool used in these applications is a perturbative expansion in the number of internal vertices but its justification is not clear.

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Section 3

Spin-foam model derived from Loop Quantum Gravity coupled to massless scalar field

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How to derive Spin-Foam model from Loop Quantum Gravity?

• M. P. Reisenberger and C. Rovelli, 'Sum over surfaces' form of loop quantum gravity, Phys. Rev. D 56 (1997) 3490–3508,arXiv:gr-qc/9612035 [gr-qc]

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- A. Henderson, C. Rovelli, F. Vidotto, and E. Wilson-Ewing, Local spinfoam expansion in loop quantum cosmology, Class. Quant. Grav. 28 (2011) 025003,arXiv:1010.0502 [gr-qc].

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- M. Domagala, K. Giesel, W. Kaminski and J. Lewandowski, *Gravity quantized:* Loop Quantum Gravity with a Scalar Field, Phys. Rev. D 82, 104038 (2010), arXiv:1009.2445 [gr-qc].

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How to derive Spin-Foam model from Loop Quantum Gravity?



The quantum constraints (gravity only)

- The Hilbert space of solutions to the Gauss constraint is spanned by spin-network states (which are SU(2)-invariant).
- The diffeomorpism constraint (vector constraint) is solved by averaging with respect to the action of the diffeomorphism group on the spin-network states.
- We use the version of the (gravitational part of the) scalar constraint operator by M. Assanioussi, J. Lewandowski, and I. Mäkinen [2017]. This scalar constraint operator \hat{C}_x^{gr} is defined on the space of solutions to the Gauss constraint and partial solutions to the vector constraint, called a vertex Hilbert space \mathcal{H}_{vtx}^{gr} . A basis of the space \mathcal{H}_{vtx}^{gr} is formed by states

$$|[\gamma,\rho,\iota]>$$

obtained from $|\gamma, \rho, \iota >$ by averaging over all diffeomorphisms that act trivially on the set of nodes of the graph γ . The operator $\hat{C}_x^{\rm gr}$ consists of an Euclidean $\hat{C}_{\rm E}$ and Lorentzian $\hat{C}_{\rm L}$ part:

$$\hat{C}_x^{\mathrm{gr}} = f(\beta) \left(\hat{C}_{\mathrm{L}x} + \lambda(\beta) \, \hat{C}_{\mathrm{E}x} \right),$$

where the value of λ is determined by the Barbero-Immirzi parameter β :

$$\lambda(eta) = rac{1}{1+eta^2}.$$

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The Lorentzian part of the scalar constraint operator

The operator \hat{C}_{Lx_l} does not change the graph nor the representation labels. What remains is to define its action on the intertwiners labelling the nodes. Let $\mathcal{R} = (\rho_1, \ldots, \rho_N)$ be a sequence of representations of the SU(2) group. Given a Hilbert space of invariant tensors

$$\mathcal{H}_{\mathcal{R}} = \operatorname{Inv}\left(\mathcal{H}_{\rho_1} \otimes \ldots \otimes \mathcal{H}_{\rho_N}\right)$$

we define operators $\hat{J}_{r\,i}, r \in \{1, \dots, N\}, i \in \{1, 2, 3\}$ by the following formula:

$$\hat{J}_{r\,i} := \mathrm{i} \ \mathbb{1} \otimes \ldots \otimes \mathbb{1} \otimes \rho'_r(\tau_i) \otimes \mathbb{1} \otimes \ldots \otimes \mathbb{1},$$

where $\tau_i = -\frac{i}{2}\sigma_i$ is the basis of the su(2) Lie algebra defined by the Pauli matrices σ_i and ρ'_r is the representation of su(2) corresponding to ρ_r . For each pair $(\mathcal{R}, \varepsilon)$ of a sequence \mathcal{R} and a symmetric function $\epsilon : \{1, \ldots, N\} \times \{1, \ldots, N\} \rightarrow \{0, 1\}$ we define an operator $\hat{C}_{L(\mathcal{R}, \varepsilon)} : \mathcal{H}_{\mathcal{R}} \rightarrow \mathcal{H}_{\mathcal{R}}$ by the following formula:

$$\hat{C}_{\mathrm{L}(\mathcal{R},\varepsilon)} = \sum_{r,s} \varepsilon_{rs} \sqrt{\sum_{i=1}^{3} (\epsilon_{ijk} \hat{J}_{rj} \hat{J}_{sk})^2} \left(\pi + \arccos\left(\frac{\sum_{i=1}^{3} \hat{J}_{ri} \hat{J}_{si}}{\sqrt{\sum_{i=1}^{3} \hat{J}_{ri}^2} \sqrt{\sum_{i=1}^{3} \hat{J}_{si}^2}}\right) \right)$$

Euclidean part of the scalar constraint operator



The Euclidean part $\hat{C}_{\rm E}$ is graph-changing. We use the prescription according to which the operator is symmetric:

$$\hat{C}_{\mathrm{E}x_{l}}|[\gamma,\rho,\iota]>:=\sum_{r,s}\varepsilon_{x_{l},rs}(\hat{C}_{\mathrm{E}x_{l},rs}+\hat{C}_{\mathrm{E}x_{l},rs}^{\dagger})|[\gamma,\rho,\iota]>.$$

Each operator $\hat{C}^{\dagger}_{\mathrm{Ex}_{l,rs}}$ adds a loop $\alpha_{x_{l,rs}}$ tangential to the links ℓ_r and ℓ_s at the node x_l oriented such that its beginning is tangent to the link ℓ_r and its end is tangent to the link ℓ_s . The new link formed by the loop $\alpha_{x_l,rs}$ is labelled with a fixed unitary irreducible representation of the SU(2) group $\rho_{(l)}$ of dimension 2l + 1. Each operator $\hat{C}_{\mathrm{Ex}_l,rs}$ removes such loop.

Euclidean part of the scalar constraint operator

The action of the Euclidean part on the spaces of invariant tensors is described by a family of operators $\hat{C}^{\dagger}_{\mathrm{E}(\mathcal{R},rs)}: \mathcal{H}_{\mathcal{R}} \to \mathcal{H}_{\mathcal{R}_{(l)}}$, where

$$\mathcal{H}_{\mathcal{R}_{(I)}} = \operatorname{Inv}\left(\mathcal{H}_{(I)} \otimes \mathcal{H}_{(I)}^* \otimes \mathcal{H}_{\rho_{\ell_1}^*} \otimes \ldots \otimes \mathcal{H}_{\rho_{\ell_M}^*} \otimes \mathcal{H}_{\rho_{\ell_{M+1}}} \otimes \ldots \otimes \mathcal{H}_{\rho_{\ell_N}}\right).$$

Let $\hat{J}_{(l)}^{i} = i \rho'_{(l)}(\tau_{i}) : \mathcal{H}_{(l)} \to \mathcal{H}_{(l)}$. We define an operator $\hat{C}_{E(\mathcal{R}, rs)}^{\dagger} : \mathcal{H}_{\mathcal{R}} \to \mathcal{H}_{\mathcal{R}_{(l)}}$ by the following formula:

$$\hat{C}^{\dagger}_{\mathrm{E}(\mathcal{R},rs)} = -\frac{3\mathrm{i}}{l(l+1)(2l+1)}\epsilon_{ijk}\hat{J}^{i}_{(l)}\hat{J}^{j}_{r}\hat{J}^{k}_{s}.$$

Let us explain in more detail how the operator $\hat{C}^{\dagger}_{\mathrm{E}(\mathcal{R},rs)}$ acts on a state $\iota \in \mathcal{H}_{\mathcal{R}}$ by using the abstract index notation:

$$(\hat{C}_{\mathrm{E}(\mathcal{R},rs)}^{\dagger}\iota)_{C_{2}}^{C_{1},B_{1}...B_{N}} = -\frac{3\mathrm{i}}{I(I+1)(2I+1)}\epsilon_{ijk}\hat{J}_{(I)C_{2}}^{i}\int_{rA_{r}}^{C_{1}}\hat{J}_{rA_{r}}^{jB_{r}}\hat{J}_{sA_{s}}^{jB_{r}}\iota^{B_{1}...B_{r-1}A_{r}B_{r+1}...B_{s-1}A_{s}B_{s+1}...B_{N}}$$

Quantization of the scalar field

For the matter part we consider the polymer quantization. The polymer Hilbert space \mathcal{H}^{mat} is spanned by functionals of a real-valued scalar field $\varphi: \Sigma \to \mathbb{R}$ given by:

$$|\pi > [\varphi] = U_{\pi}(\varphi) = e^{\mathrm{i}(\pi(x_1)\varphi(x_1)+\ldots+\pi(x_n)\varphi(x_n))},$$

where $\pi : \Sigma \to \mathbb{R}$ is a function with finite support $\operatorname{supp}(\pi) = \{x_1, \ldots, x_n\} = V$. It is equipped with a scalar product

$$\langle U_{\pi}|U_{\pi'}\rangle = \delta_{\pi,\pi'}$$

where δ is the Kronecker delta.

The states are eigenvectors of the momentum operator

$$\hat{\pi}(V)|\pi>=\left(\sum_{x\in V}\pi(x)
ight)|\pi>.$$

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Gravity + massless scalar field

We consider the Hilbert space

$$\mathcal{H} = \mathcal{H}_{\mathrm{vtx}}^{\mathrm{gr}} \otimes \mathcal{H}^{\mathrm{mat}}.$$

The scalar constraint operator is defined in this space and takes the form:

$$\hat{C}_{x_I} = \hat{\pi}_{x_I}^2 - \hat{C}_{x_I}^{\mathrm{gr}}.$$

The physical scalar product is given by:

$$\left\langle \Psi_{\mathrm{out}} | \Psi_{\mathrm{in}} \right\rangle_{\mathrm{phys}} = \int \sum_{\pi_{x_1}, \dots, \pi_{x_n}} \prod_{l=1}^n \delta_{\pi_{x_l}, \sqrt{c_{x_l}}} \left\langle \tilde{\eta}(\Psi_{\mathrm{out}}) | P_{\pi_{x_1} \dots \pi_{x_n}}^{\mathrm{mat}} \otimes dP_{c_{x_1} \dots c_{x_n}}^{\mathrm{gr}} \Psi_{\mathrm{in}} \right\rangle,$$

where $\tilde{\eta}$ corresponds to averaging with respect to the remaining diffeomorphisms $\operatorname{Diff}/\operatorname{Diff}_V = S_n$. By choosing $\delta_{\pi_{N_f},\sqrt{c_{N_f}}}$ we restricted to positive frequencies (momenta).

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The physical scalar product

We found a perturbative expansion of the physical scalar product in the parameter λ :

$$\langle \Psi_{\text{out}} | \Psi_{\text{in}} \rangle_{\text{phys}} = \\ \sum_{M=0}^{\infty} \lambda^{M} \int \mathcal{D}\varphi_{\text{out}} \mathcal{D}\varphi_{\text{in}} \sum_{[s_{\text{out}}], [s_{\text{in}}]} \overline{\Psi_{\text{out}}}(\varphi_{\text{out}}, [s_{\text{out}}]) A_{M}([s_{\text{out}}], \varphi_{\text{out}}; [s_{\text{in}}], \varphi_{\text{in}}) \Psi_{\text{in}}(\varphi_{\text{in}}, [s_{\text{in}}]).$$

Each coefficient $A_M([s_{out}], \varphi_{out}; [s_{in}], \varphi_{in})$ in this expansion can be written as a sum over spin foams and residual diffeomorphisms:

$$\begin{split} \mathcal{A}_{\mathcal{M}}([\mathsf{s}_{\mathrm{out}}],\varphi_{\mathrm{out}};[\mathsf{s}_{\mathrm{in}}],\varphi_{\mathrm{in}}) &= \\ &= \lim_{\epsilon \to 0^+} \frac{1}{|V|!} \sum_{\sigma \in S_n} \sum_{F_M} \langle [\mathsf{s}_{\mathrm{out}}] | U_{\sigma}^{\mathrm{gr}} | [\mathsf{s}_1(F_M)] \rangle \, \mathcal{A}_{F_M} \left\langle [\mathsf{s}_0(F_M)] | [\mathsf{s}_{\mathrm{in}}] \right\rangle, \end{split}$$

where $s_1(F)$ and $s_0(F)$ are spin networks induced on the boundary of F.

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Spin foams

A spin foam $F = (\kappa, \rho, \iota, p, s)$ is a 2-complex κ (a history of a graph) embedded in $\mathcal{M} = \Sigma \times I$ equipped with a coloring of its faces and edges. We assume that our coloring satisfies:

•
$$\hat{C}_{L(\mathcal{R}_e,\epsilon_e)}\iota_e = C_{Le}\iota_e, \ C_{Le} \in \mathbb{R},$$

•
$$p_e = p_{e'} = p_x \in \mathbb{R}$$
,

•
$$s_e = s_{e'} = s_x \in \{-1, 1\}.$$



Spin-foam amplitudes

To each internal edge e we assign an edge amplitude:

$$\mathcal{A}_e = rac{e^{-\mathrm{i} p_e(\varphi(t_e) - \varphi(s_e))}}{p_e^2 - C_{\mathrm{L}e} + \mathrm{i} \mathrm{s}(e) \epsilon},$$

where t_e is the target of e and s_e is its source, $\varphi:\mathcal{M}\to\mathbb{R}$ is a function such that

$$\varphi(x) = \varphi_{\mathrm{in}}(x) \text{ for } x \in \mathrm{Nodes}(\gamma_0), \quad \varphi(x) = \varphi_{\mathrm{out}}(x) \text{ for } x \in \mathrm{Nodes}(\gamma_1).$$

To each internal vertex v we assign a vertex amplitude defined by the following rules: If a loop is created at the vertex v:

$$\mathcal{A}_{\mathbf{v}} = \left\langle \iota_{\mathbf{e}'_{\mathbf{v}}} | \hat{C}^{\dagger}_{\mathrm{E}(\mathcal{R}_{\mathbf{e}_{\mathbf{v}}}, \mathbf{rs})} \iota_{\mathbf{e}_{\mathbf{v}}} \right\rangle.$$

If a loop is annihilated at the vertex v:

$$\mathcal{A}_{\nu} = \frac{1}{2l+1} \left\langle \iota_{e'_{\nu}} | \hat{C}_{\mathrm{E}(\mathcal{R}_{e_{\nu}}, rs)} \iota_{e_{\nu}} \right\rangle.$$

Finally, to each spin foam $F = (\kappa, \rho, \iota, p, \mathsf{s})$ we assign a spin-foam amplitude

$$\mathcal{A}_{F} = \prod_{e \in \mathrm{Edges}(\mathrm{int}\,\kappa)} \mathcal{A}_{e} \prod_{v \in \mathrm{Vertices}(\mathrm{int}\,\kappa)} \mathcal{A}_{v}.$$

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Section 4

Summary and outlook

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Summary and outlook

- We have generalized the EPRL spin-foam model to all Loop Quantum Gravity states.
- We have derived a spin-foam model from Loop Quantum Gravity coupled to massless scalar field.
 - The model provides perturbative expansion for the physical scalar product.
 - The coefficients of the expansion are finite order by order. There are no Lorentzian or sum-over-spin divergences. The number of foams is under control.

But ...

- The problem of convergence of the series is still open.
- The relation we found is not complete because we do not know how our model relates to the spin-foam models derived using disretizations (e.g. Barrett-Crane, EPRL, Freidel-Krasnov, Baratin-Oriti).

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Thank you for your attention!

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Relation between the EPRL spin-foam model and Loop Quantum Gravity Spin-foam model derived from Loop Quantum Gravity coupled to massless scalar

Summary and outlook



Thank you and happy birthday!

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