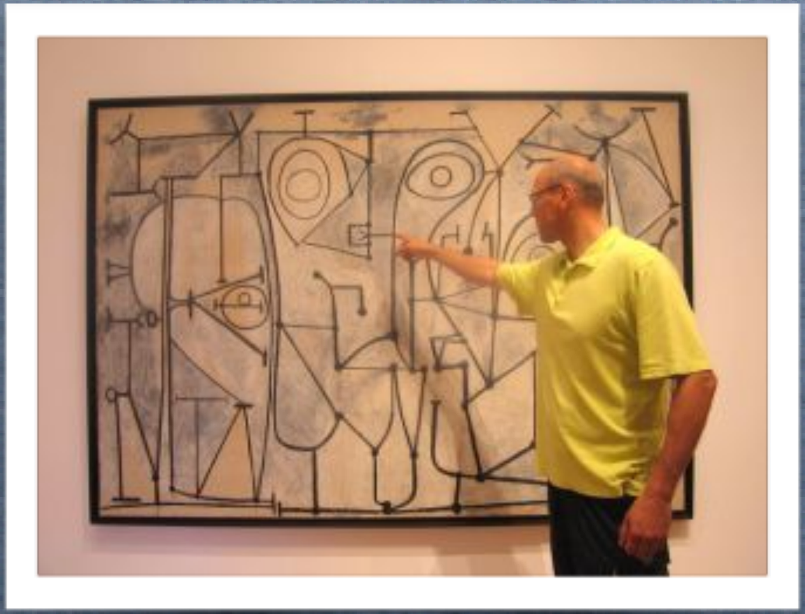
# Reduced Loop Quantum Gravity with Scalar fields



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ERLANGEN CENTRE FOR ASTROPARTICLE PHYSICS

FRIEDRICH-ALEXANDER UNIVERSITÄT ERLANGEN-NÜRNBERG

IQG - FAU Erlangen

# Marseille 2004


**<u>Loops and Spinfoams</u>**

May 3rd to 7th 2004
CPT Luminy, Marseille

# Dynamics in Loop Quantum Gravity

Start with classical general relativity Ashtekar-Barbero variables

In canonical approach: Apply canonical quantization

End up with a quantum version of Einstein's classical equations: Quantum Einstein Equations

We can use either Dirac or reduced quantization

In both approaches quantum dynamics crucially depends on choices on makes in step of quantization

Different models exists for dynamics: Physical properties?

Associated Spin foam models [Kieselowksi, Lewandowski ,19]

# Reduced Quantization: LQG

Three tasks to perform:

1.) Derive physical phase space:
Construct Dirac observables for GR

2.) Derive gauge invariant version of Einstein‘s equations on physical phase space: Determine physical Hamiltonian

3.) Quantize reduced system:
Quantum Einstein Equations on $\mathcal{H}_{\mathrm{Phys}}$

# Relational Formalism: Observables

[Rovelli, Dittrich]

Start with constrained theory

$$(q_A, P_A), \{C_I\}, \text{I labelset}$$

Choose for each constraint a so called reference fields (clock)

$$\{T^I\}\,\text{s.t.}\,\{T^I, C_J\} \approx \delta^I_J$$

Then given phase space function f, associated observable is:

$$O^C_{f,T}(\tau) = \sum_{n=0}^{\infty} \frac{1}{n!}(\tau^I - T^I)^n \{f, C_I\}_{(n)}$$

Matter clocks

Choose clocks from matter dof

Dirac observables:

$$\{O_f, C_I\} \approx 0$$

Algebra of observables:

$$\{O_f, O_g\} \approx O_{\{f,g\}^*}$$

Gauge invariant dynamics on reduced phase space:

$$\frac{dO_{f,T}}{d\tau} = \{O_f, \mathrm{H}_{\text{phys}}\}$$

# Which Reference Matter?

Introduce additional scalar fields coupled to gravity

Distinguish between 2 classes of models:

Type I and type II models [K.G., T. Thiemann `12]

Alternatively one can choose geometrical dof as reference field: 'geometrical clocks' —> quantization more complicated

Geometrical clocks have been considered in the context of linear cosmological perturbation theory

[K.G., Herzog`17, K.G., Herzog, Singh '18, K.G., Singh, Winnekens '19]

# Reference Matter:

Lagrangian can obtain up to 8 scalar fields: $\boxed{(T_I, \rho, W_j)}$

$$\boxed{\mathcal{L}_D = -\frac{1}{2}\sqrt{|g|}\,(g^{\mu\nu}[\rho\nabla_\mu T_0 \nabla_\nu T_0 + \alpha(\rho)V_\mu V_\nu + 2\beta(\rho)(\nabla_\mu T_0)V_\nu] + \Lambda(\rho))}$$

with $\boxed{V_\mu := W_j \nabla_\mu T^j \quad \alpha(\rho), \beta(\rho), \Lambda(\rho)}$ arbitrary functions of $\rho$

matter can be interpreted as dust, has same $\boxed{T_{\mu\nu}}$

Particular models considered so far:

$\mathcal{L}_{TD}$ : $\alpha = \beta = \Lambda = \rho$ [Brown, Kuchar `95]

$\mathcal{L}_{ND}$ : $\alpha = 1, \beta = \Lambda = \rho = 0$ [Bícak, Kuchar `97]

$\mathcal{L}_{NRD}$ : $\alpha = \beta = 0, \Lambda = \rho$ [Brown, Kuchar `95]

$\mathcal{L}_{GD}$ : $\alpha = 0, \beta = 1, \Lambda = \rho$ [Kuchar, Torre `91]

# Canonical Analysis of Type I & II

We distinguish between cases: $\boxed{\rho \neq 0}$

(I) $\alpha(\rho) \neq 0$ or $\beta(\rho) \neq 0$ (II) $\alpha(\rho) = \beta(\rho) = 0$

In both cases one obtains system with 2nd class constraints

$\boxed{\rho, W_j}$ are determined by solving 2nd class constraints strongly

We end up with:

(I) $(A, E), (T_0, P_0), (T_j, P_j) \longrightarrow$ 4 additional dof

(II) $(A, E), (T_0, P_0) \longrightarrow$ 1 additional dof

Particular cases:

(I) $\mathcal{L}_{TD} : h_0(A, E, T_0) \longrightarrow h_0(A, E)$

all other models $h_0(A, E, T_0)$ depends on $\partial_a T_0$

(II) $h_0(A, E, T_0)$ depends only on $q^{ab}\partial_a T_0 \partial_b T_0 \xrightarrow{\text{BK-M}} h_0(A, E)$

# Example: Type II

Lagrangian obtains 1 scalar field: $T_0$

$$\boxed{\mathcal{L}_S = \sqrt{|g|}L(I), \quad I := -\frac{1}{2}g^{\mu\nu}(\nabla_\mu T_0)(\nabla_\nu T_0)}$$

Particular models considered so far:

Klein-Gordon field: $\mathcal{L}_S = \sqrt{|g|}I$ [Rovelli, Smolin `93] [Kuchar, Romano`95]

General case: $\mathcal{L}_S = \sqrt{|g|}L(I)$ [Thiemann `06]

In both cases constraints are of the form:

$$\boxed{\widetilde{C}_0 = P_0 + h_0(A, E) = 0 \quad \widetilde{C}_a = P_0 T_{0,a} + C^{\text{geo}}_a(A, E)}$$

# Summary: Reference Matter

Type I models:

(i) Reduction wrt to Diffeo and Hamilton in classical theory

Type II models:

Reduction wrt Hamilton in classical theory, Diffeo via Dirac quantization in quantum theory

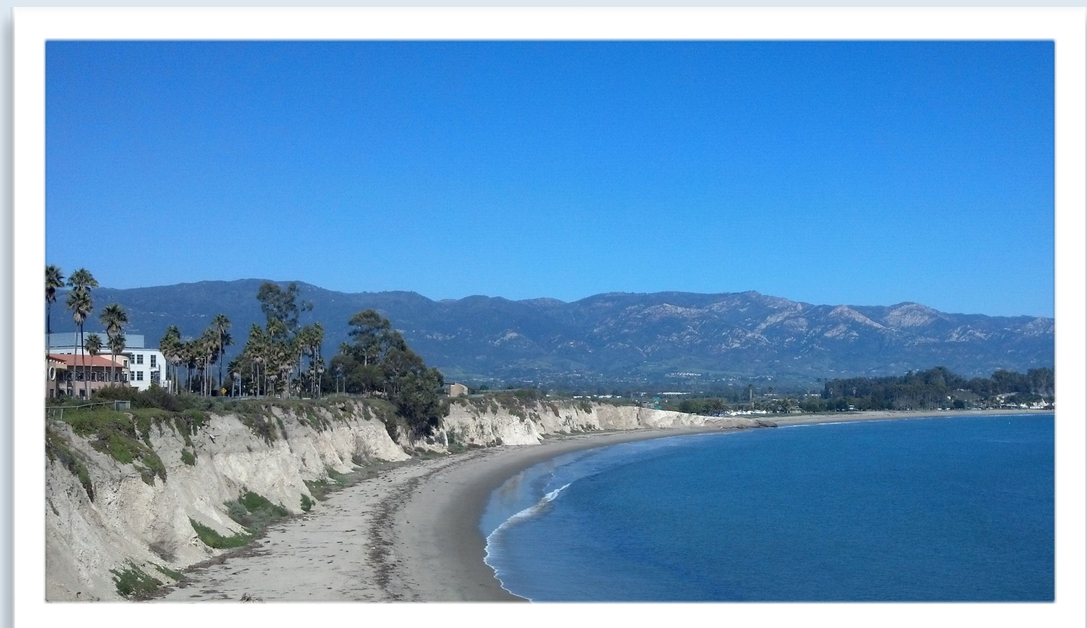
⇒ Difference relevant once quantization is considered

# Beautiful Beaches

KITP Workshop Santa Barbara:

Fishbowl @ KITP:

Beach next to KITP:



# Reduced Dynamics

We have derived (partially) reduced phase space of GR

$$\boxed{O_{A,T^I}(\tau,\sigma^k), \quad O_{E,T^I}(\tau,\sigma^k) \qquad (O_{A,T_0}(\tau), \quad O_{E,T_0}(\tau))}$$

$\tau$ can be interpreted as physical time parameter

Aim: Gauge invariant version of Einstein's eqn:

$$\frac{d}{d\tau}O_{A,T^I}(\tau,\sigma^k) = \{O_{A,T^I}(\tau,\sigma^k), H_{\text{phys}}\}$$

$$\frac{d}{d\tau}O_{E,T^I}(\tau,\sigma^k) = \{O_{E,T^I}(\tau,\sigma^k), H_{\text{hys}}\}$$

Question: How does $H_{\text{phys}}$ look like for different models?

# Reduced Dynamics

One can show that for all considered models:

$$\mathrm{H}_{\mathrm{phys}} = \int_{\mathcal{S}} d^3\sigma H(\sigma)$$

Type I: (i) $H(\sigma) := O_{h_0,T_I}(\sigma)$

Type I: (ii) $H(\sigma) := O_{\tilde{h}_0,T_0}(\sigma)$

Type II: $H(\sigma) := O_{h_0,T_0}(\sigma)$

<u>Particular Models:</u>

| | |
|---|---|
| $\mathcal{L}_{TD}$ | $H(\sigma) = \sqrt{(C^{\mathrm{geo}})^2 - q^{ab}C_a^{\mathrm{geo}}C_{\mathrm{b}}^{\mathrm{geo}}}$ |
| $\mathcal{L}_{NRD}$ | $H(\sigma) = \lvert C^{\mathrm{geo}}\rvert(\sigma)$ |
| $\mathcal{L}_{GD}$ | $H(\sigma) = C^{\mathrm{geo}}(\sigma)$ |
| $\mathcal{L}_{S}$ | $H(\sigma) = \sqrt{-\sqrt{q}C^{\mathrm{geo}} + \sqrt{q}\sqrt{(C^{\mathrm{geo}})^2 - q^{ab}C_a^{\mathrm{geo}}C_b^{\mathrm{geo}}}}$ |

# Reduced Quantization: LQG

What kind of current models exist for LQG? [K.G., T. Thiemann `12]

Type I models: 4 scalar fields

Examples:
Brown-Kuchar dust model [K.G., T. Thiemann `07]
Gaussian dust model [K.G., T. Thiemann `12]
4 KG scalar field [K.G., Vetter `16, K.G., Vetter '19]

Type II models: Partial reduction, only reference field associated with Hamiltonian constraint

Examples:
1 KG scalar field [Domagala, K.G., Kaminski, Lewandowski `10]
1 Gaussian dust field [Husein, Pawlowski `11]

# Two scalar field Models

In this talk we focus on two particular models

Type II: One massless Klein-Gordon scalar field

Refer to as 'Warsaw model', Dirac quantization

Type I: Four massless Klein-Gordon scalar fields

Refer to as '4 scalar fields model', Reduced Quantization

Allows comparison of different models and in particular allows first steps of comparison between Dirac and reduced quantization

Both can be seen as generalizations of the APS model to full LQG [Ashtekar, Pawlowski, Singh 2006]

# Warsaw Model: Reference Matter

Idea: Use one scalar field to reduce wrt Hamiltonian constraint

$$S = \int d^4X \left( \sqrt{g}R - \frac{1}{2}\sqrt{g}g^{\mu\nu}\varphi_{,\mu}\varphi_{,\nu} \right)$$

Diffeos are solved at the quantum level, Quantum Dirac observables

Reference field is one massless scalar field

[Ashtekar, Pawlowski, Singh 2006]

In order to formulate the model we need:

$\mathcal{H}_{\rm diff}$ diffeomorphism invariant Hilbert space

geometric operators on $\mathcal{H}_{\rm diff}$ to construct quantum Dirac observables

$\hat{\rm H}_{\rm phys}$ on (a suitable domain of) $\mathcal{H}_{\rm diff}$

# The birth of Time: Quantum Loops describe the evolution of the universe

Faculty of Physics University of Warsaw > Press releases > Press release

**The birth of time: Quantum loops describe the evolution of the Universe**

2010-12-16

Prof. Jerzy Lewandowski standing by The Kitchen, 1948 by Picasso at the Museum of Modern Art in Manhattan. The lines in the painting are fairly similar to graphs showing the evolution of quantum states of the gravitational field in loop quantum gravity. (Credit: Elżbieta Perlińska-Lewandowska)

Physicists from the Faculty of Physics, University of Warsaw have put forward – on the pages of Physical Review D – a new theoretical model of quantum gravity describing the emergence of space-time from the structures of quantum theory. It is not only one of the few models describing the full general theory of relativity advanced by Einstein, but it is also completely mathematically consistent. "The solutions applied allow to trace the evolution of the Universe in a more physically acceptable manner than in the case of previous cosmological models," explains Prof. Jerzy Lewandowski from the Faculty of Physics, University of Warsaw (FUW).

While the general theory of relativity is applied to describe the Universe on a cosmological scale, quantum mechanics is applied to describe reality on an atomic scale. Both theories were developed in the early 20th century. Their validity has since been confirmed by highly sophisticated experiments and observations. The problem lies in the fact that the theories are mutually exclusive.

According to the general theory of relativity, reality is always uniquely determined (as in classical mechanics). However, time and space play an active role in the events and are themselves subject to Einstein's equations. According to quantum physics, on the other hand, one may only gain a rough understanding of nature. A prediction can only be made with a probability; its precision being limited by inherent properties. But the laws of the prevailing quantum theories do not apply to time and space. Such contradictions are irrelevant under standard conditions – galaxies are not subject to quantum phenomena and quantum gravity plays a minor role in the world of atoms and particles. Nonetheless, gravity

effects need to merge under conditions close to the Big Bang.

mological models describe the evolution of the Universe withi
theory of relativity itself. The equations at the core of the theo
a dynamic, constantly expanding creation. When theorists at
erse was like in times gone by, they reach the stage where der
the model become infinite – in other words, they lose their ph
ities may only be indicative of the weaknesses of the former th
Big Bang does not have to signify the birth of the Universe.

at least some knowledge of quantum gravity, scientists cons
els, known as quantum cosmological models, in which space-t
in a single value or a few values alone. For example, the moc
wald, Lewandowski, Pawłowski and Singh predicts that quant
crease of matter energy density from exceeding a certain criti
anck density). Consequently, there must have been a contract
Bang. When matter density had reached the critical value, th
n – the Big Bang, known as the Big Bounce. However, the mo
nodel.

er to the mystery of the Big Bang lies in a unified quantum the
tempt at developing such a theory is loop quantum gravity (LC
ce is weaved from one-dimensional threads. "It is just like in th
gh it is seemingly smooth from a distance, it becomes evident
of a network of fibres," describes Wojciech Kamiński, MSc frc
onstitute a fine fabric – an area of a square centimetre would

cin Domagała, Wojciech Kamiński and Jerzy Lewandowski, to
from the University of Louisiana (guest), developed their moc
oop quantum gravity. The starting points for the model are two
vitational field. "Thanks to the general theory of relativity we kr
etry of space-time. We may, therefore, say that our point of de
pace," explains Marcin Domagała, PhD (FUW).

arting point is a scalar field – a mathematical object in which a
every point in space. In the proposed model, scalar fields are
rm of matter. Scalar fields have been known in physics for yea
g others, to describe temperature and pressure distribution in
alar field as it is the typical feature of contemporary cosmologi
evelop a model that would constitute another step forward in c
erves Prof. Lewandowski.

eveloped by physicists from Warsaw, time emerges as the rel
al field (space) and the scalar field – a moment in time is giver

the scalar field. "We pose the question about the shape of space at a given value of the scalar field and Einstein's quantum equations provide the answer," explains Prof. Lewandowski. Thus, the phenomenon of the passage of time emerges as the property of the state of the gravitational and scalar fields and the appearance of such a state corresponds to the birth of the well-known space-time. "It is worthy of note that time is nonexistent at the beginning of the model. Nothing happens. Action and dynamics appear as the interrelation between the fields when we begin to pose questions about how one object relates to another," explains Prof. Lewandowski.

Physicist from FUW have made it possible to provide a more accurate description of the evolution of the Universe. Whereas models based on the general theory of relativity are simplified and assume the gravitational field at every point of the Universe to be identical or subject to minor changes, the gravitational field in the proposed model may differ at different points in space.

The proposed theoretical construction is the first such highly advanced model characterized by internal mathematical consistency. It comes as the natural continuation of research into quantization of gravity, where each new theory is derived from classical theories. To that end, physicists apply certain algorithms, known as quantizations. "Unfortunately for physicists, the algorithms are far from precise. For example, it may follow from an algorithm that a Hilbert space needs to be constructed, but no details are provided," explains Marcin Domagała, MSc. "We have succeeded in performing a full quantization and obtained one of the possible models."

There is still a long way to go, according to Prof. Lewandowski: "We have developed a certain theoretical machinery. We may begin to ply it with questions and it will provide the answers." Theorists from FUW intend, among others, to inquire whether the Big Bounce actually occurs in their model. "In the future, we will try to include in the model further fields of the Standard Model of elementary particles. We are curious ourselves to find out what will happen," says Prof. Lewandowski.

The scientific paper "Gravity quantized" published in Physical Review D is the crowning achievement of research conducted at the Faculty of Physics, University of Warsaw within the framework of the MISTRZ Programme by the Foundation for Polish Science. One of the objectives of the programme is to award grants to professors who successfully combine scientific research with training young academic staff.

**Full bibliographic information**

"Gravity quantized: Loop quantum gravity with a scalar field"; Marcin Domagała, Kristina Giesel, Wojciech Kamiński, Jerzy Lewandowski; Phys. Rev. D 82, 104038 (2010); arXiv:1009.2445

'But we do not have quantum gravity', a phrase that is often used….

fraze: Meaning of:

a small **milling cutter** used to cut down the ends of canes or rods to receive a ferrule

# Warsaw Model: Observables

Starting point: $\mathcal{H}_{\mathrm{diff}}$ already at the SU(2) gauge invariant and spatially diffeomorphism invariant level

$\mathcal{H}_{\mathrm{diff}}$ can be obtained using group averaging techniques

Quantum Dirac observables necessary for Hamiltonian constraint

$\hat{\mathrm{L}}$ is already SU(2) gauge and spatially diff-invariant

$[\hat{\mathrm{C}}, \hat{\mathrm{L}}] = 0$ with $\hat{C} = \hat{\pi} + \hat{h}$ and $\hat{\mathrm{h}} = \mathrm{h}(\hat{\mathrm{A}}, \hat{\mathrm{E}})$

formal expression $\mathcal{O}(\hat{L}) = \sum_{n=0}^{\infty} \frac{i^n}{n!} [\hat{\mathrm{L}}, \hat{h}_{\varphi_0}]_{(n)} = e^{i\hat{h}_{\varphi_0}} \hat{\mathrm{L}} e^{-i\hat{h}_{\varphi_0}}$

with $\hat{\mathrm{h}}_{\varphi_0} = \int d^3x \varphi_0 \hat{h}$

# Warsaw Model: Dynamics

Classical physical Hamiltonian, sector

$$\mathrm{H}_{\mathrm{phys}} = \int d^3x \sqrt{-\sqrt{q}\mathrm{C}^{\mathrm{geo}} + \sqrt{q}\sqrt{(\mathrm{C}^{\mathrm{geo}})^2 - q^{ab}\mathrm{C}^{\mathrm{geo}}_a \mathrm{C}^{\mathrm{geo}}_b}}$$

$\hat{\mathrm{H}}_{\mathrm{phys}}$ needs to be implemented on $\mathcal{H}_{\mathrm{diff}}$, suitable operator ordering

$$\widehat{\mathrm{H}}_{\mathrm{phys}} = \int d^3x \sqrt{-2\widehat{\sqrt{\mathrm{q}}\mathrm{C}^{\mathrm{geo}}}}$$

Symmetry: $\{\mathrm{h(x)}, \mathrm{h(y)}\} = 0 \quad \longrightarrow \quad [\hat{\mathrm{h}}(x), \hat{\mathrm{h}}(y)] = 0$

[Properties of phys. Ham:
Zhang,Lewandowksi, Ma '18
Zhang,Lewandowski, Li, Ma '19 ]

On $\mathcal{H}_{\mathrm{diff}}$ $$\hat{\mathrm{H}}_{\mathrm{phys}} = \int d^3x\,\hat{\mathrm{h}}(x) = \sum_{\mathrm{x}\in\mathrm{M}} \sqrt{-2\widehat{\sqrt{\mathrm{q_x}}}^{\frac{1}{2}}\hat{\mathrm{C}}^{\mathrm{geo}}_x \widehat{\sqrt{\mathrm{q_x}}}^{\frac{1}{2}}}$$

$\mathcal{H}_{\mathrm{phys}}$ is unitarily isomorphic via $e^{i\hat{\mathrm{h}}_{\varphi_0}}\Psi \mapsto \Psi$

# 4 Scalar Fields Model: Reference Matter

Idea: Use 4 scalar fields to reduce wrt Hamiltonian & diffeo constraints

$$S = \int d^4X \left(\sqrt{g}R - \frac{1}{2}\sqrt{g}\delta_{IJ}g^{\mu\nu}\varphi^I_{,\mu}\varphi^J_{,\nu}\right) \qquad I = 0,1,2,3$$

Only SU(2) gauge constraint is solved at the quantum level

Reference fields are four massless scalar fields

In order to formulate the model we need:

Observables wrt to spatial diffeom. & Hamiltonian constraint

Use $\varphi^0$ as time and $\varphi^j$ as spatial reference fields

Dynamics: Physical Hamiltonian

Representation of reduced phase space: $\mathcal{H}_{\mathrm{phys}}$ with $\hat{\mathrm{H}}_{\mathrm{phys}}$

# 4 Scalar Fields Model: Observables

Need to construct: $O_{A,\varphi^I}(\tau,\sigma^k), \quad O_{E,\varphi^I}(\tau,\sigma^k)$ [Rovelli '90, Dittrich '05]

Reduced algebra: $\{O_{A,\varphi^I}(\tau,\sigma^k), O_{E,\varphi^I}(\tau,\tilde{\sigma}^k)\} = \delta^{(3)}(\sigma,\tilde{\sigma})$

$\mathcal{H}_{\rm phys}$ can be obtained using standard LQG techniques

However, classical physical Hamiltonian: $\tilde{C}^{\rm tot} = \pi_0 - \mathrm{h}$

$$\mathrm{H}_{\rm phys} = \int d^3\sigma \mathrm{H}(\sigma)$$

with physical Hamiltonian density

$$\mathrm{H}(\sigma) = \sqrt{-2\sqrt{q}\mathrm{C}^{\rm geo} - q^{jk}\delta_{jk} - \delta^{jk}\mathrm{C}^{\rm geo}_j\mathrm{C}^{\rm geo}_k}$$

Result consistent with Kuchar's 8 scalar field model [Kuchar 1991]

Realize: $\delta^{jk}\mathrm{C}^{\rm geo}_j\mathrm{C}^{\rm geo}_k$ cannot be quantized using LQG techniques!

Hence: Dirac quantization Warsaw model works

Reduced 4 scalar fields model: No quantum dynamics!

# Generalize 4 Scalar Fields Model

Idea: Use 4 scalar fields to reduce wrt Hamiltonian & diffeo constraints

$$\boxed{S = \int_M d^4X \sqrt{g} R^{(4)} - \frac{1}{2} \int_M d^4X \sqrt{g} M_{IJ} g^{\mu\nu} \varphi^I_{,\mu} \varphi^J_{,\nu} \quad I, J = 0, 1, 2, 3}$$

Assume particular form:

$$\boxed{M_{IJ} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & M_{11}(x) & 0 & 0 \\ 0 & 0 & M_{22}(x) & 0 \\ 0 & 0 & 0 & M_{33}(x) \end{pmatrix}}$$

Get 6 new dof: $(M_{jj}, \Pi^{jj})$

However, also 3 new primary constraints: $\Pi^{jj} \simeq 0$

Turns out add. 3 secondary constraints: $\tilde{c}^{jj} \simeq 0$

Realize: $\tilde{c}^{jj}, \Pi^{jj}$ build second class pair

Partially reduced phase space wrt $\tilde{c}^{jj}, \Pi^{jj}$ has original number of dof

# Generalize 4 Scalar Fields Model

Furthermore: Need to consider Dirac bracket wrt $\tilde{c}^{jj}, \Pi^{jj}$

Fortunately, Dirac bracket coincides with Poisson bracket for all variables but $(\mathrm{M}_{jj}, \Pi^{jj})$

Thus: in partially reduced phase space can work with Poisson brackets

Implementing $\tilde{c}^{jj} = 0$ strongly modifies physical Hamiltonian

We end up with an LQG quantizable: [K.G., Vetter, '16 and '19]

$$\mathrm{H}_{\mathrm{phys}} = \int d^3\sigma \mathrm{H}(\sigma)$$

with physical Hamiltonian density

$$H(\sigma) = \sqrt{-2\sqrt{Q}C^{\mathrm{geo}} + 2\sqrt{Q}\sum_{j=1}^{3}\sqrt{Q^{jj}C_j^{\mathrm{geo}}C_j^{\mathrm{geo}}}(\sigma)}$$

# Quantization of Physical Hamiltonian

[K.G., Vetter, '16 and '19]

Physical Ham. density: $$H(\sigma) = \sqrt{-2\sqrt{Q}C^{\rm geo} + 2\sqrt{Q}\sum_{j=1}^{3}\sqrt{Q^{jj}C_j^{\rm geo}C_j^{\rm geo}}(\sigma)}$$

Quantization of $\boxed{2\sqrt{Q}C^{\rm geo}}$ standard volume and Ham. constr. operator

Quantization of the second part: $$2\sqrt{Q}\sum_{j=1}^{3}\sqrt{Q^{jj}C_j^{\rm geo}C_j^{\rm geo}}$$

Rewrite this in terms of $$O_J^{(j)} = F_{jk}^L E_J^j E_L^k$$ (no contraction over j)

Then we have $$\sum_{j=1}^{3}\sqrt{O_J^{(j)}O_K^{(j)}\delta^{JK}}$$

Consider quantisation of $$O_J^{(j)} = F_{jk}^L E_J^j E_L^k$$

# Quantization of Physical Hamiltonian

Point splitting regularization for $O^{(j)}_J = F^L_{jk} E^j_J E^k_L$

label j: $\boxed{\dot{e}^a_{(1)} := \delta^a_1 \dot{e}^1(t)}$

curvature term: $\boxed{F^M_{1m}(e'(t'))\dot{e}'(t')\dot{e}^m(t') = F^M_{nm}(e'(t')\dot{e}'^n_{(1)}(t')\dot{e}^m(t)}$

We end up with the operator: [K.G., Vetter, '16 and '19]

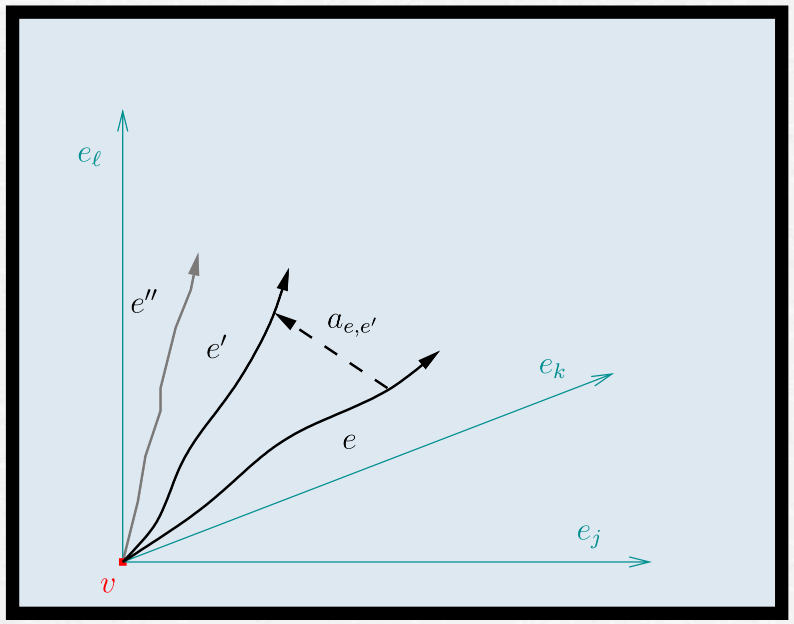
$$\hat{\mathbf{h}}_{\text{phys},\gamma,\text{v}} = \Big[2\Big| - \frac{1}{2}\left(\widehat{\sqrt{Q}}_{\gamma,v}\hat{C}^{\text{geo}}_{\gamma,v} + (\hat{C}^{\text{geo}}_{\gamma,v})^\dagger \widehat{\sqrt{Q}}_{\gamma,v}\right) + \sum_{j=1}^{3}\Big[\left(\frac{(+i)^2\ell_P^4}{4}\right)^2 \delta^{JK}$$

$$\left(\frac{1}{16}\sum_{e\cap e'=v}\mathrm{Tr}\Big(h_{\alpha_{e'(j)e}}\tau^M\Big) X^{e'}_J X^e_M + \frac{i}{8}\delta_{JM}\sum_{b(e)=v}\mathrm{Tr}\big(h_{\alpha_{e(j)e}}\tau^M\big) X^e_0\right)^\dagger$$

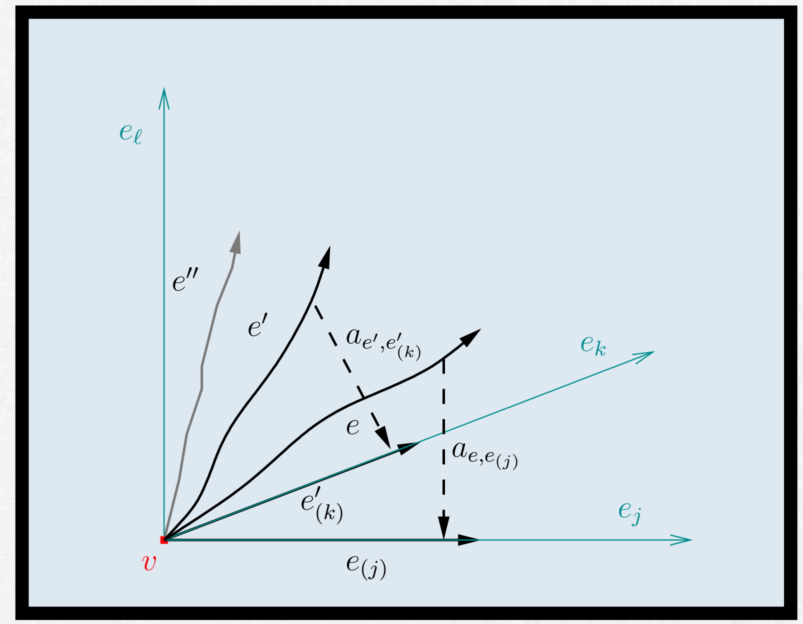
$$\left(\frac{1}{16}\sum_{e''\cap e'''=v}\mathrm{Tr}\Big(h_{\alpha_{e'''(j)e''}}\tau^M\Big) X^{e'''}_K X^{e''}_N + \frac{i}{8}\delta_{KN}\sum_{b(e'')=v}\mathrm{Tr}\Big(h_{\alpha_{e''(j)e''}}\tau^N\Big) X^{e''}_0\right)\Big]^{\frac{1}{2}}\Big|\Big]^{\frac{1}{2}}$$

# Action of Hamiltonian operator

Action of the first term:




Action of the second term:




second term: LQG: embedding dependent, can have trivial contribution

Quantization within AQG framework: graph-preserving, second term does not contribute.

Contributions of the second term can be interpreted as deviations from one scalar field model.

Possible conclusion: prefer models with covariant form of $\hat{\mathrm{H}}_{\mathrm{phys}}$

## III. Summary and Conclusions

Have discussed Dirac and reduced phase space quantization for LQG

LQG program can be completed in such models

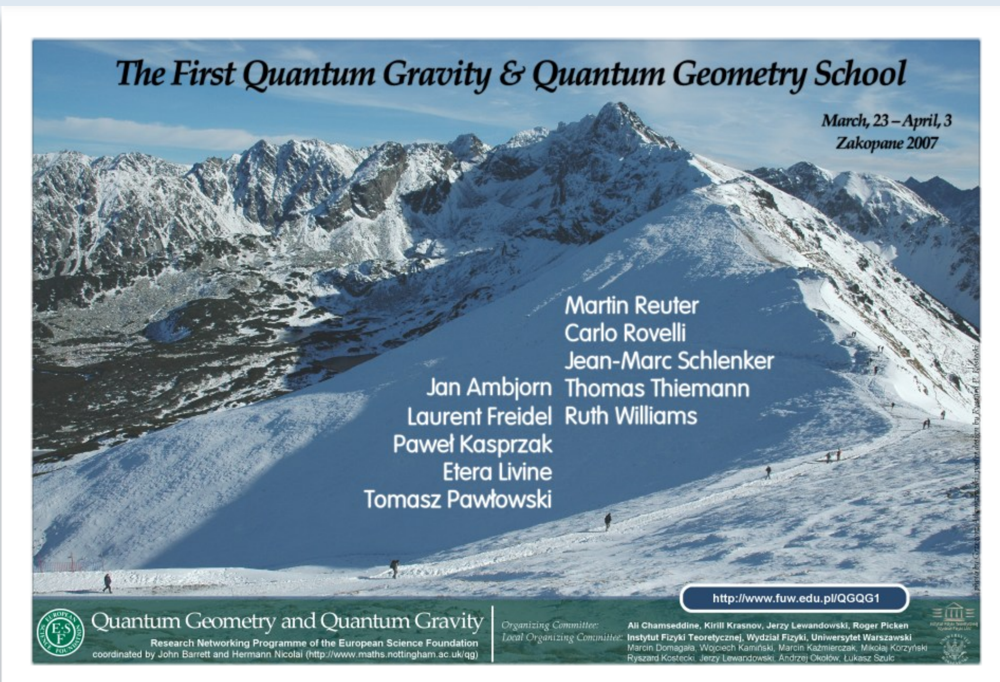
As expected particular form of Quantum Einstein Equations depends on choices, in particular gauge fixing

Important to analyze models in detail and compare them:

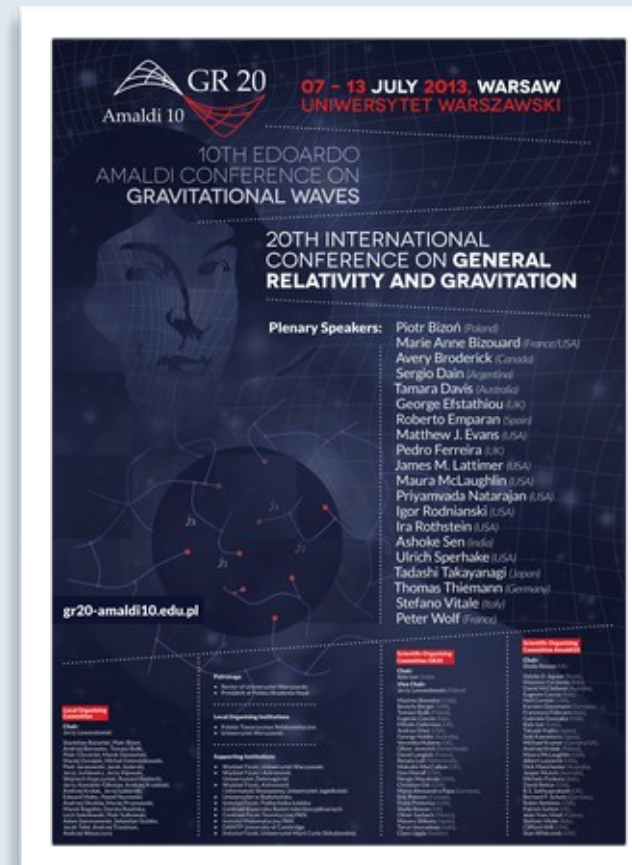
Choice of operator ordering, also consider second natural option
Consider symmetry reduced models where differs are non-vanishing (work in progress)

Also want to understand Dirac versus reduced quantization and how this effects physical properties of models not only for these particular two models.

# III. Summary and Conclusions








Wszystkiego najlepszego z okazji urodzin Jurek!

Sto Lat!