Uniqueness of kinematics and minimal dynamics in loop quantum cosmology

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Mostly based on:

CQG 34 014001 (2017) (E., Hanusch)
CMP 354 231-246 (2017) (E., Hanusch, Thiemann)
PRD 98 023505 (2018) (E., Vilensky)
arXiv:1902.01386 (2019) (E., Vilensky)

The scientific process and the role of uniqueness theorems

Scientific Principles
+
Choices

Mathematically precise
Theories

Observable
predictions

Comparison with
data

Modify choices if necessary, or scientific
principles if *really* necessary

Uniqueness theorems: Say that the theory is uniquely determined by physical principles and a certain minimum set of choices. Isolates the minimal assumptions, so that if there is mismatch with data, we know exactly what to try to modify.

General relativity

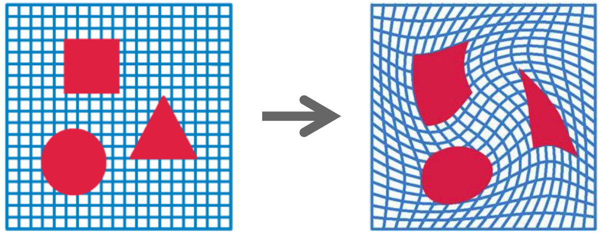
General Principle of Relativity

The laws of physics take the same form in all reference frames – whether inertial or not.

Are all equivalent

General Covariance

The laws of physics are invariant under arbitrary space-time coordinate transformations



Diffeomorphism transformations
"move stuff around."

Background independence

No background space-time structure enters the laws of physics.

"Background": Something fixed a priori and not subject to dynamics

- Most solid hope for **observing** quantum gravity effects: **Effects from Big Bang**
- In classical Einstein Gravity: Curvature and Energy density become infinite at Big Bang. Equations break down.
- Application of Loop Quantum Gravity to cosmology (with simplifying assumptions): **Infinities at Big Bang are removed.** Big Bang is replaced by "Big Bounce."
- LQG predicts modifications to cosmology near Big Bang which should have effects: on the CMB and on the large scale distribution of galaxies in the universe. **Work to calculate these effects has been done and is being done (but not by me).**

Loop quantum cosmology

(Agullo, Ashtekar, Bojowald, Lewandowski, Pawlowski, Singh ...)

Quantum `Kinematics': Hilbert space of quantum states and action of quantum analogues of basic phase space functions
Quantum `Dynamics': Definition of Hamiltonian operator/constraint

Uniqueness of LQG and LQC from diffeomorphism-invariance: kinematics

(LQG: Lewandowski, Okolow, Sahlmann, Thiemann 2005, Fleischhack 2004
LQC: E., Hanusch, Thiemann 2017, Ashtekar, Campiglia 2012)

Algebraic formulation of quantum mechanics

- **Given** a classical phase space Γ, **choose** a space of functions $\mathcal{F} \ni f : \Gamma \to \mathbb{C}$ that is
  - **closed under addition, multiplication by constants, complex conjugation,** and **Poisson brackets** $\{f, g\}$, and
  - **separates points** of Γ.

  $\mathcal{F}$ is the **Classical Poisson algebra of pre-observables.**

- From $\mathcal{F}$, construct **Quantum ($\star$-)algebra of pre-observables** $\mathfrak{A} := \left(\oplus_{n=0}^{\infty} \overset{n}{\otimes} \mathcal{F} \right) / \mathcal{I}$ with $a^\star := \overline{a}$ and with $\mathcal{I}$ generated by

$$1 - \mathbb{I}, \qquad a \otimes b - b \otimes a - i\hbar\{a, b\}, \qquad a_{(1} \otimes a_2 \otimes \cdots \otimes a_{n)} - (a_1 a_2 \cdots a_n)$$

  for all $a, b, a_1, \ldots a_n \in \mathcal{F}$ with $(a_1 a_2 \cdots a_n) \in \mathcal{F}$. (Ashtekar: 1991, 1980)

Example: $\Gamma = \{(x, p)\}$, $\mathcal{F} = \mathrm{span}\{1, x, p\}$, $\mathfrak{A} = \{\sum_{n,m} A_{n,m} \hat{x}^n \hat{p}^m\}$, $(A\hat{x}^n\hat{p}^m)^\star = \overline{A}\hat{p}^m\hat{x}^n$, $[\hat{x}, \hat{p}] = i\hbar\widehat{\{x, p\}} = i\hbar$.

Algebraic formulation of quantum mechanics

- **Primary object** in algebraic QM is the **quantum algebra of pre-observables** $\mathfrak{A}$.
- **States** are represented as **positive linear functionals** on $\mathfrak{A}$.
  That is, $\omega : \mathfrak{A} \to \mathbb{C}$ such that
  $$\omega(\hat{f}^\star \hat{f}) \geq 0 \qquad \omega(\hat{f} + \lambda \hat{g}) = \omega(\hat{f}) + \lambda \omega(\hat{g}) \qquad \omega(\hat{1}) = 1$$
  Physical meaning of $\omega(\hat{f})$: expectation value of $\hat{f}$ in the state ω.

Hilbert space representation of a quantum algebra

A *representation* of a quantum $\star$-algebra $\mathfrak{A}$ is a Hilbert space $\mathcal{H}$ together with a map ρ from $\mathfrak{A}$ to linear operators on $\mathcal{H}$ such that $\rho(a_1 \cdot a_2) = \rho(a_1)\rho(a_2)$ and $\rho(a^\star) = \rho(a)^\dagger$.

- Elements of the **quantum algebra** correspond to **operators** on $\mathcal{H}$ via ρ.
- **States** ω on $\mathfrak{A}$ then correspond to **density matrices** (i.e., **possibly *mixed* states**) on $\mathcal{H}$.

Is important to construct a Hilbert space representation of the algebra because the **operators defining dynamics** (Hamiltonian and constraints) **rarely lie in the basic quantum algebra**.

Conditions one can impose on a representation:

- Can require that $\mathcal{H}$ have ***some*** **'vacuum' state** ψ_o such that $\mathcal{H} = \overline{\textbf{span}\{\hat{f}\psi_o\}_{\hat{f}\in\mathfrak{A}}}$. That is, we can require that the **representation be 'cyclic'**. (Cyclic implies irreducible, but not the converse.)

In this case the representation is uniquely determined by the choice of vacuum ψ_o as a state on the algebra (Gel'fand-Naimark-Segal construction).

- A **symmetry group** G of the algebra $\mathfrak{A}$ will then **act unitarily** on $\mathcal{H}$ iff the vacuum state ψ_o is **invariant under** G.

Example:

Loop Quantum Gravity: There is a unique cyclic representation of the basic quantum algebra in which all diffeomorphisms act unitarily (The Ashtekar-Lewandowski representation). This is the 'L.O.S.T.' theorem (Lewandowski, Okolow, Sahlmann, and Thiemann; Fleischhack).

Loop quantum cosmology (LQC) and the residual diffeomorphisms

Copernican principle: On the largest scales, no location or direction in the universe is `special'.

Mathematically, we impose invariance of A_a^i, E_i^a under translations and rotations. That is, we impose homogeneity and isotropy.

Yields: $A_a^i = c\delta_a^i, \quad E_i^a = p\delta_i^a, \quad$ with $\{c, p\} = \frac{8\pi\gamma G}{3V_o}$
where V_o is the coordinate volume of a cell which is fixed and used for integrating the Lagrangian to make it finite.

Only diffeomorphism freedom remaining:

parity	$P : x^{\mathbf{a}} \mapsto -x^{\mathbf{a}},$	$(c, p) \mapsto (-c, -p)$
dilations	$\Lambda_\lambda : x^{\mathbf{a}} \mapsto e^\lambda x^{\mathbf{a}},$	$(c, p) \mapsto (e^{-\lambda}c, e^{-2\lambda}p)$

LQC: Classical Poisson Algebra

The Classical Poisson algebra of basic observables used in LQG:

$$\mathcal{F} = \mathrm{span}\{E(S)_i\}_{S,i} \oplus \mathrm{span}\left\{\prod_{i=1}^{N} A(\ell_i)^{A_i}{}_{B_i}\right\}_{N,\ell_i,A_i,B_i}$$

Restriction of this Poisson algebra to $(A_a^i, E_j^b) = (c\,\delta_a^i, p\,\delta_j^b)$ yields

$$\mathcal{F}_S = C_{AP}(\mathbb{R}) \oplus C_0(\mathbb{R}) \oplus \{zp\},$$

where $C_{AP}(\mathbb{R})$,$C_0(\mathbb{R})$ are the spaces of almost periodic functions, and functions vanishing at $\pm\infty$, of c. (Fleischhack 2010).

Problem:

- To define quantum analogue of a classical transformation T:
  Usually one would first define $T : \sum_i a_{i1} \otimes \cdots \otimes a_{in_i} \mapsto \sum_i (T a_{i1}) \otimes \cdots \otimes (T a_{in_i})$,
  and then lift the action to $\mathfrak{A}_S = \left(\sum_n \overset{n}{\otimes} \mathcal{F}_S \right) / \mathcal{I}$.
- But the lift to $\mathfrak{A}_S$ exists only if T preserves the P.B. $\{c, p\} = \frac{8\pi\gamma G}{3}$.
- The only residual diffeo satisfying this is parity. **No dilation satisfies this.**

Solution: Define the action directly on the quantum algebra $\mathfrak{A}$

- For algebra elements of form p^n and $\varphi \in C_{AP}(\mathbb{R}) \oplus C_0(\mathbb{R})$, definition of the action is obvious:

$$p^n \mapsto (e^{-2\lambda} p)^n \qquad \varphi(c) \mapsto \varphi(e^{-\lambda} c).$$

- For mixed algebra elements, must choose an ordering. For example:

$$\sum_{n=0}^{\infty} (\varphi_n(c) p^n + p^n \varphi_n(c)) \quad \mapsto \quad \sum_{n=0}^{\infty} \left(\varphi_n(e^{-\lambda} c)(e^{-2\lambda} p)^n + p^n \varphi_n(e^{-\lambda} c) \right), \quad \text{or}$$

$$\sum_{n=0}^{\infty} \varphi_n(c) p^n \varphi_n(c) \quad \mapsto \quad \sum_{n=0}^{\infty} \varphi_n(e^{-\lambda} c)(e^{-2\lambda} p)^n \varphi_n(e^{-\lambda} c)$$

LQC: uniqueness of the vacuum/representation

There exists a unique cyclic representation $(\mathcal{H}_S, \rho_S)$ of $\mathfrak{A}_S$ in which dilations act unitarily.

(Equivalently, $\exists!$ state (vacuum) on $\mathfrak{A}_S$ which is invariant under dilations).

Remarkably, this uniqueness and the selected representation $(\mathcal{H}_S, \rho_S)$ **are independent** of the **choice of ordering** used to define the action of dilations on mixed algebra elements!

Reason for this: Uniqueness proof **only** uses the action of dilations on p^n and $C_{AP}(\mathbb{R}) \oplus C_0(\mathbb{R})$. The action on mixed elements is irrelevant for the uniqueness proof.

$\mathcal{H}_S$ is the **(Harald) Bohr** Hilbert space of almost periodic functions.
Eigenstates of $\hat{p}$: $\psi_{\mathfrak{p}}(c) = \exp\left(\frac{\mathfrak{p}\, c}{8\pi\gamma\ell_P^2}\right)$ form an orthonormal basis $\langle \psi_{\mathfrak{p}}, \psi_{\mathfrak{p}'} \rangle = \delta_{\mathfrak{p},\mathfrak{p}'}$
General state: $\psi = \sum_{\mathfrak{p}} \tilde{\psi}(\mathfrak{p}) \psi_{\mathfrak{p}}$ with $\sum_{\mathfrak{p}} |\tilde{\psi}(\mathfrak{p})|^2 < \infty$.

Compare with standard algebra and representation used in LQC since 2003 (Ashtekar, Bojowald, Lewandowski):

- Standard algebra $\underline{\mathfrak{A}}_S$ is the subalgebra of $\mathfrak{A}_S$ generated by p and $C_{AP}(\mathbb{R})$ (that is, the algebra you get if you restrict straight edges).
- Standard representation is just the restriction of $(\mathcal{H}_S, \rho_S)$ to this subalgebra.

That is, the Hilbert spaces are the same, and the action of the operators $\underline{\mathfrak{A}}_S \subset \mathfrak{A}_S$ are the same. The only difference is that the representation ρ_S defines an action for more operators, namely those corresponding to parallel transports along non-straight edges.

Is non-trivial: Is **not** what one would guess based on naïve `Schrodinger' quantization.

Uniqueness of LQC from diffeomorphism-invariance: dynamics

(E., Vilensky 2018, 2019. Earlier work Corichi, Singh 2008)

In Brief

- **Corichi and Singh 2008**: Show that, from a few different possible quantizations, independence of effective dynamics from choice of fiducial cell/fiducial metric selects uniquely the "improved dynamics" of Ashtekar, Pawlowski, and Singh (2006).
- Besides, this, no work prior to ours on uniqueness of dynamics in quantum gravity/quantum cosmology of which I am aware.

In this work, we consider covariance of the exact quantum Hamiltonian constraint under the action of dilations (the `active' equivalent to imposing independence of cell/fiducial metric).

This condition, together with other physical criteria, and **a single "minimality" assumption**, will enable us to select the APS dynamics uniquely **among *all* operators** on $\mathcal{H}_S$.

Specifically, we require of $\hat{H}$:

1. Covariance under dilations (to be defined shortly)
2. Invariance under parity
3. Hermiticity
4. That its domain contain at least one volume eigenstate
   (I strongly suspect this assumption can be dropped - help?)
5. The correct classical limit (to be defined shortly)
6. ***'Minimality'***: That $\hat{H}$ have a minimal number of terms (to be defined shortly)

Classical Hamiltonian constraint with lapse $N = |v|^n$, with $v := \mathrm{sgn}(p)|p|^{3/2}$:

$$H = \frac{-3}{8\pi G\gamma^2}|p|^{\frac{3n+1}{2}}c^2$$

$n = 0$: proper time
$n = 1$: harmonic time

Covariance under dilations

- Action of dilations defined on $\mathfrak{A}_S$ cannot be obviously extended to all operators on $\mathcal{H}_S$.
- Instead, we start from the observation that, even though dilations are not canonical, their flow ***is proportional*** to a canonical flow:

$$\dot{F} = \omega\{\Lambda, F\}.$$

The correct flow for dilations is $\dot{c} = \frac{d}{dt}(e^{-t}c(0)) = -c(t)$, $\dot{p} = \frac{d}{dt}(e^{-2t}p(0)) = -2p(t)$. The most general ω and Λ leading to this are

$$\omega = -Mv \qquad \Lambda = \frac{6}{8\pi G\gamma}\left(M^{-1}b + \ell\right)$$

where $v := \mathrm{sgn}(p)|p|^{3/2}$, $b := c|p|^{-1/2}$, for some $M, \ell \in \mathbb{R}$. But for now leave ω, Λ general.

- $\hat{b}$ and hence $\hat{\Lambda}$ are not defined on $\mathcal{H}_S$, only their exponentiation. Hence we rewrite the flow as

$$\dot{F} = \omega \frac{1}{i\mu} e^{-i\mu\Lambda}\{e^{i\mu\Lambda}, F\}.$$

- We quantize the flow equation:

$$\dot{\hat{F}} = \hat{\omega} \star \left(\frac{-1}{\mu\hbar} \widehat{e^{-i\mu\Lambda}} \left[\widehat{e^{i\mu\Lambda}}, \hat{F} \right] \right)$$

where $\star$ is a choice of operator product. Choose for concreteness Weyl ordering: $\hat{v} \star \hat{O} := \frac{1}{2} \left(\hat{v}\hat{O} + \hat{O}\hat{v} \right)$. However, final result will be independent of this choice.

- For the case $\omega =$const., we want this to yield standard unitary flow generated by $\hat{\Lambda}$. Will be true only if $\mu \to 0$ limit is taken:

$$\dot{\hat{F}} := \hat{\omega} \star \lim_{\mu \to 0} \left(\frac{-1}{\mu\hbar} \widehat{e^{-i\mu\Lambda}} \left[\widehat{e^{i\mu\Lambda}}, \hat{F} \right] \right)$$

- Consider now dilations $\omega = -Mv$, $\Lambda = \frac{6}{8\pi G\gamma} \left(M^{-1} b + \ell \right)$. Classically, under this flow, $\dot{H} = -3(n+1)H$. Using the above definition to impose $\dot{\hat{H}} = -3(n+1)\hat{H}$ yields an ODE independent of the arbitrary constants M, ℓ:

$$-3(n+1)\hat{H} = \dot{\hat{H}} := \frac{6}{8\pi G\hbar\gamma} \hat{v} \star \lim_{\tilde{\mu} \to 0} \left(\hat{H} - e^{-i\tilde{\mu}b} \hat{H} e^{i\tilde{\mu}b} \right)$$

- Classically $\{v, b\} = 4\pi G\gamma/V_o$, so action of $\widehat{e^{i\mu b}}$ on eigenstates of $\hat{v}$ shift eigenvalue: $|v\rangle \mapsto |v + 4\pi G\hbar\gamma\rangle$. Therefore most convenient to solve for $\hat{H}$ by solving for its matrix elements $\langle v''|\hat{H}|v'\rangle =: f_{v''-v'}(v')$. Equation becomes:
$$\frac{w+2u}{2} f'_w(u) = (n+1) f_w(u),$$
an ODE in u for each w.

Parity inv., Hermiticity, and Domain condition

- Solving the above equation, and imposing that $\hat{H}$ be parity invariant, Hermitian, and have at least one volume eigenstate in its domain, one proves the general form
$$\hat{H} = \sum_{i=1}^{N} \widehat{e^{i\frac{\tilde{A}_i}{2}b}} \left(\tilde{a}_i + i\tilde{b}_i \mathrm{sgn}(\hat{v})\right) |\hat{v}|^{n+1} \widehat{e^{i\frac{\tilde{A}_i}{2}b}} + \text{h.c.} + \tilde{a}_0 |\hat{v}|^{n+1}$$
which suggests the quantization map $\widehat{g(v)e^{iAb}} := \widehat{e^{iAb/2}} g(\hat{v}) \widehat{e^{iAb/2}}$, in terms of which the above reads:
$$\hat{H} = \sum_{i=1}^{N} \widehat{\left(\tilde{a}_i + i\tilde{b}_i \mathrm{sgn}(\hat{v})\right) |\hat{v}|^{n+1} e^{i\tilde{A}_i b}} + \text{h.c.} + \tilde{a}_0 |\hat{v}|^{n+1}$$

Single length scale

- One deduces $\tilde{A}_i$ to have dimensions of length, and $\tilde{a}_i, \tilde{b}_i$ to have dimensions of inverse area over G. Thus, if ℓ_P is to be the only fundamental length scale in the theory, $\tilde{A}_i = A_i \ell_P$, $\tilde{a}_i = a_i/(G\ell_P^2)$, $\tilde{b}_i = b_i/(G\ell_P^2)$ with A_i and a_i dimensionless:

$$\boxed{\hat{H} = \frac{\ell_p^{-2}}{G}\left(\sum_{i=1}^{N}\widehat{(a_i + ib_i \mathrm{sgn}(v))\,|v|^{n+1}e^{i\ell_p A_i b}} + \text{h.c.} + a_0|\hat{v}|^{n+1}\right)}$$

Correct classical limit

- Define the 'classical analogue' of an operator to be an element of its inverse image under the quantization map on prior slide. A classical analogue of the above general form of $\hat{H}$ is then:

$$H_{\ell_p} = \frac{\ell_p^{-2}}{G}\left(\sum_{i=1}^{N}(a_i + ib_i \mathrm{sgn}(v))\,|v|^{n+1}e^{i\ell_p A_i b} + \text{h.c.} + a_0|v|^{n+1}\right)$$

- Requiring $\lim_{\ell_p \to 0} H_{\ell_p} = H$ is then equivalent to:

$$\boxed{a_0 + \sum_i 2a_i = 0 \qquad \sum_i A_i b_i = 0 \qquad \sum_i A_i^2 a_i = \frac{3}{8\pi\gamma^2}}$$

These three conditions, together with the form from the last slide,

$$\boxed{\hat{H} = \frac{\ell_p^{-2}}{G}\left(\sum_{i=1}^{N}\left(a_i + ib_i \widehat{\mathrm{sgn}(v))\,|v}|^{n+1} e^{i\ell_p A_i b} + \text{h.c.} + a_0 |\hat{v}|^{n+1}\right)}$$

give the **most general quantum Hamiltonian constraint** which is **covariant under all residual diffeomorphisms**, is **Hermitian**, has a **single length scale**, and has the **correct classical limit**.

- APS (Ashtekar-Pawlowski-Singh) Hamiltonian has been standard in LQC since 2006.
- Other Hamiltonians since then have also been proposed (Yang-Ding-Ma 2009, Dapor-Liegener 2017).

> Of these, all '$\overline{\mu}$'-type Hamiltonians are included in the above class.
> All 'μ_0'-type Hamiltonians are excluded.

Minimality

Is it possible to impose a further condition to obtain a unique quantum Hamiltonian from among this class?

YES: If we require that **N be minimal (`minimality'),** then the quantum Hamiltonian is **unique** up to a single parameter A!

$$\boxed{\hat{H} = \frac{3}{4\pi A^2 \gamma^2 G \ell_p^2}\left(\widehat{|v|^{n+1} e^{i\ell_p A b}} + \text{h.c.} + a_0 |\hat{v}|^{n+1}\right)}$$

If we choose A to be $2\sqrt{\Delta}$ with $\Delta \ell_p^2$ the LQG area gap, the above is *exactly* the APS Hamiltonian, including ordering!

<u>Is strongest uniqueness we could have hoped for:</u> In LQC this parameter could only be fixed by `importing' the area gap from full theory.

Independence of operator product $\star$

Consider more general family of operator products, defined for arbitrary operator $\hat{O}$:

$$\hat{v} \star \hat{O} = \sum_i \alpha_i \hat{v}^{\lambda_i} \hat{O} \hat{v}^{1-\lambda_i}$$

If we instead use this $\star$ to define dilation flow, we obtain a class of quantum Hamiltonians $\hat{\hat{H}}$. In the limit of large cell volume, holding the one physically meaningful pure gravity quantity, the Hubble rate b/γ, constant — i.e., the limit of infrared regulator removal — this class is equivalent to the foregoing class of quantum Hamiltonians, in two senses:

1. The classical analogues and hence effective Hamiltonians are asymptotic to each other in this limit, and
2. The exact operators are also equivalent in this limit, in the sense that

$$\lim_{(|v''|,|v'|)\to(\infty,\infty)} \frac{\langle v''|\hat{\hat{H}}|.v'\rangle + C}{\langle v''|\hat{H}|v'\rangle + C} = 1$$

for any non-zero C.

Is intuitively clear from $1/V_o$ in basic P.B.: $\{c,p\} = \frac{8\pi\gamma G}{3V_o}$, $\{b,v\} = \frac{4\pi\gamma G}{V_o}$. The P.B. and hence basic commutators vanish, and hence operator orderings do not matter, in the $V_o \to \infty$ limit!

Discussion

Viewpoint: Significance of uniqueness theorem is not to close discussion, but to clarify it.

More precisely: This work shows that, besides basic physical symmetries, the only assumptions going into LQC are

(1.) **parallel transports and fluxes** as basic variables (and actually only a small part of this is used), and
(2.) the **minimality principle** – minimal number of terms.

Assumption (1.) is what connects LQC to LQG: **Cannot be removed** if goal is to obtain predictions of LQG for cosmology.
Assumption (2.), however, **can be removed**. This work shows the exact class of operators allowed when (2.) is removed.

- For example: Two alternative LQC Hamiltonians (Yang, Ding, Ma 2009) which quantize the 'Lorentzian term' as in full theory, falls into this larger class of constraints selected (specifically corresponding to N=5), in addition to Ashtekar-Singh-Pawlowski (2006). These are so-called $\overline{\mu}$-schemes.

- By contrast, all 'μ_o'-schemes, such as Dapor, Liegener 2017 or the pre-APS dynamics of Ashtekar, Bojowald, and Lewandowski 2003, are **not** in this larger class - they are excluded by basic physical principles.

Extensions

Inclusion of (homogeneous) scalar matter

- Classical action of dilations: $(\varphi, \pi) \mapsto (\varphi, e^{-3\lambda}\pi)$
- Basic classical algebra descending from LQG: A.P. functions of φ plus π.
- Situation is almost exactly the same as in gravitational sector, except no action of dilations on π! Can we still get unique representation? If so, is it again the same Bohr Hilbert space representation?
- If so, uniqueness of dynamics as above will be easy, and considering the large volume limit, and using the work of Domagala, Dziendzikowski, and Lewandowski (2012), it is equivalent to APS matter.

Non-isotropic models

- All results on prior slides extend to Bianchi I case (Ashtekar and Campiglia; E., Hanusch, and Thiemann; E. and Vilensky).
- However, for example, in isotropic $k = 1$ model, no residual diffeomorphisms - neither infinitesimal nor large. Above strategy is inapplicable. Other strategies to get uniqueness?

Further tasks, thoughts and questions:

- Find phenomenological consequences of some different constraints without minimality (Has started for Hamiltonians with Lorentzian term (Assanioussi, Dapor, Liegener, Pawlowski 2018; Li, Singh, Wang 2018; Agullo 2018; de Haro 2018; Saini, Singh 2018)). Qualitatively different from the minimal case? Do we find that some options are not viable for reasons not considered?

- LQG is diffeomorphism invariant. Thus, the dynamics it implies for LQC must also be diffeomorphism invariant, and hence belong to the above class, possibly without minimality. <u>This makes the problem of directly calculating LQC dynamics from LQG more controllable.</u> Which among this class corresponds to the Thiemann constraint? To the EPRL model? Proper vertex?

- Have action of dilations on basic algebra and for operators satisfying certain differentiability (enough to impose covariance of an operator), and these are consistent. Can we extend the definition to all operators?

- Dilation invariant states – i.e. density matrices? Will certainly be mixed states! Relation to Alesci's mixed coherent states (Alesci, 2017) ? Could this simplify removal of infrared cut-off?

Thank you for your time, and happy birthday, Jurek!