Effective Dynamics of Loop Quantum Gravity: Bouncing Black Holes and Gravitational Phonons

Andrea Dapor*

Louisiana State University

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*with Mehdi Assanioussi, Wojciech Kamiński, Klaus Liegener and Tomasz Pawłowski

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outline

1. introduction: what is effective dynamics?
2. semiclassical states in quantum gravity on a graph
3. example: homogeneous cosmology
4. example: spherical black hole interior
5. example: linearized Einstein equations
6. conclusion

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A "semiclassical state" in QM:

$$\psi_{(x_o,p_o)}(x) = \frac{1}{\sqrt{\epsilon\sqrt{\pi}}} e^{-\frac{(x-x_o)^2}{2\epsilon^2} + i p_o (x - x_o)}$$

Peakedness:

$$\langle \psi_{(x_o,p_o)} | \hat{X} | \psi_{(x_o,p_o)} \rangle = x_o, \qquad \langle \psi_{(x_o,p_o)} | \hat{P} | \psi_{(x_o,p_o)} \rangle = p_o$$

and

$$\delta X := \sqrt{\frac{\Delta X^2}{\langle \hat{X} \rangle^2}} = \frac{\epsilon}{\sqrt{2} x_o} \ll 1, \qquad \delta P := \sqrt{\frac{\Delta P^2}{\langle \hat{P} \rangle^2}} = \frac{1}{\sqrt{2}\epsilon p_o} \ll 1$$

which is achieved if $p_o \gg \epsilon^{-1} \gg 1$.

For any polynomial operator $\hat{A} = A(\hat{X}, \hat{P}, [\cdot,\cdot])$, we have

$$\langle \psi_{(x_o,p_o)} | \hat{A} | \psi_{(x_o,p_o)} \rangle = A(x_o, p_o, i\{\cdot,\cdot\}) \left[1 + \mathcal{O}(\delta X + \delta P)\right]$$

What do we meen by "effective dynamics"?

Quantum dynamics:

$$\langle\psi^t_{(x_o,p_o)}|\hat{A}|\psi^t_{(x_o,p_o)}\rangle$$

where

$$\psi^t_{(x_o,p_o)} := e^{-i\hat{H}t}\psi_{(x_o,p_o)}$$

"Classical" dynamics on phase space:

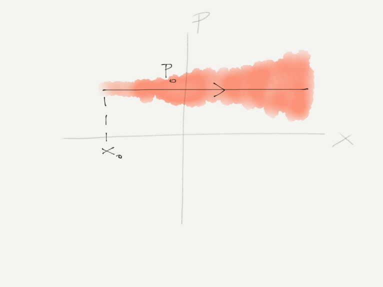
$$\dot{a}(t) = \{H_{\text{eff}}, a(t)\}, \qquad a(0) = A(x_o, p_o)$$

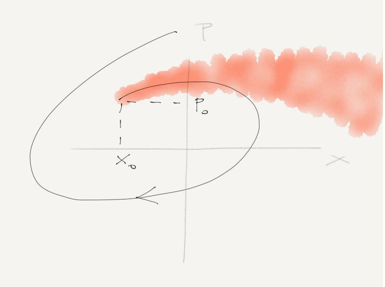
where

$$H_{\text{eff}}(x,p) := \langle\psi_{(x,p)}|\hat{H}|\psi_{(x,p)}\rangle$$

Effective Dynamics

$$\langle\psi^t_{(x_o,p_o)}|\hat{A}|\psi^t_{(x_o,p_o)}\rangle = a(t)[1 + \mathcal{O}(\delta X + \delta P)]$$





a good example (free particle) and a bad one (just everything else)

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Effective dynamics in LQG so far:

- Successfull in LQC **[Ashtekar, Pawlowski and Singh, 2006; Taveras, 2008]**
- Conjectured in QRLG **[Alesci, Bahrami, Botta, Cianfrani, Luzi, Pranzetti, Stagno]**

What about the full theory (at least on a fixed graph)?

Fix Γ (e.g., cubic lattice) with N edges:

- kinematical Hilbert space

$$\mathcal{H} = L_2(SU(2)^N, \mathrm{d}g_1..\mathrm{d}g_N)$$

  with operators, $\hat{h}_e$ (multiplication by g_e) and $\hat{E}^I_e$ (right-invariant v.f. R^I_e)
- discrete "geometry": Collection $(u_e, \xi_e) \in SU(2) \times \mathfrak{su}_2$ for every edge e.

Inspired on complexifier coherent states **[Sahlmann, Thiemann and Winkler, 2002; Bahr and Thiemann, 2007]**, we construct a class of "generalized coherent states".

Generalized coherent state: $\Psi_{(u,\xi)} \in \mathcal{H}$ given by

$$\Psi_{(u,\xi)}(g_1, .., g_N) = \prod_{e=1}^{N} \psi_{(u_e,\xi_e)}(g_e), \qquad \psi_{(u_e,\xi_e)}(g) = \frac{1}{N_e} f_e(g) e^{-S_e(g)/\epsilon}$$

where S_e satisfies:

- $\mathrm{Re}(S_e)$ has single minimum at $g_{e,o}$
- Hessian $R_e^I R_e^J S_e$ is non-degenerate at $g_{e,o}$

$\Rightarrow$ "approximate peakedness" wrt $\hat{h}_e$ and $\hat{E}_e^I$:

$$\langle \Psi_{(u,\xi)} | \hat{h}_e | \Psi_{(u,\xi)} \rangle = u_e \left[1 + \mathcal{O}(\epsilon)\right], \qquad \langle \Psi_{(u,\xi)} | \hat{E}_e^I | \Psi_{(u,\xi)} \rangle = \xi_e^I \left[1 + \mathcal{O}\left(\frac{1}{\epsilon |\xi_e|^2}\right)\right]$$

$$\delta h_e \propto \sqrt{\epsilon}[1 + \mathcal{O}(\epsilon)], \qquad \delta E_e^I \propto \frac{1}{\sqrt{\epsilon}|\xi_e|}\left[1 + \mathcal{O}\left(\frac{1}{\sqrt{\epsilon}|\xi_e|}\right)\right]$$

where $u_e = g_{e,o}$ and $\xi_e^I = \frac{1}{\epsilon}\mathrm{Im}(R_e^I S_e)(g_{e,o})$.

So, if we take $|\xi_e|^2 \gg \epsilon^{-1} \gg 1$, we find that $\delta h_e, \delta E_e^I \ll 1$, and hence $\Psi_{(u,\xi)}$ is peaked on the discrete "geometry" (u_e, ξ_e).

Effective dynamics conjecture [AD, Kamiński and Liegener]

Two observations:

- If $\hat{A}$ is a pdo[1] with principal symbol a, then

$$\langle \Psi_{(u,\xi)} | \hat{A} | \Psi_{(u,\xi)} \rangle = a(u,\xi)[1 + \mathcal{O}(\delta h + \delta E)]$$

- Egorov's Theorem: Let $\hat{B}$ be a positive, self-adjoint, elliptic pdo. Then

$$\hat{A} \text{ is a pdo} \quad \Longrightarrow \quad \hat{A}_t := e^{it\hat{B}} \hat{A} e^{-it\hat{B}} \text{ is a pdo}$$

If true, then

$$\langle \Psi^t_{(u,\xi)} | \hat{A} | \Psi^t_{(u,\xi)} \rangle = \langle \Psi_{(u,\xi)} | \hat{A}_t | \Psi_{(u,\xi)} \rangle = a_t(u,\xi) \left[1 + \mathcal{O}(\delta h + \delta E)\right]$$

with a_t principal symbol of $\hat{A}_t$. It follows

$$\frac{d}{dt} a_t(u,\xi) \approx \frac{d}{dt} \langle \Psi^t_{(u,\xi)} | \hat{A} | \Psi^t_{(u,\xi)} \rangle = i \langle \Psi_{(u,\xi)} | [\hat{B}, \hat{A}_t] | \Psi_{(u,\xi)} \rangle \approx -\{b(u,\xi), a_t(u,\xi)\}$$

with $b \approx \langle \Psi_{(u,\xi)} | \hat{B} | \Psi_{(u,\xi)} \rangle$ principal symbol of $\hat{B}$.

[1] See e.g. L. Hörmander, *The Analysis of Linear Partial Differential Operators*, (1987).

Summary of the conjecture

a_t given by

$$\langle\Psi^t_{(u,\xi)}|\hat{A}|\Psi^t_{(u,\xi)}\rangle = a_t(u,\xi)\ \left[1+\mathcal{O}(\delta h+\delta E)\right]$$

satisfies

$$\dot{a}_t = \{a_t, b\}, \qquad a_0 = \langle\Psi_{(u,\xi)}|\hat{A}|\Psi_{(u,\xi)}\rangle \left[1+\mathcal{O}(\delta h+\delta E)\right]$$

with

$$b = \langle\Psi_{(u,\xi)}|\hat{B}|\Psi_{(u,\xi)}\rangle \left[1+\mathcal{O}(\delta h+\delta E)\right]$$

This is exactly effective dynamics with effective Hamiltonian b!

Expectation value of $\hat{H}_{LQG}$ **[Giesel, Thiemann, 2006]** on generalized coherent state:

$$H_{\rm eff}(u,\xi) := \langle\Psi_{(u,\xi)}|\hat{H}_{LQG}|\Psi_{(u,\xi)}\rangle = \langle\Psi_{(u,\xi)}|H_{LQG}(\hat{h},\hat{E},[\cdot,\cdot])|\Psi_{(u,\xi)}\rangle =$$

$$= H^{\mu}_{\rm GR}(u,\xi,i\{\cdot,\cdot\})[1+\mathcal{O}(\delta h+\delta E)]$$

Examples:

1. homogeneous spacetimes (cosmology)
2. spherical black hole interior (BH's)
3. linearized Einstein equations (GW's)

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$$ds^2 = -dt^2 + p(t)[dx^2 + dy^2 + dz^2]$$

- Ashtekar-Barbero variables: $A_a^I = c\delta_a^I$ and $E_I^a = p\delta_I^a$
- fix the graph: cubic lattice embedded in space along the coordinate axes
- read off the classical holonomy and flux on each edge:

$$u_e = e^{-c\mu\tau_e}, \qquad \xi_e^I = \delta_e^I \alpha\mu^2 p$$

with μ the *coordinate length* of each edge

By construction, $\Psi_{(u,\xi)}$ is peaked on this "geometry":

$$\langle\Psi_{(u,\xi)}|\hat{h}_e|\Psi_{(u,\xi)}\rangle \approx e^{-c\mu\tau_e}, \qquad \langle\Psi_{(u,\xi)}|\hat{E}_e^I|\Psi_{(u,\xi)}\rangle \approx \delta_e^I\mu^2 p$$

Note – Scale μ is independent of (c, p): we are in μ_o-scheme.

Effective Hamiltonian **[AD and Liegener, 2017]**:

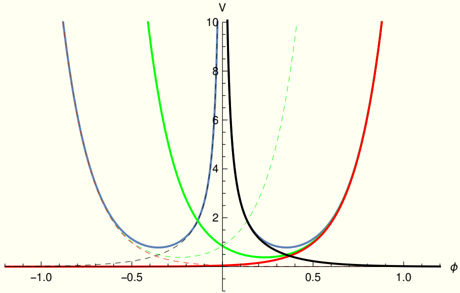
$$H_{\text{eff}} = -\frac{3}{8\pi G\gamma^2\mu^2}\sqrt{p}\left[\sin^2(\mu c) - (1+\gamma^2)\sin^4(\mu c) + \mathcal{O}(\delta h + \delta E)\right]$$
$$\approx H^{eff}_{LQC}\left[1 - (1+\gamma^2)\sin^2(\mu c)\right]$$

If it were $\bar{\mu}$-scheme, then:

- example of general LQC effective Hamiltonian **[Engle and Vilensky, 2018]**
- Equations of motion analytically solvable **[Assanioussi, AD, Liegener and Pawłowski, 2018 and 2019]**

Volume:

$$p^{\frac{3}{2}}(\phi) \propto \frac{1 + \gamma^2\cosh^2(\sqrt{12\pi G}\phi)}{|\sinh(\sqrt{12\pi G}\phi)|}$$



Pre-bounce branch: contracting de Sitter with effective cosmological constant

$$\Lambda_{\text{eff}} = \frac{3}{\Delta(1+\gamma^2)}$$

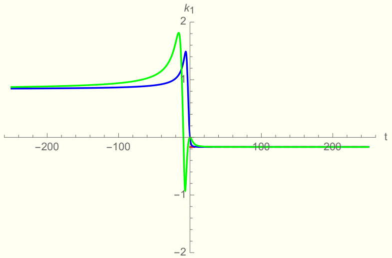
Notes:

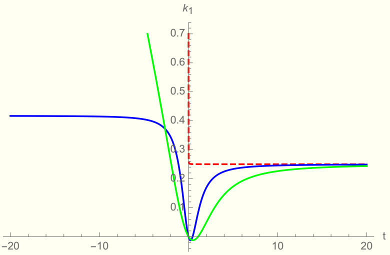
- quantum LQC-like model first proposed in **[Yang, Ding and Ma, 2009]**
- effective dynamics consistent with quantum dynamics **[Assanioussi, AD, Liegener and Pawłowski, 2018]**
- inclusion of inflaton **[Agullo, 2018; Li, Singh and Wang, 2018]**

Generalization to Bianchi I **[García-Quismondo and Mena Marugán, 2019]**

$$H_{\text{eff}} = \frac{1}{8\pi G}\sqrt{\frac{p_2 p_3}{p_1}}\frac{\sin(c_2\mu_2)}{\mu_2}\frac{\sin(c_3\mu_3)}{\mu_3}\left[1 - \frac{1+\gamma^2}{4\gamma^2}A(c_3, c_1)A(c_1, c_2)\right] + \text{cyclic}$$

where $A(c_i, c_j) := \cos(c_i\mu_i) + \cos(c_j\mu_j)$.





Notes:

- Vacuum case: only small differences between LQC and the new model.
- Matter case: no Kasner transition, pre-bounce dS phase (with same Λ_{eff})

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Studied within loop context from various perspecives **[Alesci, Ashtekar, Bodendorfer, Boehmer, Bojowald, Campiglia, Corichi, Gambini, Mele, Modesto, Münch, Olmedo, Pranzetti, Pullin, Saini, Singh, Vandersloot, ...]**

Schwartzschild interior = Kantowski-Sachs

$$ds^2 = -dT^2 + \frac{p_b(T)^2}{4|p_a(T)|} dR^2 + |p_a(T)| d\Omega^2$$

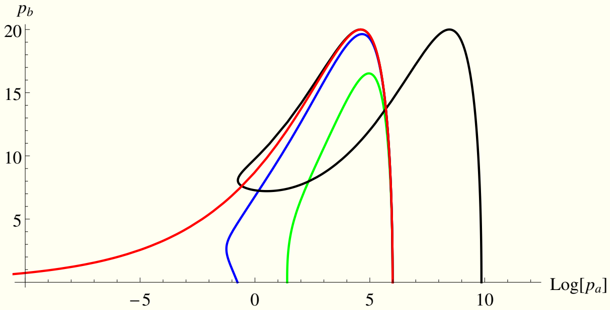
Ashterkar-Barbero variables written in terms of canonical variables a, b, p_a, p_b.

After an appropriate choice of graph, we construct the discrete geometry (u_e, ξ_e). For example, holonomies are

$$u_R = \exp[\gamma a \tau_1 \mu_1], \quad u_\theta = \exp[\gamma b \tau_2 \mu_2], \quad u_\phi = \exp[(\gamma b \tau_3 \sin\theta - \tau_1 \cos\theta)\mu_3]$$

The state $\Psi_{(u,\xi)}$ is peaked on this geometry, and we can compute $H_{\rm eff}$.

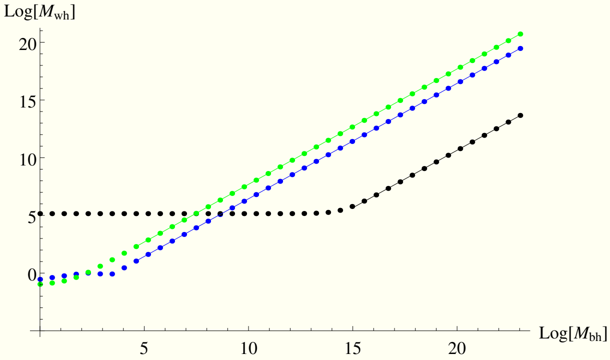
After solving the equations of motion **[Assanioussi, AD and Liegener, 2019]** ...



- horizons: $p_b = 0$ and $p_a = 4M^2$
- singularity: $p_a = 0$

In all cases, singularity replaced by BH $\to$ WH transition.

M_{WH} as a function of M:



For M large enough, $M_{WH} \propto\!\!\!\!\sim M$ in all cases.

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$$ds^2 = -dt^2 + [1 + h_+(t,z)]dx^2 + [1 - h_+(t,z)]dy^2 + 2h_\times(t,z)\,dxdy + dz^2$$

Lattice with N nodes along z direction.

Repeat construction... arrive at H_{eff}. From $\dot{h}_A = \{h_A, H_{\text{eff}}\}$, we find

$$h_A(t,z) = \frac{1}{N}\sum_k e^{ikz}\tilde{h}_{A,k}(t), \qquad \text{where } \ddot{\tilde{h}}_{A,k} + \omega_k^2 \tilde{h}_{A,k} = 0$$

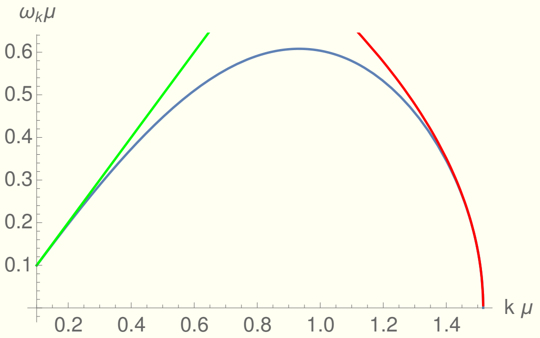
with

$$\omega_k = \frac{|\sin(k\mu)|}{\mu}\sqrt{(1+\gamma^2)\cos(k\mu) - \gamma^2}$$

Modified dispersion relation due to lattice.

Reminiscent of **[Sahlmann, Thiemann, 2002]**.

$$\omega_k = \frac{|\sin(k\mu)|}{\mu}\sqrt{(1+\gamma^2)\cos(k\mu) - \gamma^2}$$



Notes:

- $\mu \to 0$ recovers the classical dispersion relation $\omega_k \to |k|$
- not all modes propagate: for $k > k_o$, ω_k is imaginary
- restricing to $k \in [0, k_o]$, the high-k behavior is different from phonons

Toy model: tensor modes in cosmology

Classical cosmology:

$$\hat{h}_A(\eta, z) \propto \sum_k \left[\hat{b}_k \xi(k,\eta) e^{ikz} + \hat{b}_k^\dagger \xi^*(k,\eta) e^{-ikz}\right]$$

For background $ds^2 = a^2(\eta)[-d\eta^2 + dx^2 + ..]$, mode functions ξ satisfy

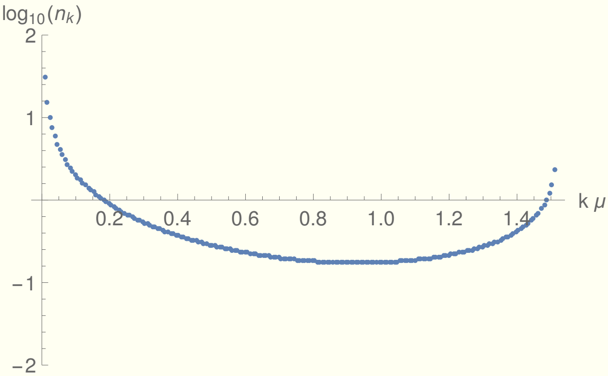
$$\xi'' + \left(k^2 - \frac{a''}{a}\right)\xi = 0$$

Generalization to our case:

$$\xi'' + \left(\omega_k^2 - \frac{a''}{a}\right)\xi = 0$$

$\Rightarrow$ amplification for low-k and high-k modes!

Number n_k of k-particles in the "far" future ($t \approx t_P \times 10^4$) starting with vacuum at bounce ($t = 0$):



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Done

- Generalized coherent state $\Psi_{(u,\xi)}$ peaked on discrete "geometry". Dynamical conjecture for $H_{\rm eff}$.
- Applications:
  - cosmology: non-symmetric bounce, pre-bounce dS (with $\Lambda_{\rm eff}$); Bianchi I deviates from LQC only if matter is present.
  - BH's: singularity replaced by BH $\to$ WH transition with $M_{WH} \propto M_{BH}$
  - GW's: modified ω_k; particle creation at both ends of k-spectrum

Next

- different regularizations $\Rightarrow$ different physics. Role of μ?
- role of matter: why Bianchi I different from LQC only if ϕ present?
- if toy model on tensor modes is serious: observable effects in CMB?

Thank you!

Happy birthday prof. Lewandowski!