Effective Dynamics of Loop Quantum Gravity: Bouncing Black Holes and Gravitational Phonons

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1. introduction: what is effective dynamics?

2. semiclassical states in quantum gravity on a graph

3. example: homogeneous cosmology

4. example: spherical black hole interior

5. example: linearized Einstein equations

6. conclusion
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A “semiclassical state” in QM:

$$\psi(x_0, p_0)(x) = \frac{1}{\sqrt{\epsilon \sqrt{\pi}}} e^{-\frac{(x-x_0)^2}{2\epsilon^2} + ip_0(x-x_0)}$$

Peakedness:

$$\langle \psi(x_0, p_0) | \hat{X} | \psi(x_0, p_0) \rangle = x_0, \quad \langle \psi(x_0, p_0) | \hat{P} | \psi(x_0, p_0) \rangle = p_0$$

and

$$\delta X := \sqrt{\frac{\Delta X^2}{\langle \hat{X} \rangle^2}} = \frac{\epsilon}{\sqrt{2x_0}} \ll 1, \quad \delta P := \sqrt{\frac{\Delta P^2}{\langle \hat{P} \rangle^2}} = \frac{1}{\sqrt{2\epsilon p_0}} \ll 1$$

which is achieved if $$p_0 \gg \epsilon^{-1} \gg 1$$.

For any polynomial operator $$\hat{A} = A(\hat{X}, \hat{P}, [\cdot, \cdot])$$, we have

$$\langle \psi(x_0, p_0) | \hat{A} | \psi(x_0, p_0) \rangle = A(x_0, p_0, i\{\cdot, \cdot\}) [1 + \mathcal{O}(\delta X + \delta P)]$$
What do we mean by “effective dynamics”?

Quantum dynamics:

\[ \langle \psi^t_{(x_0, p_0)} | \hat{A} | \psi^t_{(x_0, p_0)} \rangle \]

where

\[ \psi^t_{(x_0, p_0)} := e^{-i\hat{H}t} \psi_{(x_0, p_0)} \]

“Classical” dynamics on phase space:

\[ \dot{a}(t) = \{ H_{\text{eff}}, a(t) \}, \quad a(0) = A(x_0, p_0) \]

where

\[ H_{\text{eff}}(x, p) := \langle \psi_{(x, p)} | \hat{H} | \psi_{(x, p)} \rangle \]

**Effective Dynamics**

\[ \langle \psi^t_{(x_0, p_0)} | \hat{A} | \psi^t_{(x_0, p_0)} \rangle = a(t)[1 + O(\delta X + \delta P)] \]
a good example (free particle) and a bad one (just everything else)
introduction: what is effective dynamics?

semiclassical states in quantum gravity on a graph

example: homogeneous cosmology

example: spherical black hole interior

example: linearized Einstein equations

conclusion
Effective dynamics in LQG so far:
- **Successfull in LQC** [Ashtekar, Pawlowski and Singh, 2006; Taveras, 2008]
- **Conjectured in QRLG** [Alesci, Bahrami, Botta, Cianfrani, Luzi, Pranzetti, Stagno]

What about the full theory (at least on a fixed graph)?

Fix $\Gamma$ (e.g., cubic lattice) with $N$ edges:
- kinematical Hilbert space
  \[ \mathcal{H} = L_2(SU(2)^N, d g_1..d g_N) \]
  with operators, $\hat{h}_e$ (multiplication by $g_e$) and $\hat{E}_e^I$ (right-invariant v.f. $R_e^I$)
- discrete “geometry”: Collection $(u_e, \xi_e) \in SU(2) \times su_2$ for every edge $e$.

Inspired on complexifier coherent states [Sahlmann, Thiemann and Winkler, 2002; Bahr and Thiemann, 2007], we construct a class of “generalized coherent states”.
Generalized coherent state: $\Psi_{(u,\xi)} \in \mathcal{H}$ given by

$$
\Psi_{(u,\xi)}(g_1, \ldots, g_N) = \prod_{e=1}^{N} \psi_{(u_e,\xi_e)}(g_e), \quad \psi_{(u_e,\xi_e)}(g) = \frac{1}{N_e} f_e(g) e^{-S_e(g)/\epsilon}
$$

where $S_e$ satisfies:

- $\text{Re}(S_e)$ has single minimum at $g_{e,o}$
- Hessian $R_e R_e^l S_e$ is non-degenerate at $g_{e,o}$

$\Rightarrow$ “approximate peakedness” wrt $\hat{h}_e$ and $\hat{E}_e^l$:

$$
\langle \Psi_{(u,\xi)} | \hat{h}_e | \Psi_{(u,\xi)} \rangle = u_e \left[ 1 + O(\epsilon) \right], \quad \langle \Psi_{(u,\xi)} | \hat{E}_e^l | \Psi_{(u,\xi)} \rangle = \xi_e^l \left[ 1 + O\left(\frac{1}{\epsilon |\xi_e|^2} \right) \right]
$$

$$
\delta h_e \propto \sqrt{\epsilon} \left[ 1 + O(\epsilon) \right], \quad \delta E_e^l \propto \frac{1}{\sqrt{\epsilon |\xi_e|}} \left[ 1 + O\left(\frac{1}{\sqrt{\epsilon |\xi_e|}} \right) \right]
$$

where $u_e = g_{e,o}$ and $\xi_e^l = \frac{1}{\epsilon} \text{Im}(R_e R_e^l S_e)(g_{e,o})$.

So, if we take $|\xi_e|^2 \gg \epsilon^{-1} \gg 1$, we find that $\delta h_e, \delta E_e^l \ll 1$, and hence $\Psi_{(u,\xi)}$ is peaked on the discrete “geometry” $(u_e, \xi_e)$. 

Effective dynamics conjecture [AD, Kamiński and Liegener]

Two observations:

- If \( \hat{A} \) is a pdo\(^1\) with principal symbol \( a \), then
  \[
  \langle \psi_{(u,\xi)} | \hat{A} | \psi_{(u,\xi)} \rangle = a(u, \xi) [1 + O(\delta h + \delta E)]
  \]

- Egorov's Theorem: Let \( \hat{B} \) be a positive, self-adjoint, elliptic pdo. Then
  \[
  \hat{A} \text{ is a pdo} \implies \hat{A}_t := e^{it\hat{B}} \hat{A} e^{-it\hat{B}} \text{ is a pdo}
  \]

If true, then

\[
\langle \psi^t_{(u,\xi)} | \hat{A} | \psi^t_{(u,\xi)} \rangle = a_t(u, \xi) [1 + O(\delta h + \delta E)]
\]

with \( a_t \) principal symbol of \( \hat{A}_t \). It follows

\[
\frac{d}{dt} a_t(u, \xi) \approx \frac{d}{dt} \langle \psi^t_{(u,\xi)} | \hat{A} | \psi^t_{(u,\xi)} \rangle = i \langle \psi_{(u,\xi)} | [\hat{B}, \hat{A}_t] | \psi_{(u,\xi)} \rangle \approx -\{b(u, \xi), a_t(u, \xi)\}
\]

with \( b \approx \langle \psi_{(u,\xi)} | \hat{B} | \psi_{(u,\xi)} \rangle \) principal symbol of \( \hat{B} \).

Summary of the conjecture

\[ a_t \text{ given by} \]
\[ \langle \Psi^t_{(u,\xi)}|\hat{A}|\Psi^t_{(u,\xi)} \rangle = a_t(u,\xi) \left[ 1 + \mathcal{O}(\delta h + \delta E) \right] \]

satisfies

\[ \dot{a}_t = \{a_t, b\}, \quad a_0 = \langle \Psi_{(u,\xi)}|\hat{A}|\Psi_{(u,\xi)} \rangle \left[ 1 + \mathcal{O}(\delta h + \delta E) \right] \]

with

\[ b = \langle \Psi_{(u,\xi)}|\hat{B}|\Psi_{(u,\xi)} \rangle \left[ 1 + \mathcal{O}(\delta h + \delta E) \right] \]

This is exactly effective dynamics with effective Hamiltonian \( b \)!

Expectation value of \( \hat{H}_{LQG} \) [Giesel, Thiemann, 2006] on generalized coherent state:

\[ H_{\text{eff}}(u,\xi) := \langle \Psi_{(u,\xi)}|\hat{H}_{LQG}|\Psi_{(u,\xi)} \rangle = \langle \Psi_{(u,\xi)}|H_{LQG}(\hat{h}, \hat{E}, [\cdot,\cdot])|\Psi_{(u,\xi)} \rangle = \]

\[ = H^\mu_{\text{GR}}(u,\xi, i\{\cdot,\cdot\})[1 + \mathcal{O}(\delta h + \delta E)] \]
Examples:

1. homogeneous spacetimes (cosmology)
2. spherical black hole interior (BH’s)
3. linearized Einstein equations (GW’s)
outline

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\[ ds^2 = -dt^2 + p(t)[dx^2 + dy^2 + dz^2] \]

- Ashtekar-Barbero variables: \( A^l_a = c\delta^l_a \) and \( E^a_l = p\delta^a_l \)
- fix the graph: cubic lattice embedded in space along the coordinate axes
- read off the classical holonomy and flux on each edge:

\[
  u_e = e^{-c\mu \tau_e}, \quad \xi^l_e = \delta^l_e \alpha \mu^2 p
\]

with \( \mu \) the coordinate length of each edge

By construction, \( \Psi_{(u, \xi)} \) is peaked on this “geometry”:

\[
  \langle \Psi_{(u, \xi)} | \hat{h}_e | \Psi_{(u, \xi)} \rangle \approx e^{-c\mu \tau_e}, \quad \langle \Psi_{(u, \xi)} | \hat{E}^l_e | \Psi_{(u, \xi)} \rangle \approx \delta^l_e \mu^2 p
\]

Note – Scale \( \mu \) is independent of \((c, p)\): we are in \( \mu_o \)-scheme.
Effective Hamiltonian [AD and Liegener, 2017]:

\[
H_{\text{eff}} = -\frac{3}{8\pi G \gamma^2 \mu^2} \sqrt{p} \left[ \sin^2(\mu c) - (1 + \gamma^2) \sin^4(\mu c) + O(\delta h + \delta E) \right]
\approx H_{\text{eff}}^{\text{LQC}} \left[ 1 - (1 + \gamma^2) \sin^2(\mu c) \right]
\]

If it were \( \bar{\mu} \)-scheme, then:
- example of general LQC effective Hamiltonian [Engle and Vilensky, 2018]
- Equations of motion analytically solvable [Assanioussi, AD, Liegener and Pawłowski, 2018 and 2019]

Volume:

\[
p^{\frac{3}{2}}(\phi) \propto \frac{1 + \gamma^2 \cosh^2(\sqrt{12\pi G\phi})}{|\sinh(\sqrt{12\pi G\phi})|^{\frac{15}{29}}}
\]
Pre-bounce branch: contracting de Sitter with effective cosmological constant

\[ \Lambda_{\text{eff}} = \frac{3}{\Delta(1 + \gamma^2)} \]

Notes:
- quantum LQC-like model first proposed in [Yang, Ding and Ma, 2009]
- effective dynamics consistent with quantum dynamics [Assanioussi, AD, Liegener and Pawłowski, 2018]
- inclusion of inflaton [Agullo, 2018; Li, Singh and Wang, 2018]
Generalization to Bianchi I [García-Quismondo and Mena Marugán, 2019]

\[
H_{\text{eff}} = \frac{1}{8\pi G} \sqrt{\frac{p_2 p_3}{p_1} \frac{\sin(c_2\mu_2)}{\mu_2} \frac{\sin(c_3\mu_3)}{\mu_3}} \left[ 1 - \frac{1 + \gamma^2}{4\gamma^2} A(c_3, c_1) A(c_1, c_2) \right] + \text{cyclic}
\]

where \( A(c_i, c_j) := \cos(c_i\mu_i) + \cos(c_j\mu_j) \).

Notes:
- Vacuum case: only small differences between LQC and the new model.
- Matter case: no Kasner transition, pre-bounce dS phase (with same \( \Lambda_{\text{eff}} \))
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Studied within loop context from various perspectives [Alesci, Ashtekar, Bodendorfer, Boehmer, Bojowald, Campiglia, Corichi, Gambini, Mele, Modesto, Münch, Olmedo, Pranzetti, Pullin, Saini, Singh, Vandersloot, ...]

Schwartzschild interior = Kantowski-Sachs

\[ ds^2 = -dT^2 + \frac{p_b(T)^2}{4|p_a(T)|} dR^2 + |p_a(T)|d\Omega^2 \]

Ashtekar-Barbero variables written in terms of canonical variables \( a, b, p_a, p_b \).

After an appropriate choice of graph, we construct the discrete geometry \((u_e, \xi_e)\). For example, holonomies are

\[ u_R = \exp[\gamma a \tau_1 \mu_1], \quad u_\theta = \exp[\gamma b \tau_2 \mu_2], \quad u_\phi = \exp[(\gamma b \tau_3 \sin \theta - \tau_1 \cos \theta) \mu_3] \]

The state \( \Psi_{(u,\xi)} \) is peaked on this geometry, and we can compute \( H_{\text{eff}} \).
After solving the equations of motion [Assanioussi, AD and Liegener, 2019] ...

- horizons: $p_b = 0$ and $p_a = 4M^2$
- singularity: $p_a = 0$

In all cases, singularity replaced by BH $\rightarrow$ WH transition.
\[ M_{WH} \text{ as a function of } M: \]

For \( M \) large enough, \( M_{WH} \propto M \) in all cases.
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\[ ds^2 = -dt^2 + [1 + h_+(t, z)]dx^2 + [1 - h_+(t, z)]dy^2 + 2h_\times(t, z) 
dx dy + dz^2 \]

Lattice with \( N \) nodes along \( z \) direction.

Repeat construction... arrive at \( H_{\text{eff}} \). From \( \dot{h}_A = \{ h_A, H_{\text{eff}} \} \), we find

\[ h_A(t, z) = \frac{1}{N} \sum_k e^{ikz} \tilde{h}_{A,k}(t), \quad \text{where} \quad \ddot{h}_{A,k} + \omega_k^2 \tilde{h}_{A,k} = 0 \]

with

\[ \omega_k = \frac{|\sin(k\mu)|}{\mu} \sqrt{(1 + \gamma^2) \cos(k\mu) - \gamma^2} \]

Modified dispersion relation due to lattice.

Reminiscent of [Sahlmann, Thiemann, 2002].
\[ \omega_k = \frac{|\sin(k\mu)|}{\mu} \sqrt{(1 + \gamma^2) \cos(k\mu) - \gamma^2} \]

Notes:
- \( \mu \to 0 \) recovers the classical dispersion relation \( \omega_k \to |k| \)
- Not all modes propagate: for \( k > k_o \), \( \omega_k \) is imaginary
- Restricting to \( k \in [0, k_o] \), the high-k behavior is different from phonons
Toy model: tensor modes in cosmology

Classical cosmology:

\[ \hat{h}_A(\eta, z) \propto \sum_k \left[ \hat{b}_k \xi(k, \eta) e^{ikz} + \hat{b}^\dagger_k \xi^*(k, \eta) e^{-ikz} \right] \]

For background \[ ds^2 = a^2(\eta)[-d\eta^2 + dx^2 + ..] \], mode functions \( \xi \) satisfy

\[ \xi'' + \left( k^2 - \frac{a''}{a} \right) \xi = 0 \]

Generalization to our case:

\[ \xi'' + \left( \omega_k^2 - \frac{a''}{a} \right) \xi = 0 \]

\( \Rightarrow \) amplification for low-\( k \) and high-\( k \) modes!
Number $n_k$ of $k$-particles in the “far” future ($t \approx t_P \times 10^4$) starting with vacuum at bounce ($t = 0$):
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Done

- Generalized coherent state $\Psi_{(u,\xi)}$ peaked on discrete “geometry”.
  Dynamical conjecture for $H_{\text{eff}}$.

- Applications:
  * cosmology: non-symmetric bounce, pre-bounce dS (with $\Lambda_{\text{eff}}$); Bianchi I deviates from LQC only if matter is present.
  * BH’s: singularity replaced by BH $\rightarrow$ WH transition with $M_{\text{WH}} \propto M_{\text{BH}}$
  * GW’s: modified $\omega_k$; particle creation at both ends of $k$-spectrum

Next

- different regularizations $\Rightarrow$ different physics. Role of $\mu$?

- role of matter: why Bianchi I different from LQC only if $\phi$ present?

- if toy model on tensor modes is serious: observable effects in CMB?
Thank you!

Happy birthday prof. Lewandowski!