Positivity of energy for asymptotically hyperbolic manifolds

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(photos of the blackboard notes from the talk given at JurekFest)

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http://jurekfest.fuw.edu.pl

Theorem A The energy-momentum of asymptotically hyperbolic manifolds is timelike future pointing unless hyperbolic space.

(1) Spacetime with $\Lambda < 0$, time symmetric if $R(g) \geq -n(n-1)$

(2) AdS $\overset{\circ}{g} = -\overset{\circ}{V}^2 dt^2 + \frac{dr^2}{\overset{\circ}{V}^2} + r^2 d\Omega^2$

$\overset{\circ}{V} = \sqrt{1 + r^2}$

$d\Omega^2$: sphere

$\overset{\circ}{g}$

(3) Not known for spin manifolds

(1) (M, g) ... $g \to \tilde{g}$ in the asymptotic region

$$R(g) \geq R(\tilde{g}) = -n(n-1)$$

n = dimension

Constraints

(M, g, K)

$$R(g) = \rho \boxed{+ 2\Lambda} + |K|^2 - (\mathrm{tr} K)^2$$

Space

energy-momentum ?

Space-time $(g) \to (\bar{g})$ translational Killing vector

$$p(V) = (\ ) \lim_{R \to \infty} \int \left(R^i_j - \frac{R}{n} \delta^i_j \right) D_i V \, dS^j$$

$n+1$ functions $\quad V^0 = \overset{r=R}{\overset{\circ}{V}}{}^0 \quad V^i = x^i$

$$V = \alpha_\mu V^\mu \qquad \alpha_\mu \in \mathbb{R}$$

$$p^\mu = p(V^\mu)$$

FACT p^μ transforms as a LORENTZ VECTOR

ASYMPT FLAT 2016 LOHKAMP: A FLAT $\Rightarrow p^\mu$ TIMELIKE FUTURE

If the dominant energy condition

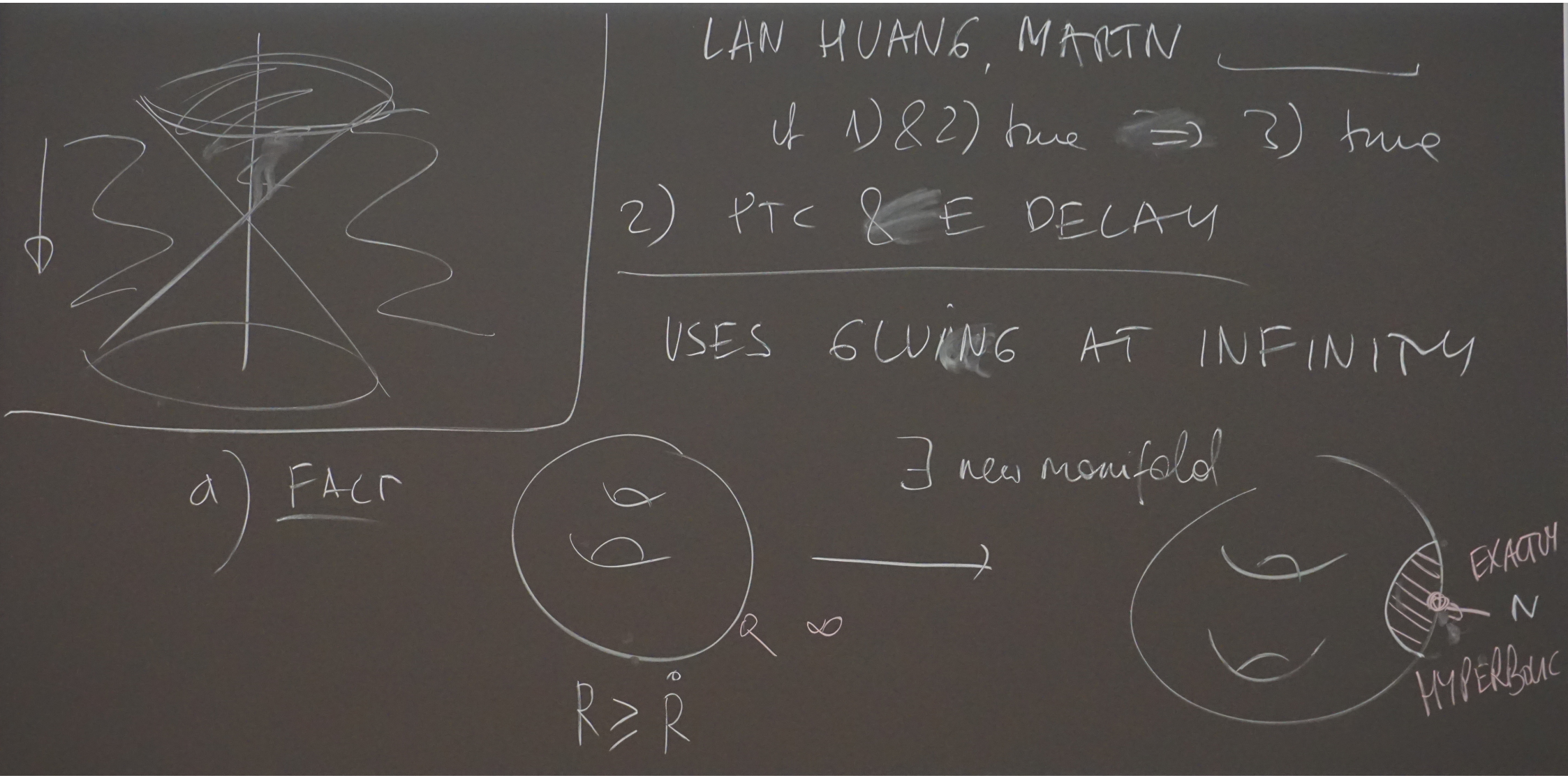
2017 SCHOEN-YAU ($K_{ij} = 0$)

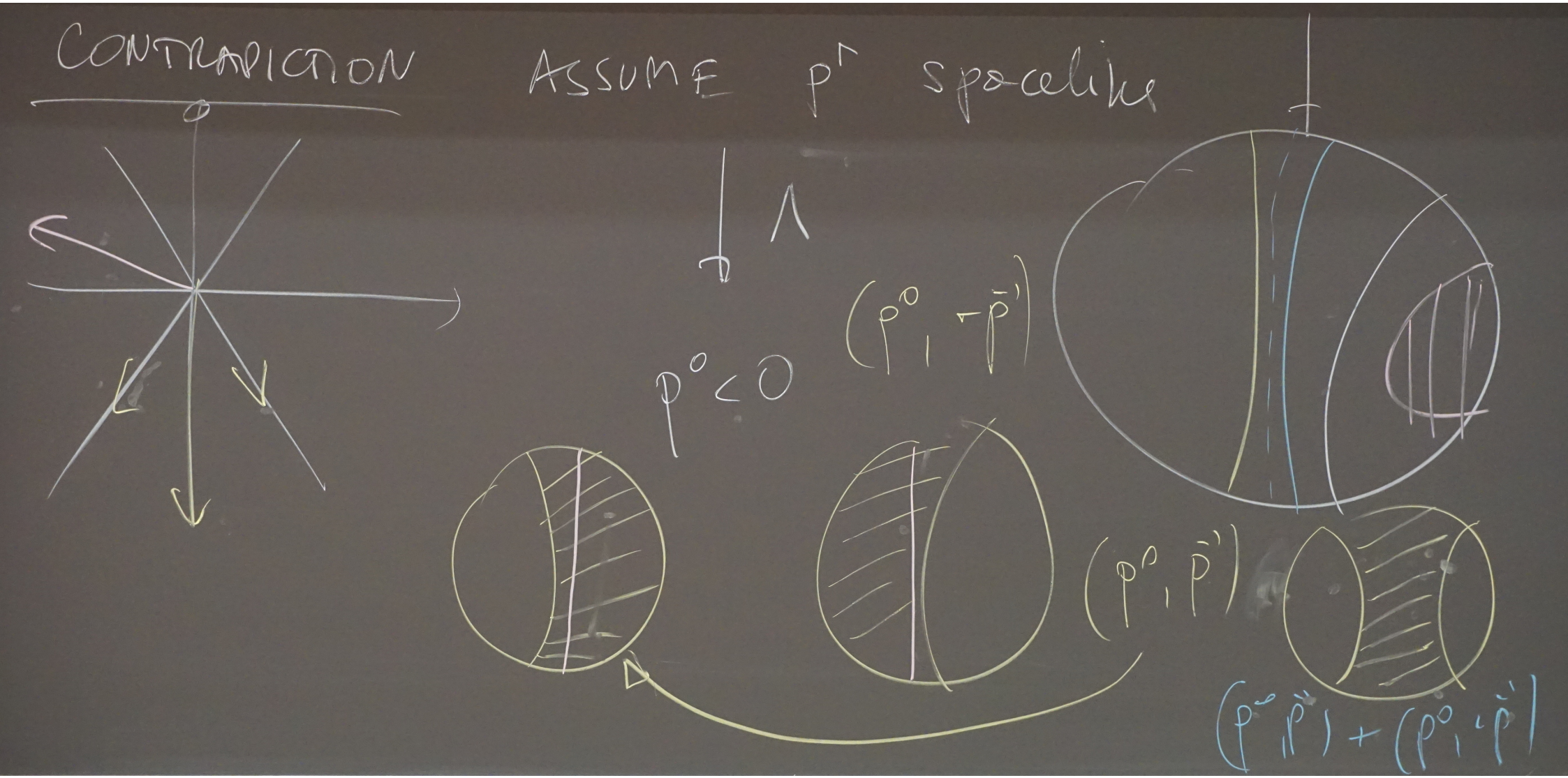
STEPS OF THM A $n = 3$ SPIN $\Rightarrow$ nothing

thus $n \geq 4$: 1) p^μ cannot be timelike past pointing

2) p^μ cannot be spacelike or null past pointing

3) ——— null future pointing


LAN HUANG, MARTN
if 1) & 2) true ⇒ 3) true
2) PTC & E DELAY
USES GLUING AT INFINITY
a) FACT
∃ new manifold
R ≥ R̊
EXACTLY
N
HYPERBOLIC


CONTRADICTION
ASSUME p^μ spacelike
$p^0 < 0$
$(p^0, -\vec{p})$
$(p^0, \vec{p})$
$(p^0, \vec{p}) + (p^0, -\vec{p})$