Operator Spin Foam models:
Coarse graining and entanglement entropy

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Jurekfest

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I Motivation

II Operator Spin Foam Models

- a. Definition
- b. Coarse graining

III Toy model: hypercuboidal OSFM

- a. RG flow & fixed point
- b. Entanglement entropy

IV Summary

I Motivation

1990's: Construction of LQG Hilbert space $\mathcal{H}_{\rm kin} = L^2(\overline{\mathcal{A}}, d\mu_{\rm AL})$

ONBasis: Spin network functions (quantised 3-geometry) $\psi_{\Gamma,\vec{j},\vec{\iota}}$

Dynamics: Constraints (canonical) $C(N),\ \vec{C}(\vec{N})$

kinematical Hilbert space $\rightarrow$ physical Hilbert space ?

physical inner product?

[Ashtekar, Lewandowski '92, ALMMT (MAFIA) '95, Rovelli, Smolin '95, Marolf '95, Ashtekar, Lewandowski '96, Thiemann ''96-'00, ...]

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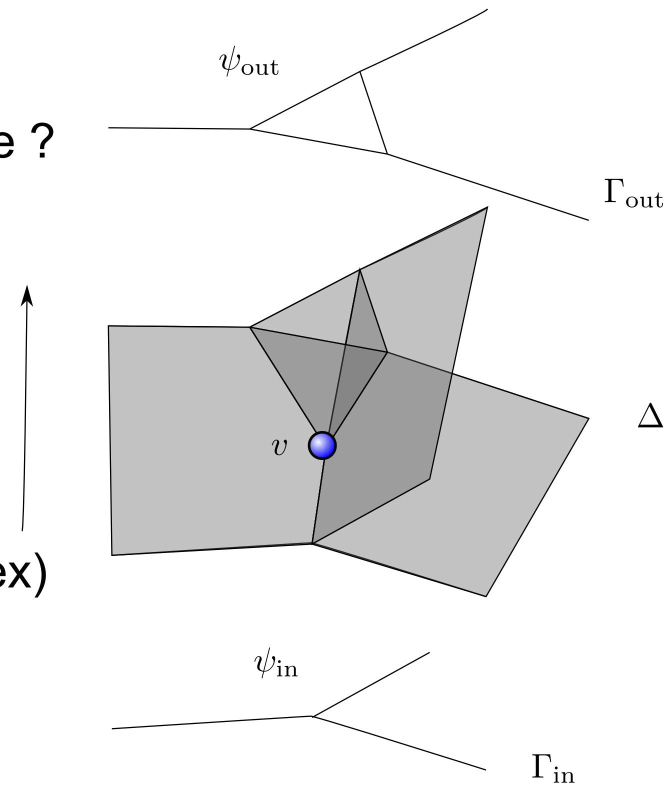
Spin Foam models as "histories of 3-geometries"

1997: Barrett Crane spin foam model

2007: Livine Speziale, EPRL-model, FK-model (4-simplex)

2008: Baratin, Flori, Thiemann (cubulation)

2009: KKL-extension of EPRL-FK (arbitrary 2-complex)



[Reisenberger '94, Barrett, Crane '99, Livine, Speziale '07, Engle, Pereira, Rovelli, Livine '07, Freidel, Krasnov '07, Baratin, Flori, Thiemann '08, Oriti Baratin '11,... Kaminski, Kisielowski, Lewandowski '09]

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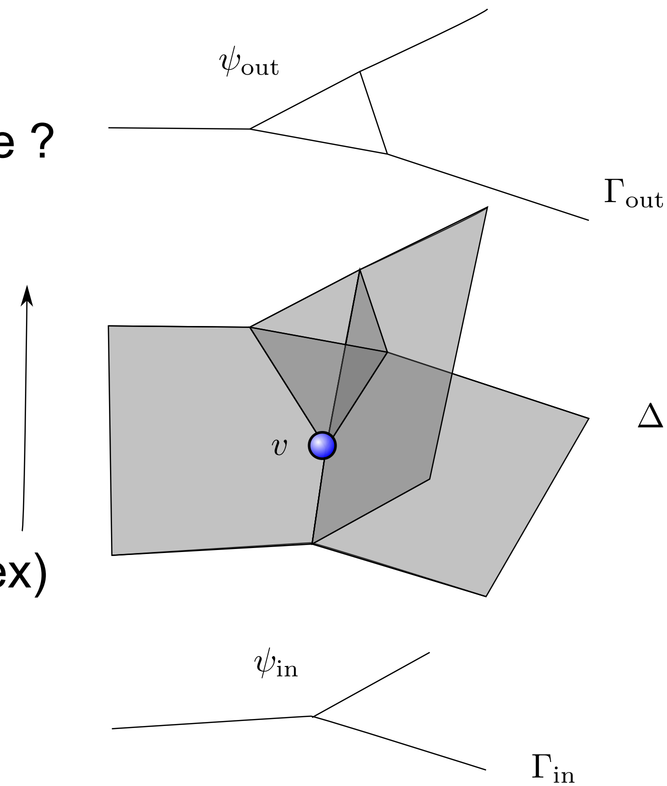
2010: General class: Operator Spin Foam models

→ useful for renormalisation



[BB, Hellmann, Kaminski, Kisielowski, Lewandowski '10]

"renormalisation"

"renormalisation"

"cylindrical consistency"

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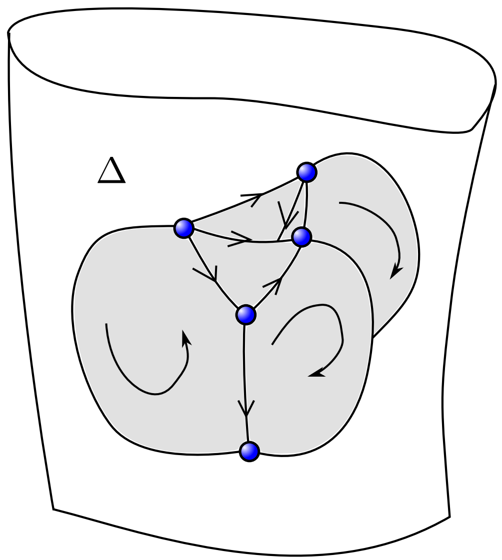
Ingredients:



- Oriented 2-complex Δ
- Compact gauge group G
- Class function $w : G \longrightarrow \mathbb{C}$
- For each tensor product

$$\rho_1 \otimes \rho_2 \otimes \cdots \rho_k \otimes \rho_{k+1}^* \otimes \cdots \rho_n^*$$

of irreducible representations (and duals) :
an operator

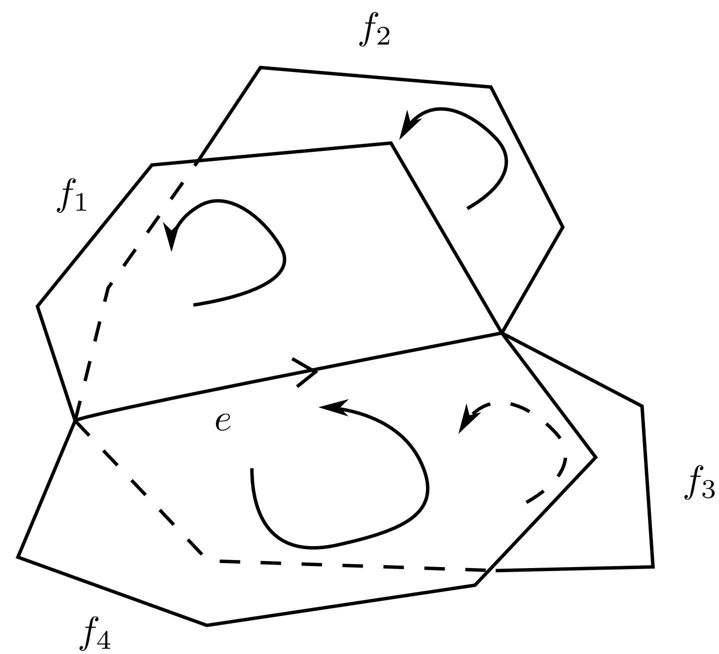
$$P : V_{\rho_1} \otimes \cdots \otimes V_{\rho_k} \otimes V_{\rho_{k+1}^*} \otimes \cdots V_{\rho_n^*} \longrightarrow V_{\rho_1} \otimes \cdots \otimes V_{\rho_k} \otimes V_{\rho_{k+1}^*} \otimes \cdots V_{\rho_n^*}$$

[as in: BB, Hellmann, Kaminski, Kisielowski, Lewandowski ‘10]

A “state” on Δ :

Distribution of irreps of G
To 2-cells (“faces”) of Δ

$$f \;:\longrightarrow\; \rho_f$$



→ “edge-Hilbert space”

$$\mathcal{H}_e \;:=\; \bigotimes_{[e,f]=1} V_{\rho_f} \otimes \bigotimes_{[e,f]=-1} V_{\rho_f^*}$$

where $[e,f] \;=\; \pm 1$ iff respective orientations agree / disagree

→ “edge-operator”

$$P_e \;:\; \mathcal{H}_e \;\longrightarrow\; \mathcal{H}_e$$

II Operator Spin Foam Models: Definition

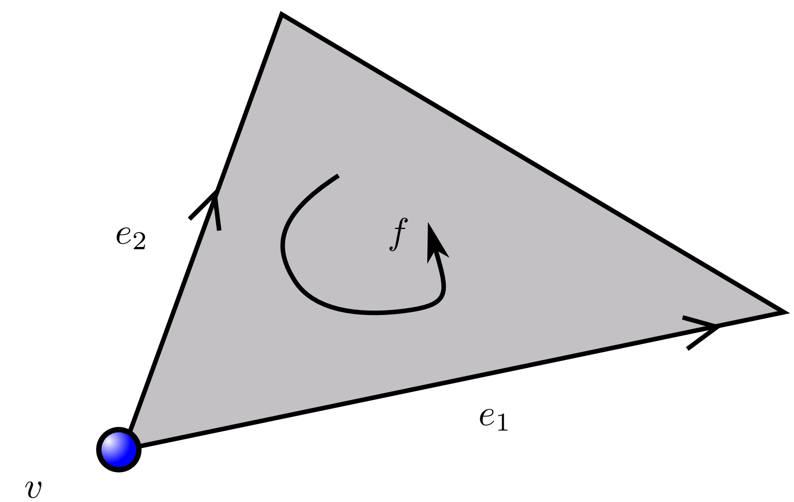
Vertex-trace: $P \,:\, V_{\rho_1} \otimes \cdots \otimes V_{\rho_k} \otimes V_{\rho^*_{k+1}} \otimes \cdots V_{\rho^*_n} \longrightarrow V_{\rho_1} \otimes \cdots \otimes V_{\rho_k} \otimes V_{\rho^*_{k+1}} \otimes \cdots V_{\rho^*_n}$

Contraction of all indices of edge operators $P_e \,:\, \mathcal{H}_e \to \mathcal{H}_e$
on edges meeting at a 0-cell (“vertex”): v

$$(P_{e_1})_{m_1 \cdots m_k}{}^{m_{k+1} \cdots m_f \cdots m_N}|^{n_1 \cdots n_k}{}_{n_{k+1} \cdots n_N}$$

$$\delta^{r_f}{}_{m_f}$$

$$(P_{e_2})_{r_1 \cdots r_f \cdots r_k}{}^{r_{k+1} \cdots r_M}|^{s_1 \cdots s_k}{}_{s_{k+1} \cdots s_N}$$



$$\longrightarrow \quad \mathcal{Z}[\Delta,\, \{\rho_f\},\, w,\, P] \;=\; \prod_f \hat{w}_{\rho_f} \prod_v \mathrm{tr}_v\big(\otimes_e P_e\big)$$

Where $w(g) \;=\; \sum_\rho \hat{w}_\rho\, \chi_\rho(g)$

And (if it converges): $\mathcal{Z}[\Delta,\, w,\, P] \;=\; \sum_{\rho_f} \mathcal{Z}[\Delta,\, \{\rho_f\},\, w,\, P]$ “Spin Foam State Sum”

(can be written as sum over irreps and intertwiners of amplitudes)

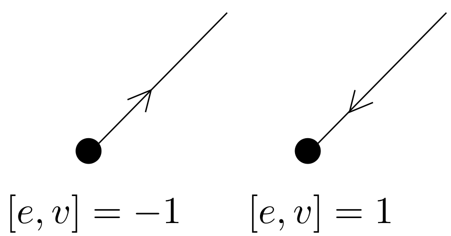
II Operator Spin Foam Models: Definition

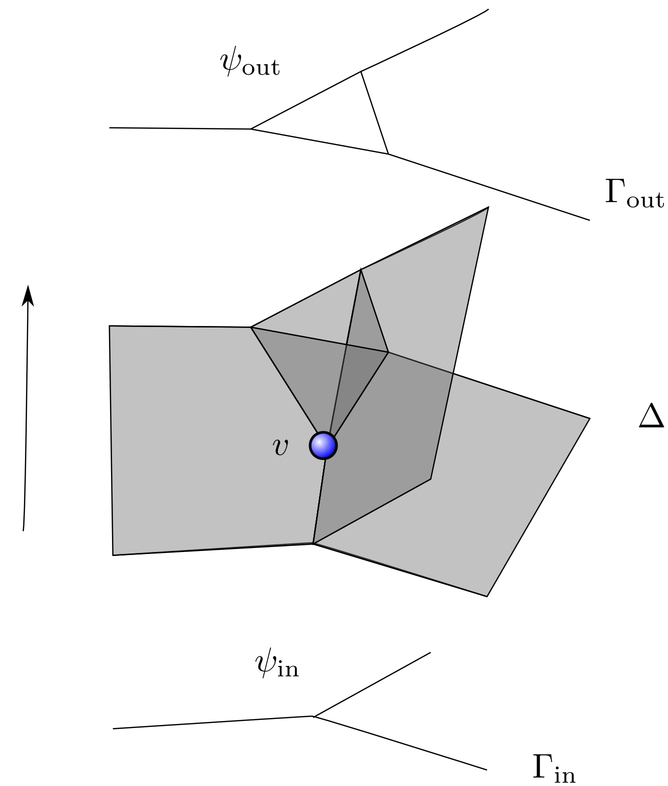
2-complex with boundary: (not necessarily connected) subgraph, $\Gamma \subset \Delta$
e.g. all edges with only one face (“link”), all vertices with only one edge (“node”)

→ orientation of links determined by that of their respective faces

Boundary Hilbert space: $\mathcal{H}_\Gamma = \bigotimes_{[e,v]=1} V_{\rho_e} \otimes \bigotimes_{[e,v]=-1} V_{\rho_e^*}$

$\sim L^2(G^L, d\mu_{\mathrm{H}})$





Spin foam state sum: linear form on boundary Hilbert space

$$\mathcal{Z}[\Delta,\, w,\, P] \,:\, \mathcal{H}_\Gamma \to \mathbb{C}$$

$$\mathcal{Z}[\Delta,\, w,\, P]\psi \;:= \sum_{\rho_f} \prod_{f:\partial\Delta} \hat{w}_{\rho_f}^{\frac{1}{2}} \prod_{f:\Delta^\circ} \hat{w}_{\rho_f} \prod_v \mathrm{tr}_v \left(\bigotimes_{e\partial\Delta} P_e \otimes \psi \right)$$

→ Boundary decomposes in “in” and “out” part: sesquilinear form on $\mathcal{H}_{\Gamma_{\mathrm{in}}}^\dagger \times \mathcal{H}_{\Gamma_{\mathrm{out}}}$

Properties:

- Operators P_e Hermitean and $\rho \sim \rho^*$: $\mathcal{Z}[\Delta, w, P]$ independent of orientations of Δ
- Additionally: $\mathrm{Img}(P_e) \subset \mathrm{Inv}_G(\mathcal{H}_e)$: linear form $\mathcal{Z}[\Delta, w, P]$ gauge-invariant

  → sum over invariant elements ("intertwiners"): $P_e = \sum_{n,m} P_{nm}\, \iota_n^\dagger \otimes \iota_m$

  → $\mathcal{Z}[\Delta, w, P]$ invariant under trivial subdivisions of faces
- Idempotent: $P_e^2 = P_e$

  → $\mathcal{Z}[\Delta, w, P]$ invariant under trivial subdivisions of edges
- Composition: $\mathcal{Z}[\Delta_1, \{\rho_f\}, w, P]\mathcal{Z}[\Delta_2, \{\rho_f\}, w, P] = \mathcal{Z}[\Delta_1 \#_\Gamma \Delta_2, \{\rho_f\}, w, P]$

Examples:

- Lattice Yang-Mills theory: 2-complex Δ dual to cubic lattice,
  Gauge group $G = SU(N)$
  Haar projectors: $P_e = P_{\text{Haar}}$
  Wilson action: $w \sim e^{-\beta S_{\text{Wilson}}}$

- BF-theory: (unregularised) TQFT, Class function $w \to \delta_G$ formally (finite for finite groups, or non-TARDIS-complexes) $P_e = P_{\text{Haar}}$

$$\mathcal{Z}[\Delta,\, \delta_G,\, P_{\text{Haar}}] = \int_{G^E} dg \prod_f \delta(H_f) \qquad H_f := \overrightarrow{\prod_{e \supset f}} g_e^{[e,f]}$$

- Euclidean Barrett-Crane model: 2-complex dual to 4d triangulation
  gauge group $G = SU(2) \times SU(2)$ class function $w \to \delta_G$
  operators $P_e = \iota_{\text{BC}}^\dagger \otimes \iota_{\text{BC}}$ projectors on 1-dim subspace,
  spanned by BC-intertwiner $\iota_{\text{BC}} \in \mathcal{H}_e = V_{(j_1^+, j_1^-)} \otimes V_{(j_2^+, j_2^-)} \otimes V_{(j_3^+, j_3^-)} \otimes V_{(j_4^+, j_4^-)}$ $\quad j_i^+ = j_i^-$

- KKL-extension of (Euclidean) EPRL-FK-model: $G = SU(2) \times SU(2)$
  operators $P_e = P_{\mathcal{V}}$ maps onto $\mathcal{V} \subset \mathcal{H}_e = V_{(j_1^+, j_1^-)} \otimes \cdots V_{(j_n^+, j_n^-)}$
  $\longrightarrow$ “solutions to simplicity constraints” $\quad j_i^\pm = \dfrac{|1 \pm \gamma|}{2} k_i$
  $\longrightarrow$ $\gamma \in \mathbb{R} \backslash \{0, \pm 1\}$ Barbero-Immirzi parameter

[Barrett, Crane, ‘99, Barrett, Naish-Guzman ‘08, Kaminski, Kisielowski, Lewandowski ‘09, ...]

II Operator Spin Foam Models: Definition

Further developments / generalisations:

- Feynman-diagrammatic approach
- Dual holonomy formulation (HSFM)
- Non-compact groups (e.g. Lorentzian signature for BC, EPRL-FK)
  $\rightarrow$ careful removal of divergencies
- Vertex trace: contraction with non-trivial operators (~cosm. const. Λ)
- Group $\rightarrow$ Quantum Group (~ cosm. const Λ , finiteness)
- different state spaces (spin networks $\rightarrow$ fusion networks, 2-groups, …)
- Sum over Δ : group field theories, tensor field theories
- Cosine issue: proper vertex
- Non-localities (e.g. volume simplicity constraint implementation)

[Kisielowski, Lewandowski, Puchta ‘11 / BB, Dittrich, Hellmann, Kaminski ‘12 / Engle, Pereira, Rovelli, Livine, ‘09 / Fairbairn, Meusburger ‘11 / Han ‘11, BB, Rabuffo ‘17 / Delcamp, Dittrich, Riello ‘16, Dittrich ‘19 / Oriti ‘06, Baratin, Oriti ‘11 / Engle ‘13, Engle, Zipfel, Vilensky ‘15 / BB Belov ‘18]

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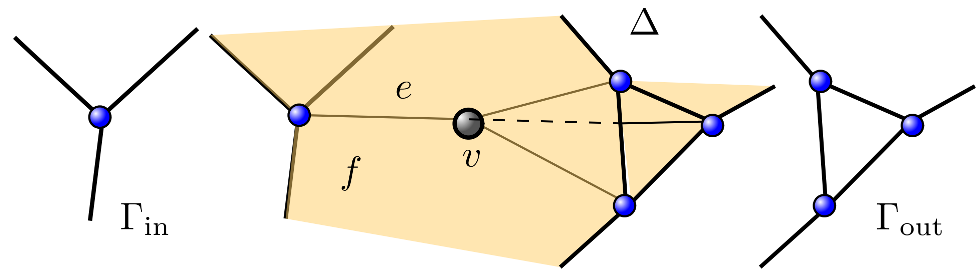
II Operator Spin Foam Models: Coarse graining

Spin foam operator so far depends on 2-complex Δ : discretisation (d.o.f. cutoff)

$$\mathcal{Z}[\Delta,\, w\, P]\; \psi_{\text{in}}^{\dagger} \otimes \psi_{\text{out}} \;\simeq\; \langle \psi_{\text{in}} \,|\, \psi_{\text{out}} \rangle_{\text{phys}}$$

$$\psi_{\text{in/out}} \;\in\; \mathcal{H}_{\Gamma_{\text{in/out}}}$$



Physical Hilbert space: contain information about all graphs Γ : continuum limit

Coarse graining / refinement of graphs: directed set $\Gamma \,\leq\, \Gamma'$

Choice of embedding maps: $\phi_{\Gamma'\Gamma} \,:\, \mathcal{H}_{\Gamma} \longrightarrow \mathcal{H}_{\Gamma'}$

→ Relation between OSFM on Δ and Δ'

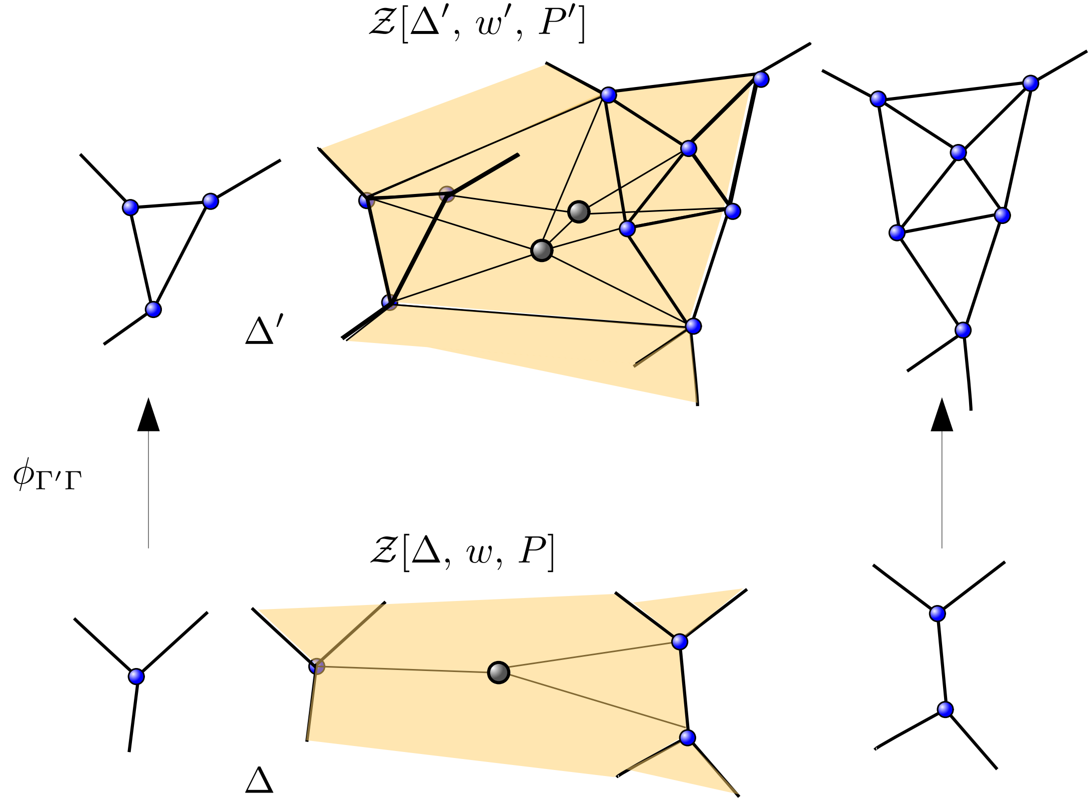
condition: $$\mathcal{Z}[\Delta',\, w',\, P'] \circ \phi_{\Gamma'\Gamma} \;=\; \mathcal{Z}[\Delta,\, w,\, P]$$

→ “Flow of coupling constants”: $w,\, P \sim$ parameters of the OSF

$\Delta' \to \Delta$ results in $w', P' \longrightarrow w, P$

[Manrique, Oeckl, Weber, Zapata ‘05, Rovelli, Smerlak ‘10, Dittrich, Eckert, Martin-Benito ‘11, BB ‘11, BB, Dittrich, Hellmann, Kaminski ‘12, Riello ‘13, Dittrich, Steinhaus ‘13, BB ‘14, Dittrich, Mizera, Steinhaus ‘14, Banburski, Chen, Freidel, Hnybida ‘14, Dittrich, Schnetter, Seth, Steinhaus ‘16, Delcamp, Dittrich ‘17, BB, Steinhaus ‘17, Lang, Liegener, Thiemann ‘17, BB, Rabuffo, Steinhaus ‘18, ...]

Schematically:



Change of $\Delta' \to \Delta$ results in $w', P' \longrightarrow w, P$ → "renormalisation"

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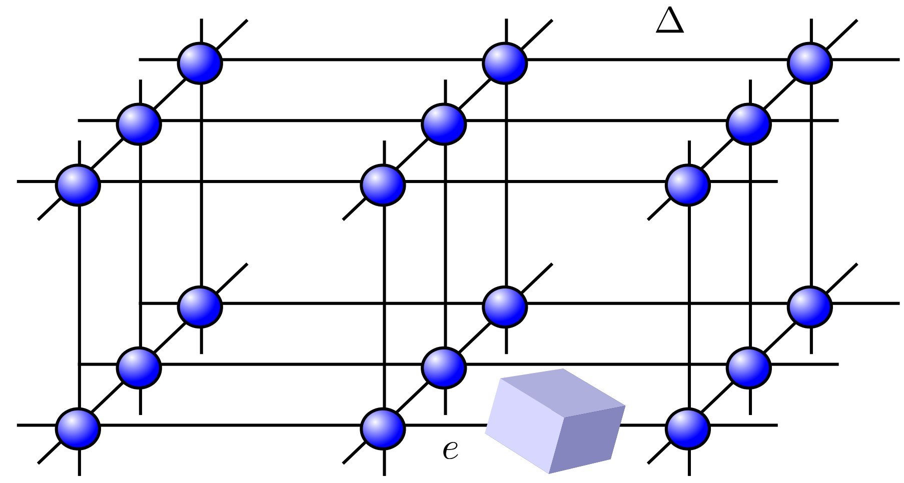
OSF toy model: (modified) EPRL-FK model truncated to hypercuboids

class function: $\hat{w}_{j^+,j^-} = \Big((2j^+ + 1)(2j^- + 1)\Big)^\alpha$

→ coupling constant α

2-complex Δ : dual to 4d hypercubic lattice

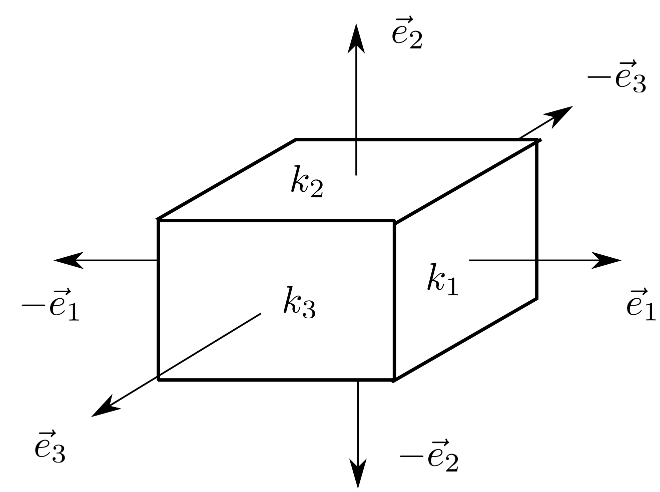


Operators: $P_e = \iota^\dagger_{k_1,k_2,k_3} Y_e^\dagger \otimes Y_e \iota_{k_1,k_2,k_3}$

$$\iota_{k_1,k_2,k_3} := \int_{SU(2)} dg\; g \triangleright \Big[|k_1,\, \vec{e}_1\rangle \otimes \dots \otimes \langle k_3,\, \vec{e}_3|\Big]$$

$k_i =$ irreps of $SU(2)$

$Y_e =$ EPRL "boosting map" $j_i^\pm = \frac{|1 \pm \gamma|}{2} k_i$



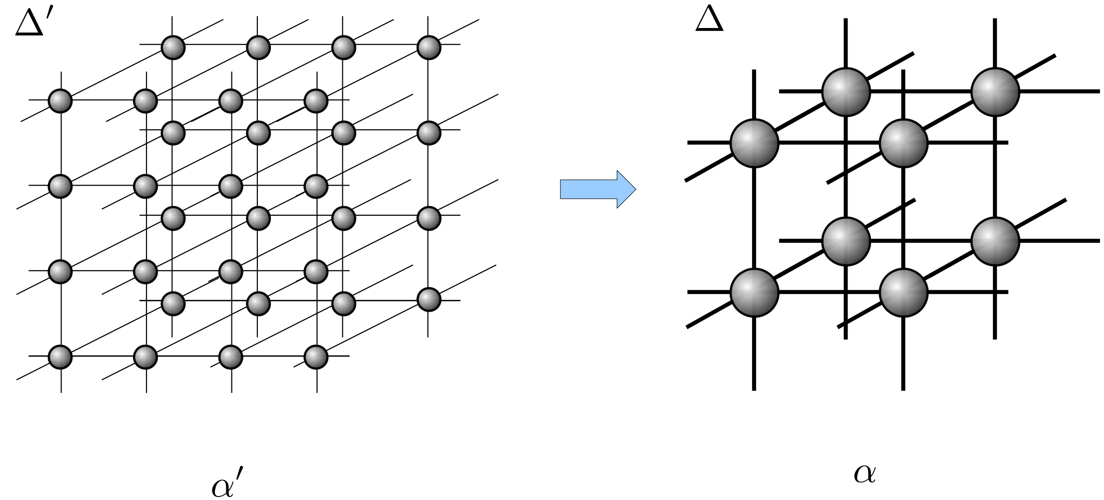
→ sum over spins and intertwiners truncated to sum over quantum cuboids

→ much simpler than EPRL-FK-KKL, but retains some interesting features

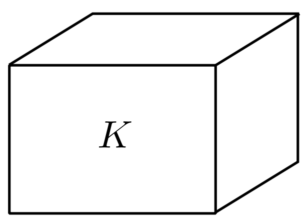
[Livine, Speziale '07, Bianchi, Dona, Speziale '10, BB, Steinhaus '15, BB, Steinhaus '16, BB, Steinhaus '17]

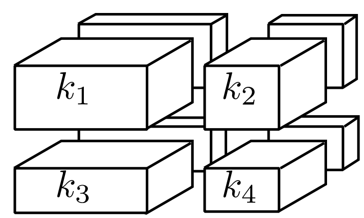
Coarse graining step:
2x2x2x2 → 1 hypercuboid

→ iterate



embedding map (not dynamical):

$\phi_{\Gamma'\Gamma}\Big($ 

$\Big) = \frac{1}{N}\sum_{k_i}$ 

$\delta\big(K - k_1 - k_2 - k_3 - k_4\big)$

EPRL-FK model amplitudes, large spin-asymptotic formula
→ α the only coupling constant in this case

III Toy model: RG fixed point

Isochoric RG flow: 32 → 2 vertices

boundary state $k_1, k_2, k_3 =$ const
4-volume $V_{\text{tot}} = V_1 + V_2 =$ const

flow: $\alpha' \longmapsto \alpha(\alpha')$

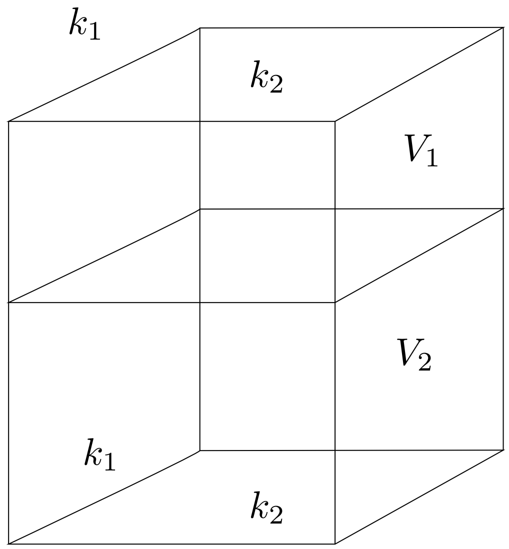
flow has a fixed point!

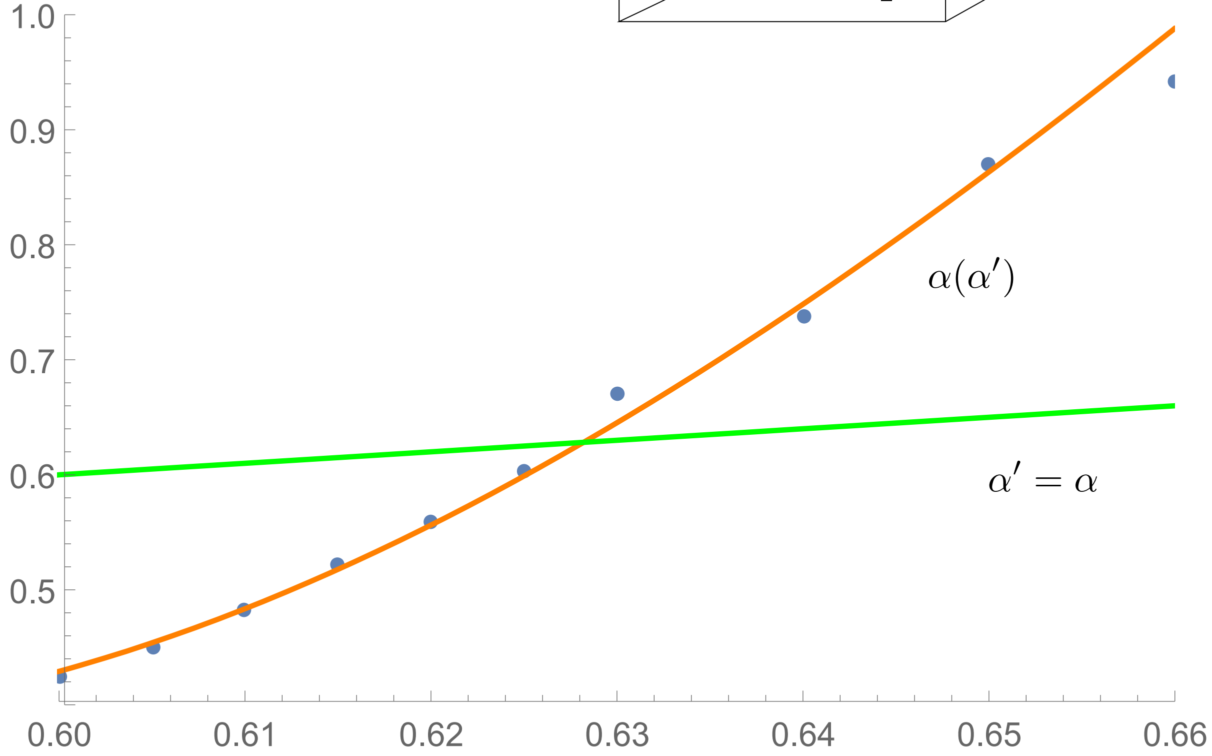
$\alpha^* \approx 0.63$

→ unstable (UV-attravtive)

→ splits phase diagram into two regions

→ beyond hypercuboids non-gaussian!





[BB, Steinhaus '17, BB, Rabuffo, Stainhaus '18]

III Toy model: fluctuations at NGFP

Finite size scaling: fluctuations $\Delta V^2_{\rm red}$ are similar for different lattice sizes N :

reduced coupling constant:

$$\overline{\alpha} \;:=\; \frac{\alpha - \alpha_0}{\alpha_0}$$

$$\alpha_0 \;\approx\; 0.59$$

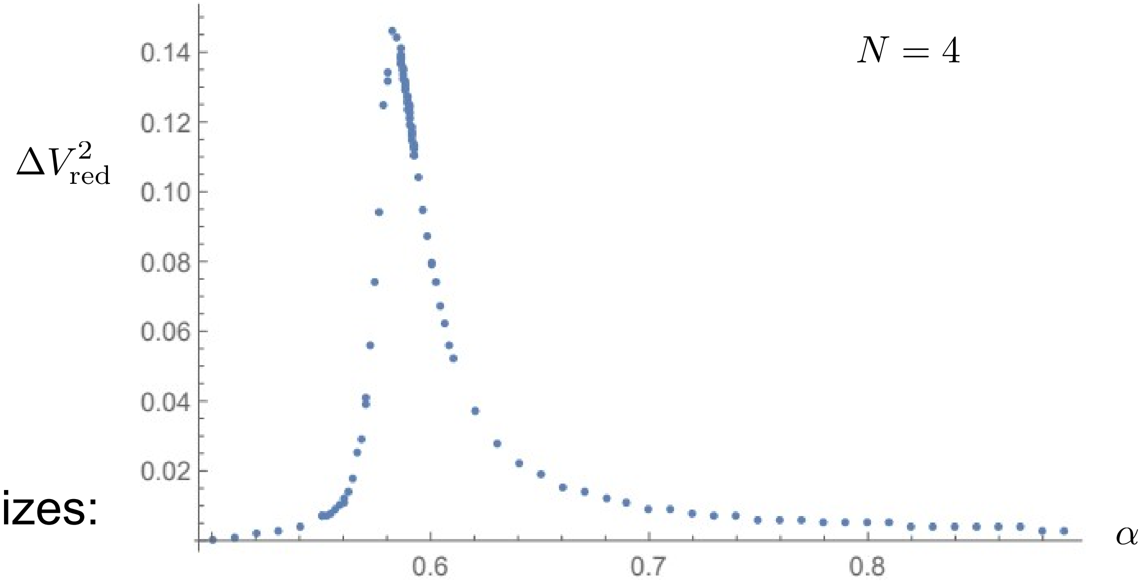
fluctuations for different lattice sizes:

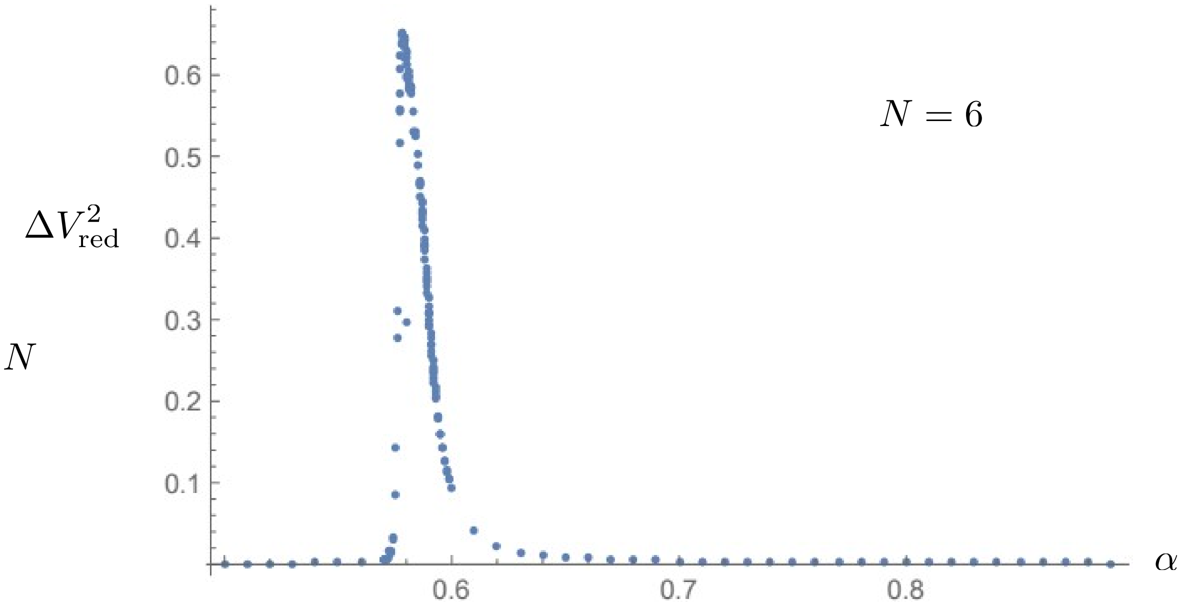
$$\Delta V^{(N)}_{\rm red}(\overline{\alpha}) \;=\; N^{\frac{\gamma}{\nu}}\, \Delta V^{(0)}_{\rm red}\left(N^{\frac{1}{\nu}}\overline{\alpha}\right)$$

read off critical exponents ν, γ
by collapsing data for different N

$$\nu = 0.33 \;-\; 0.40$$

$$\gamma = 1.35 \;-\; 1.50$$





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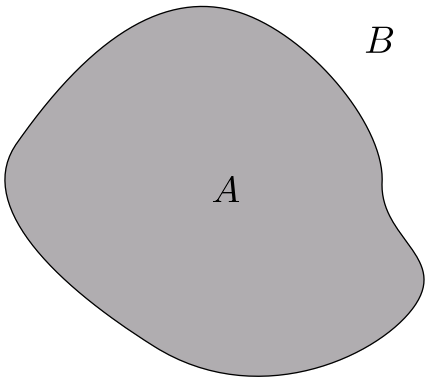
III Entanglement entropy

States → entanglement property between complementary regions A, B



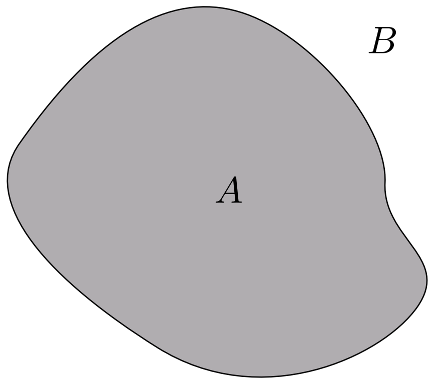
→ Measures entanglement of d.o.f. inside A with those inside B.

→ Generically scales with #d.o.f. in region (~volume law),

ground states: area law

→ Interesting quantity in LQG (BH entropy?).

[Rovelli ‘96, Donnelly ‘08, Rovelli, Vidotto ‘10, Engle, Noui, Perez, Pranzetti ‘11, Ghosh, Perez ‘11, Ghosh, Noui, Perez ‘13, Chirco, Rovelli, Ruggiero ‘14, Wang, Ma, Zhao ‘14, Han, Hung ‘16, Feller, Livine ‘17, Bianchi, Dona, Vilensky ‘18, Grüber, Sahlmann, Zilker ‘18, Bianchi, Dona ‘19, ...]



General framework:

Factorising Hilbert space: $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$

State: $\psi \in \mathcal{H}$ $\longrightarrow$ reduced density matrix $\hat{\rho}_A := \mathrm{tr}_B\left(\psi^\dagger \otimes \psi\right)$

$\longrightarrow$ entanglement entropy: $S_{\mathrm{EE}}(\psi) = -\mathrm{tr}_A\left(\hat{\rho}_A \ln \hat{\rho}_A\right)$

Non-factorising Hilbert space: $\mathcal{H} = \bigoplus_i \mathcal{H}_A^i \otimes \mathcal{H}_B^i$

State: $\psi = \sum_i q_i \, \psi_i$

$\longrightarrow$ entanglement entropy: $S_{\mathrm{EE}}(\psi) := \sum_i p_i S_{\mathrm{EE}}(\psi_i) - \sum_i p_i \ln p_i$ $\qquad p_i := |q_i|^2$

[e.g. Bianchi, Dona ‘19]

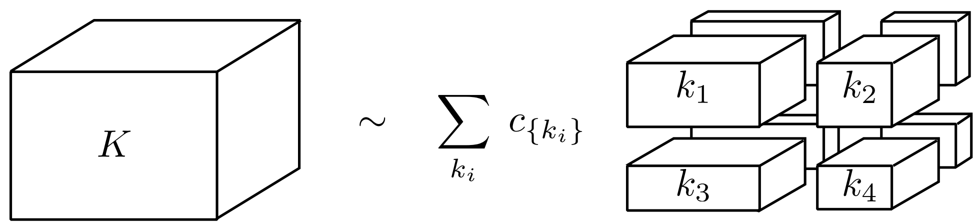
III Entanglement entropy of physical states in hypercuboidal OSFM

In the following:

entanglement entropy of physical states in hypercuboidal OSFM

In the physical Hilbert space $\mathcal{H}_{\text{phys}}$, due to cylindrical consistency, several kinematical states on different graphs are identified.

One quantum cuboid equivalent to several ones:

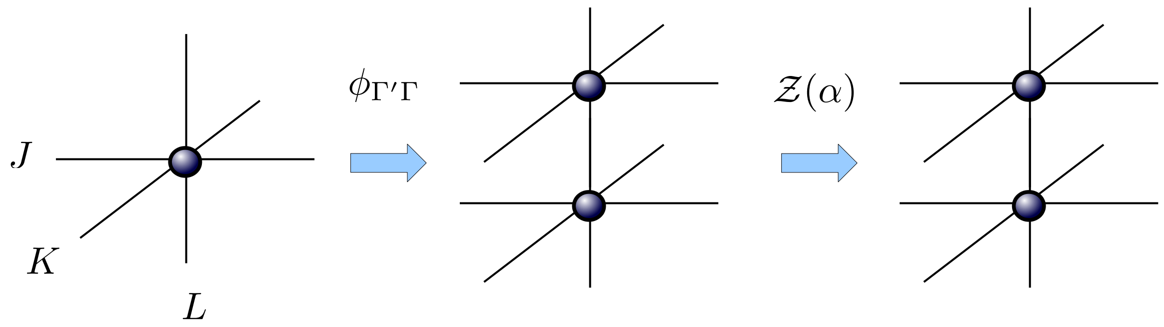


Coefficient is the physical inner product $\psi \sim \sum_{k_i} c_{\{k_i\}} \psi_{\{k_i\}}$ $\qquad c_{\{k_i\}} = \langle \psi | \psi_{\{k_i\}} \rangle_{\text{phys}}$

Embedding map + spin foam transition → “physical embedding map”

Simple case: subdivision of one quantum cuboid into two:

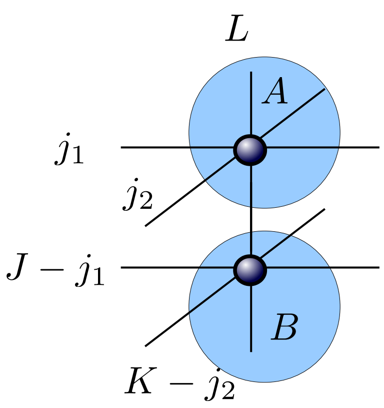
Coarse state: $\psi_{\text{in}} = \iota_{J,K,L}$



Fine state: $\psi_{\text{out}} = \sum_{j_i} c^{(\alpha)}_{j_i} \, \iota_{j_1,j_2,j_3} \otimes \iota_{j_4,j_5,j_3}$

(graphs toroidally compactified)

Bipartition of the system:



Coarse state: $\psi_{\text{in}} = \iota_{J,K,L}$

Hypercuboidal symmetries

$\longrightarrow \quad J = j_1 + j_3,\ K = j_2 + j_4,\ L = j_3$

Additional condition: Volume-simplicity constraint

$\longrightarrow \quad j_1(K - j_2) = j_2(J - j_1)$

(excludes non-metric degrees of freedom)

Isochoric transition: 4-volume V constant

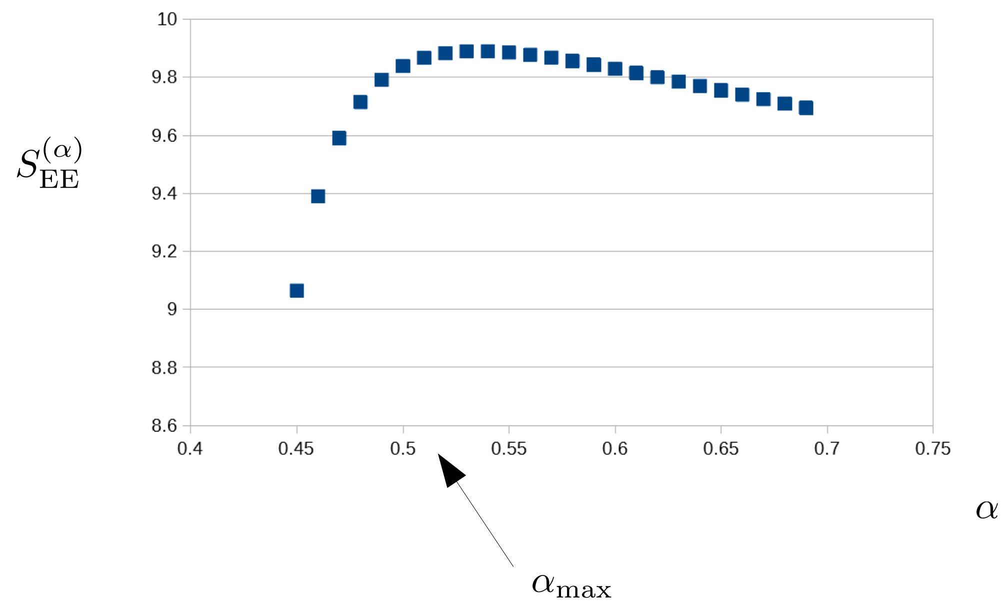
$$\begin{aligned}\psi_{\text{out}} &= \sum_{j_i} c_{j_i}^{(\alpha)}\, \iota_{j_1,j_2,j_3} \otimes \iota_{j_4,j_5,j_3} \\ &= \sum_{j_1} c_{j_1}^{(\alpha)} \iota_{j_1,j_1 K/J,L} \otimes \iota_{J-j_1,(J-j_1)K/J,L}\end{aligned}$$

[BB, Belov ‘17, Dona, Fanizza, Sarno, Speziale, 17]

Entanglement entropy: depends on α

$$S_{\mathrm{EE}}^{(\alpha)} = -\sum_j |c_j^{(\alpha)}|^2 \ln |c_j^{(\alpha)}|^2$$

$$c_j^{(\alpha)} = \prod_v \hat{\mathcal{A}}_v^{(\alpha)}$$



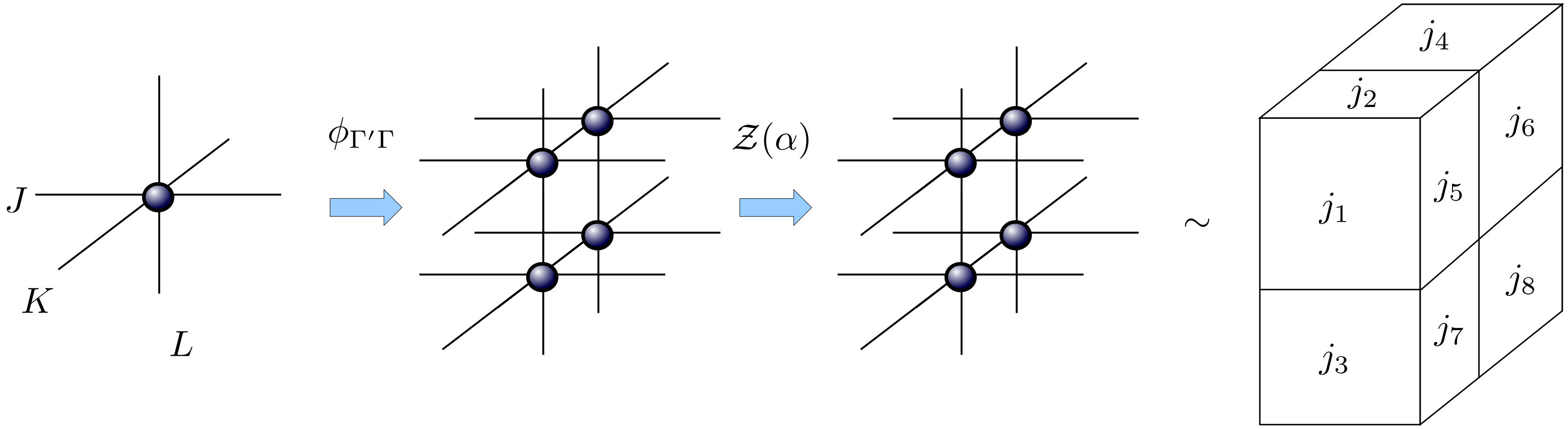
Dressed EPRL-FK amplitude: $\hat{\mathcal{A}}_v^{(\alpha)} = \mathcal{A}_v \left(\prod_e \mathcal{A}_e \right)^{\frac{1}{2}} \left(\prod_f \mathcal{A}_f \right)^{\frac{1}{4}}$

(face-, edge- and vertex amplitudes $\mathcal{A}_f, \mathcal{A}_e, \mathcal{A}_v \sim w, P$)

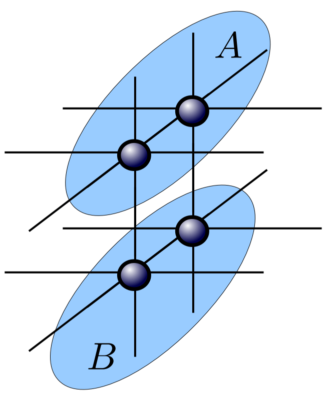
$\longrightarrow$ Maximum of entanglement entropy!

Example: $J = K = 2 \cdot 10^5, L = 10^5, V/\ell_P^4 \sim 10^{10}$ $\longrightarrow$ $\alpha_{\mathrm{max}} \approx 0.51$

More complicated case: subdivision into 4 quantum cuboids



Bipartition of system: $$\psi_{\text{out}} = \sum_{j_1,j_2} c^{(\alpha)}_{j_1,j_2}\, \iota_{j_1,j_2,j_5} \otimes \iota_{j_1,j_4,j_6} \otimes \iota_{j_2,j_3,j_7} \otimes \iota_{j_3,j_4,j_8}$$



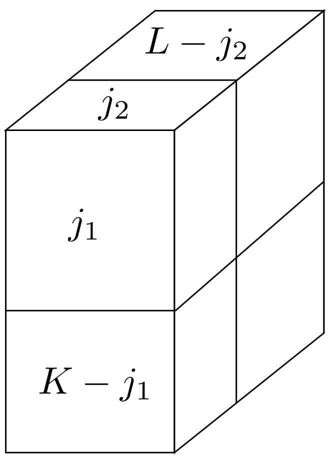
isochoric flow & geometricity: relations among spins
→ only j_1, j_2 remain as variables

$$S^{(\alpha)}_{\text{EE}} = -\sum_{j_1,j_2} |c_{j_1,j_2}|^2 \ln |c_{j_1,j_2}|^2$$

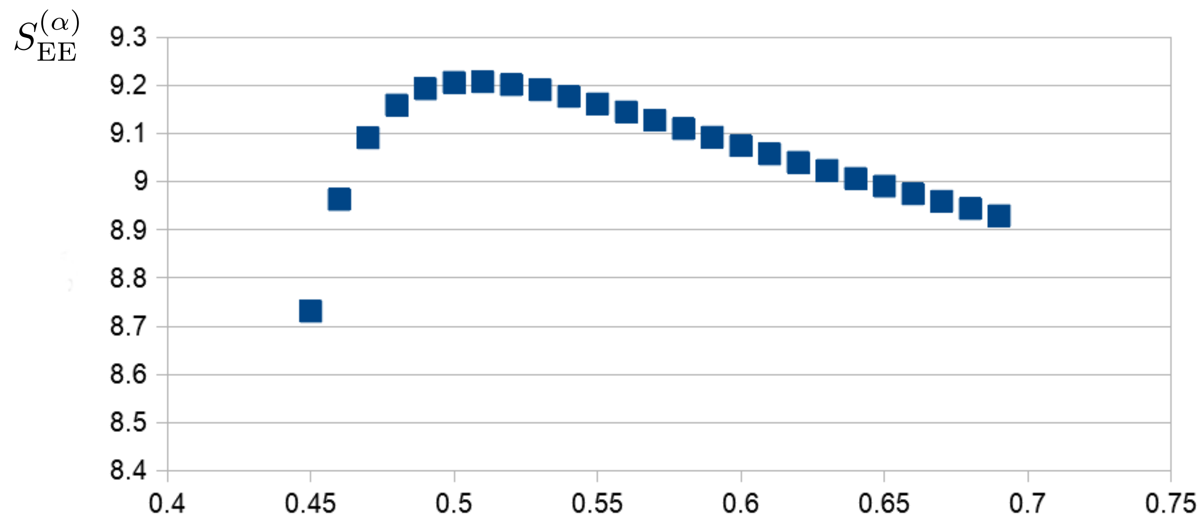
III Entanglement entropy of physical states in hypercuboidal OSFM

Isochoric model: fix 4-volume, & volume-simplicity constraints (~geometricity):

$$S_{\mathrm{EE}}^{(\alpha)} = \sum_{j_1,j_2} |c_{j_1,j_2}^{(\alpha)}|^2 \ln |c_{j_1,j_2}^{(\alpha)}|^2$$



α



$J = K = 2 \cdot 10^5, L = 10^5, V/\ell_P^4 \sim 10^{10}$ α

Again, EE maximal for specific value of coupling constant: RG fixed point!

→ RG fixed point characterised by maximum of entanglement entropy of physical states!

→ Effect expected to be more pronounced on larger lattices

III Entanglement entropy of physical states: interpretation

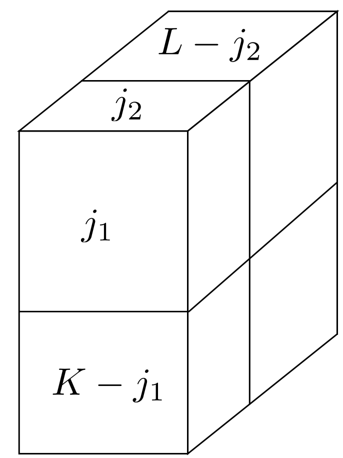
Vertex translation symmetry:

→ Change of spins according to deformation of flat polytopes.

↔ remnant of diffeomorphism symmetry in Regge calculus

All final kinematic states for different j_1, j_2 can be related by Vertex translations (different subdivisions of the same cuboid)

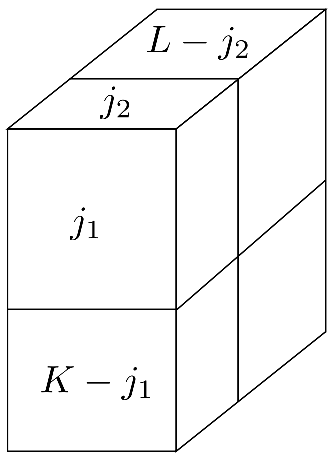
Summation ↔ gauge orbit of vertex translation symmetry

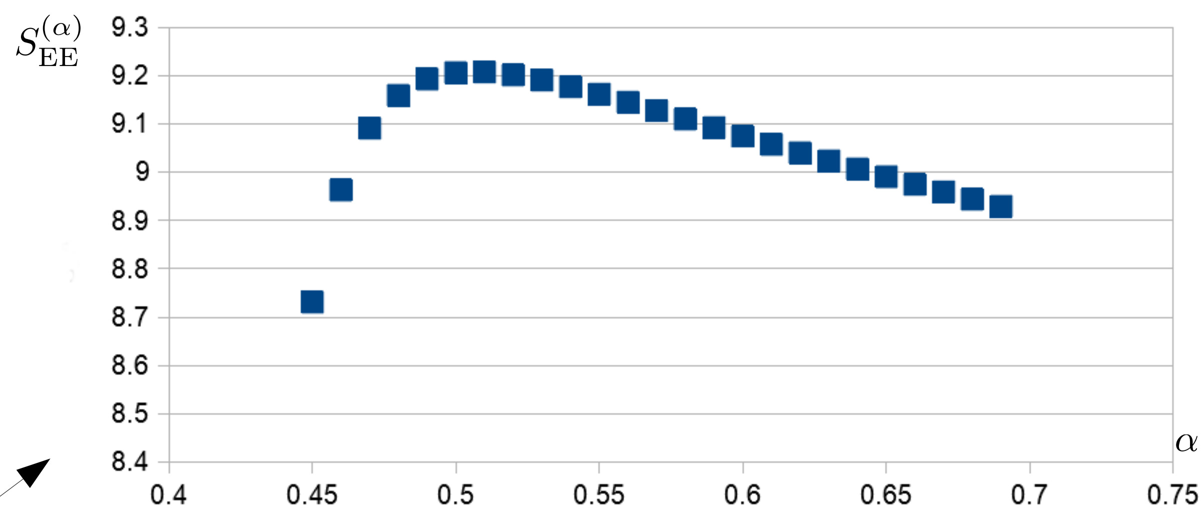


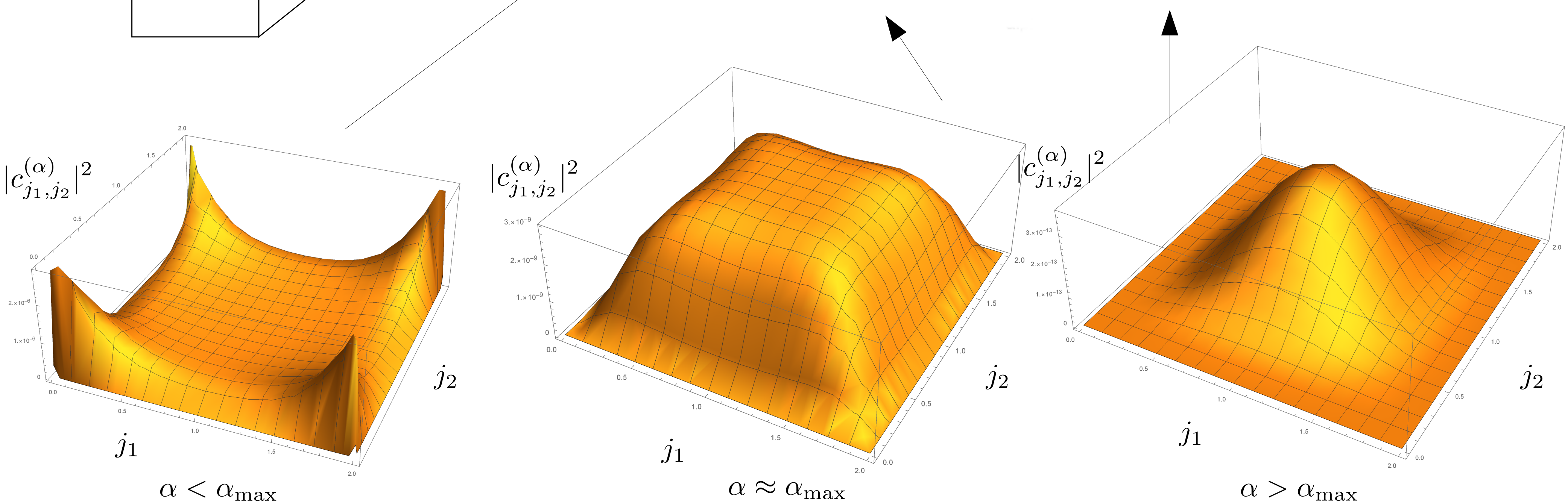
[Regge '61, Freidel, Louapre '03, Dittrich '08, BB, Dittrich '08-09', BB, Steinhaus '15, ...]

III Entanglement entropy of physical states: interpretation

Interpretation:







→ Diffeomorphically equivalent d.o.f. are getting <u>entangled</u> at RG fixed point!

I Motivation

II Operator Spin Foam Models
   a. Definition
   b. Coarse graining

III Toy model: hypercuboidal OSFM
   a. RG flow & fixed point
   b. Entanglement entropy

IV Summary

IV Summary

Review of operator spin foam models:

- Class of models to construct transitions between (spin) network states
- → KKL-extension of EPRL-FK model is an example
- → suitable for coarse graining: cylindrical consistency ↔ RG flow of model

Example: hypercuboidal OSF → toy model for EPRL-FK model

- Large spin → only one coupling constant α
  → related to face amplitude

$$\hat{w}_{j^+,j^-} = \Big((2j^+ + 1)(2j^- + 1)\Big)^\alpha$$

- Flow in α : UV fixed point.
- → at FP: restoration of broken diffeo-symmetry in SFM
- Feature of FP: Entanglement Entropy increases: diffeo-d.o.f. become entangled

  → Feature chances to remain in the full EPRL-FK model
  → Sign of restoration of broken diffeo symmetry at FP
  → Neat new method to identify interesting points in parameter space

Happy birthday Jurek!