GRAPH COHERENT STATES IN LOOP QUANTUM GRAVITY

Mehdi Assanioussi

II. Institute for Th. Physics, University of Hamburg

JurekFest, September 2019
Problems to solve

I. Dynamics in LQG
II. Coherent states & semi-classical limit in LQG
III. Matter fields coupling in LQG
IV. LQG cosmology
V. Black holes in LQG
VI. ...
Warsaw LQG Commando

On the move...
KEEP CALM AND ABANDON SHIP
... We are not done yet!
Plan of the talk

I. Motivation

II. Graph coherent states
   i. Construction concept
   ii. Example 1: closed loops in the U(1) gauge theory
   iii. Example 2: closed loops in non-Abelian gauge theories

III. Concluding comments
I. Motivation
Graphs in LQG

Graph change?
Graph superposition

Graph change?

Graph superposition

Physical solution?

Semi-classical limit?

Continuum limit?
Coherent states in LQG

**Intertwiners**
- e.g.: Livine–Speziale coherent intertwiners

**Spins**
- e.g.: complexifier (Area) coherent states

Coherent states

Graphs

***
II. Graph coherent states

1. Construction concept
Construction concept

1. Define a graph change on a colored graph:

\[ |n\rangle_\omega \equiv \]

2. Construct the structure of a harmonic oscillator space:

\[ \text{New coherent states (graph coherent states);} \]
Construction concept

Key observation:
A (generic) graph change (with a “finite structure”) provides a decomposition of the $\mathcal{H}_{\text{Diff}}$ into separable subspaces $\mathcal{H}_{\Gamma^A}$, which are stable under the action of the operators inducing such graph change.

$\Gamma^A$ – ancestor graphs: colored graphs with no graph “excitation”

\[
\mathcal{H}^{\Gamma^A} \cong \bigotimes_{s \in \Gamma^A} \mathcal{H}^{\Gamma^A}_s
\]

\[
\mathcal{H}^{\Gamma^A}_s \cong \bigotimes_{i=1}^{w_s} L^2_i(\mathbb{R})
\]
II. Graph coherent states

ii. Example 1: closed loops in the U(1) gauge theory
Example 1: Closed loops in the U(1) gauge theory

Closed loop graph excitation

\[ \text{Tr}[h_{\alpha \omega}] \]

\[ |n\rangle_\omega \equiv \]

\[ \mathcal{L}_\omega^2(\mathbb{R}) = \overline{\text{Span}\{|n\rangle_\omega, n \in \mathbb{N}\}} \]
Example 1: closed loops in the U(1) gauge theory

Canonical structure:

- **Annihilation & creation operators**
  \[ \forall \omega_i \in \mathcal{W}_v, \quad a_i |0_i\rangle = 0, \quad a_i |n_i\rangle = \sqrt{n_i} |n_i - 1\rangle, \quad a_i^\dagger |n_i\rangle = \sqrt{n_i + 1} |n_i + 1\rangle \]

- **Wedge coherent states**
  \[ \forall \omega_i \in \mathcal{W}_v, \quad [a_i, a_j] = [a_i^\dagger, a_j^\dagger] = 0, \quad [a_i, a_j^\dagger] = \delta_{ij} I \]

- **Graph coherent states**
  \[ |Z_v\rangle := \bigotimes_{i=1}^{w_v} |z_i\rangle, \quad Z_v := \{z_i\} \in \mathbb{C}^{w_v} \]
Example 1: closed loops in the U(1) gauge theory

Operators correspondence:

① Closed loop holonomies

\[ \text{Tr}[h^{(l)}_{\alpha, \omega}] = a^\dagger_{\omega} \mathcal{V}_\omega \]
\[ \text{Tr}[h^{(l)\dagger}_{\alpha, \omega}] = : \mathcal{V}_\omega a_{\omega} \]

\[ \mathcal{V}_{K\bar{L}} := (\mathcal{N}_{K\bar{L}} + I)^{-1/2} = (a_{K\bar{L}} a^\dagger_{K\bar{L}})^{-1/2} \]

② Fluxes

\[ X_{\bar{I}} := \sum_{I \in \bar{I}} X_I = \sum_{I_0 \in \bar{I}} j_{\bar{I}} I + l \sum_{\bar{Y}} \mathcal{N}_{\bar{X}_{\bar{Y}}} - \mathcal{N}_{\bar{Y}_{\bar{X}}} \]
Graph coherent states I: Maxwell Field

Coherence properties:

① Expectation values:
\[
\langle h_{\alpha \omega} \rangle = \bar{z} \sum_{n} \frac{|z|^{2n}}{n! \sqrt{n+1}} \\
\langle (h_{\alpha \omega})^{\dagger} \rangle = z \sum_{n} \frac{|z|^{2n}}{n! \sqrt{n+1}}
\]

② Relative variances:
\[
\Delta_r(h_{\alpha \omega}) \xrightarrow{|z|\gg 1} 0 \quad , \quad \Delta_r((h_{\alpha \omega})^{\dagger}) \xrightarrow{|z|\gg 1} 0
\]

\[
\Delta_r(X[I]) \xrightarrow{|z|\gg 1} 0
\]
Example 1: closed loops in the U(1) gauge theory

Example of an observable:

+ Maxwell field Hamiltonian:

\[ H_{YM}(N) := \int_{\Sigma} N \frac{q_{ab}}{2g^2 \sqrt{q}} (E^a_i E^b_i + B^a_i B^b_i) \]

\[ \rightarrow \frac{\sqrt{q}}{\sqrt{q}} \left( j_{IJ} j_{I\Pi} + Q_{M}(v) \sum_{\omega, \omega'} (a^\dagger_\omega \mathcal{V}_{\omega} + \mathcal{V}_{\omega} a^\dagger_\omega - 2\Pi)(a^\dagger_{\omega'}, \mathcal{V}_{\omega'} + \mathcal{V}_{\omega'} a_{\omega'} - 2\Pi) \right) \]

"The action of the Hamiltonian at a vertex is reformulated as an action in a space of a finite number of harmonic oscillators"
II. Graph coherent states

11. Example 2: closed loops in non-Abelian gauge theories
Example 2: closed loops in non-Abelian gauge theories

Hilbert space structure:

\[ \mathcal{H}_v^{\Gamma A} := \bigoplus_{n=0}^{\infty} \mathcal{H}_{v,n}^{\Gamma A} \]

\[
\begin{align*}
\includegraphics[width=0.3\textwidth]{example2}
\end{align*}
\]
Example 2: closed loops in non-Abelian gauge theories

Canonical structure:

1. Generalized annihilation & creation operators

\[ \forall \omega_i, \omega_j \in \mathcal{W}_v, \quad [a_i, a_j] = [a_i^\dagger, a_j^\dagger] = 0, \quad [a_i, a_j^\dagger] = \delta_{ij} \mathbb{I} \]

\[ \forall i \in \mathcal{W}_v, \quad a_i(\mathcal{H}^{\Gamma A}_{v,0}) = \{0\}, \quad \forall n \geq 1, \quad a_i(\mathcal{H}^{\Gamma A}_{v,n}) = \mathcal{H}^{\Gamma A}_{v,n-1} \]

\[ |\nu_{\{n_i\}}^\alpha\rangle \]
Example 2: closed loops in non-Abelian gauge theories

Canonical structure:

Generalized annihilation & creation operators

Imposing \( \forall \omega_i, \omega_j \in \mathcal{W}_v, \quad [a_i, a_j] = [a_i^\dagger, a_j^\dagger] = 0 \), \( [a_i, a_j^\dagger] = \delta_{ij} \mathbb{I} \)

\[ \forall i \in \mathcal{W}_v, \quad a_i(\mathcal{H}_{v,0}^\Gamma) = \{0\} , \quad \forall n \geq 1, \quad a_i(\mathcal{H}_{v,n}^\Gamma) = \mathcal{H}_{v,n-1}^\Gamma \]

\[ |\ell_{\{n_i\}}^\alpha\rangle \]

\[ \forall \omega_j \in \mathcal{W}_v, \quad a_j |\ell_{\{n_i\};\{m_i\}}^\alpha\rangle = \sqrt{n_j - m_j} |\ell_{\{n_1,\ldots,n_j-1,\ldots,n_{w_v}\};\{m_i\}}^\beta\rangle \]

\[ a_j^\dagger |\ell_{\{n_i\};\{m_i\}}^\alpha\rangle = \sqrt{n_j - m_j + 1} |\ell_{\{n_1,\ldots,n_j+1,\ldots,n_{w_v}\};\{m_i\}}^\gamma\rangle \]
Example 2: closed loops in non-Abelian gauge theories

**Canonical structure:**

**Abelian case**

|2\rangle

|1\rangle

|0\rangle

**Non-Abelian case**

|\ell^1_{\{2_1,0_j\};\{0_i\}}\rangle

|\ell^1_{\{1_1,0_j\};\{0_i\}}\rangle

|0^1_{\{0_i\};\{0_i\}}\rangle

|0^2_{\{2_1,0_i\};\{2_1,0_i\}}\rangle

|0^2_{\{1_1,0_i\};\{1_1,0_i\}}\rangle

|0^1_{\{0_i\};\{0_i\}}\rangle
Example 2: closed loops in non-Abelian gauge theories

**Choice of canonical structure:**

- **Non-uniqueness of vacuum states**
- **Non-uniqueness of the mapping between G-tensors**

**Graph coherent states:**

\[ |Z_v\rangle := \prod_{i=1}^{w_v} e^{z_i a_i^\dagger - \bar{z}_i a_i} |0^\alpha_v,\{m_i\}\rangle, \quad |0^\alpha_v,\{m_i\}\rangle \in \mathcal{K}_v(\{a_i\}) \]

\[ \forall i \in \mathcal{W}_v, \quad a_i |Z_v\rangle = z_i |Z_v\rangle, \quad Z_v = \{z_i\} \]
Example 2: Closed loops in non-Abelian gauge theories

Example of canonical structure:

\[ \mathcal{V}_i a_i := \text{Tr}_N^{(l)} [\tau^k h_{\alpha_i}]^\dagger, \]  
OR  
\[ \mathcal{V}_i a_i := \text{Tr}_N^{(l)} [h_{\alpha_i}]^\dagger, \ldots \]

\[ \mathcal{V}_i := (a_i a_i^\dagger)^{-1/2} \]

Example:

\[ \langle \mathcal{V}_i a_i \rangle = z_i \sum_n \frac{|z_i|^{2n}}{n! \sqrt{n + 1}} \]

\[ \langle (a_i^\dagger \mathcal{V}_i)^\dagger \rangle = \bar{z}_i \sum_n \frac{|z_i|^{2n}}{n! \sqrt{n + 1}} \]

Relative variances:

\[ \Delta_r(\mathcal{V}_i a_i) \xrightarrow{|z| \gg 1} 0, \quad \Delta_r(a_i^\dagger \mathcal{V}_i) \xrightarrow{|z| \gg 1} 0 \]
III. Concluding comments
Concluding comments

Comments on the construction:

- The graph change

  \[ \mathcal{H}^{\Gamma^A} \cong \bigotimes_{s \in \Gamma^A} \mathcal{H}^s \]

  \[ \mathcal{H}^s \cong \bigotimes_{i=1}^{w_s} \mathcal{L}_i^2 \]

  **Various valid graph changes, even “non-local” ones**

- Freedom in choice of vacuum states & tensor mappings

  **Adapting the choice of CS to the choice of observables**

- Canonical structure with fluxes

  \[ V_i a_i := \text{Tr}_N^{(l)} \left[ X_{I,k} \tau^k h_{\alpha i} \right]^\dagger \]
**Concluding comments**

**Summary:**
- ✔ New coherent states which exhibit graph coherence;
- ✔ The construction method applies to various graph excitations;
- ✔ Flexibility of the mapping on the intertwiner space;

**Outlook:**
- Explore the dynamics of such states;
- Explore the possible choices of the intertwiners mapping;
- Explore different graph excitations for the purposes of investigating semi-classical and continuum limits
Thank you & Happy birthday!!!