Graph Coherent States in Loop Quantum Gravity

Mehdi Assanioussi

II. Institute for Th. Physics, University of Hamburg

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Problems to solve

I. Dynamics in LQG
II. Coherent states & semi-classical limit in LQG
III. Matter fields coupling in LQG
IV. LQG cosmology
V. Black holes in LQG
VI. …

Warsaw LQG Commando

On the move...

KEEP
CALM
AND
ABANDON
SHIP

… We are not done yet!

Plan of the talk

I. Motivation

II. Graph coherent states

  i. Construction concept

  ii. Example 1: closed loops in the U(1) gauge theory

  iii. Example 2: closed loops in non-Abelian gauge ths.

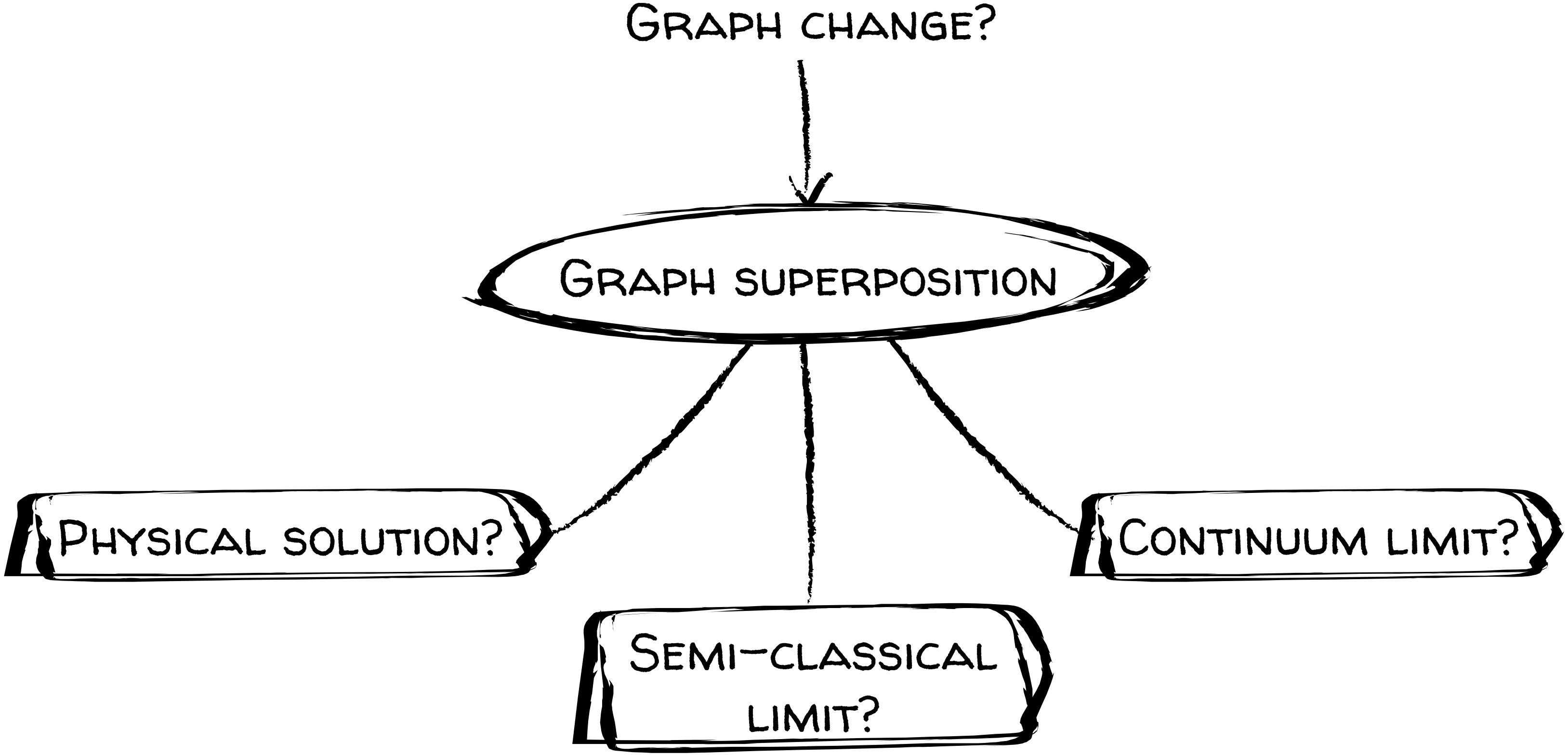
III. Concluding comments

I. Motivation

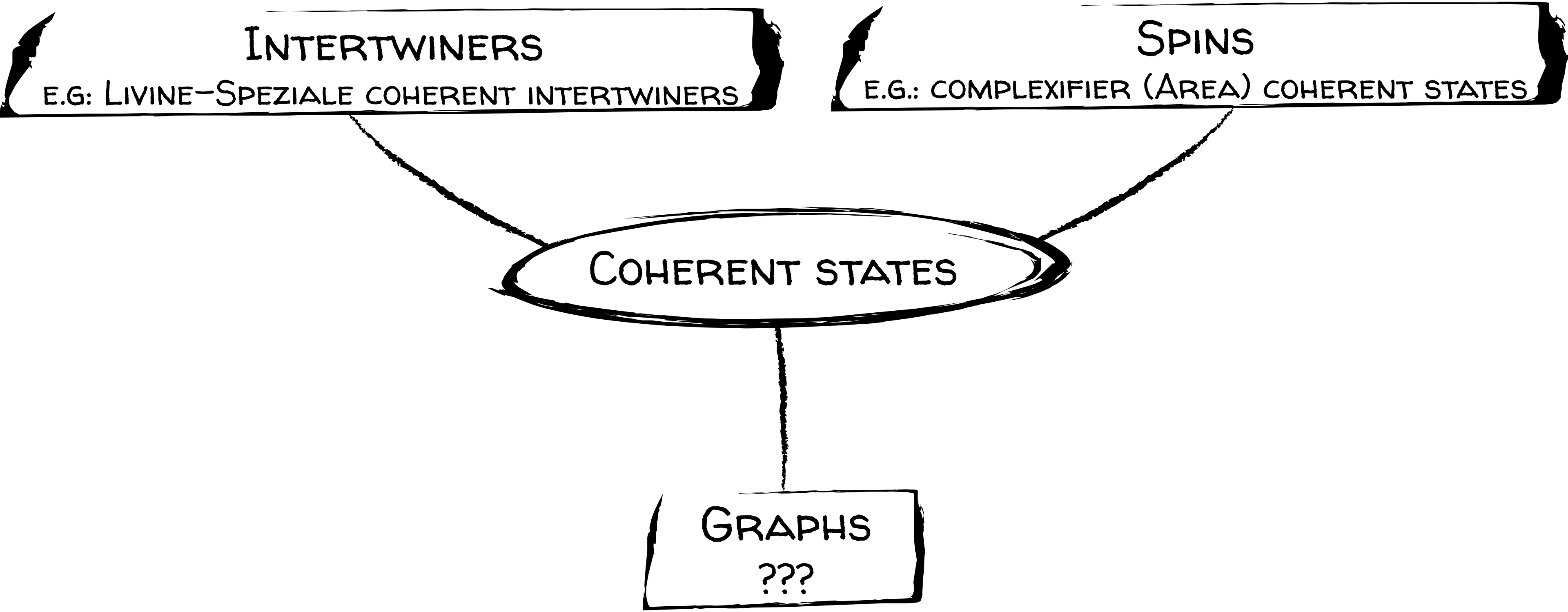
Graphs in LQG

Graph change?

Graphs in LQG



Coherent states in LQG



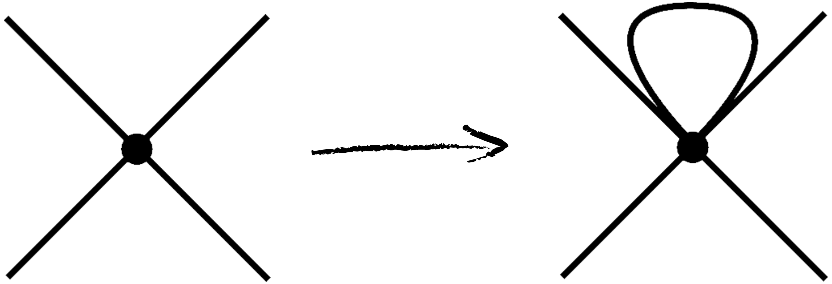
II. Graph coherent states

1. Construction concept

Construction concept

① Define a graph change on a colored graph:

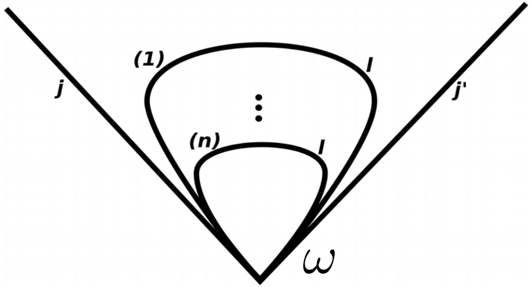
exp.:



② Construct the structure of a harmonic oscillator space:

exp.:

$$|n\rangle_\omega \equiv$$



⟶ New coherent states (graph coherent states);

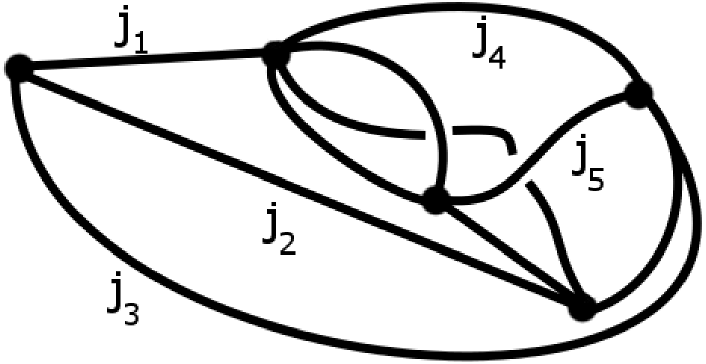
Construction concept

⊕ Key observation:
A (generic) graph change (with a "finite structure") provides a decomposition of the $\mathcal{H}_{\mathrm{Diff}}$ into separable subspaces $\mathcal{H}^{\Gamma^A}$, which are stable under the action of the operators inducing such graph change.

Γ^A– ancestor graphs: colored graphs with no graph "excitation"

$$\mathcal{H}^{\Gamma^{\mathcal{A}}} \cong \bigotimes_{s\in\Gamma^{\mathcal{A}}} \mathcal{H}_s^{\Gamma^{\mathcal{A}}}$$

$$\mathcal{H}_s^{\Gamma^{\mathcal{A}}} \cong \bigotimes_{i=1}^{w_s} \mathcal{L}_i^2(\mathbb{R})$$



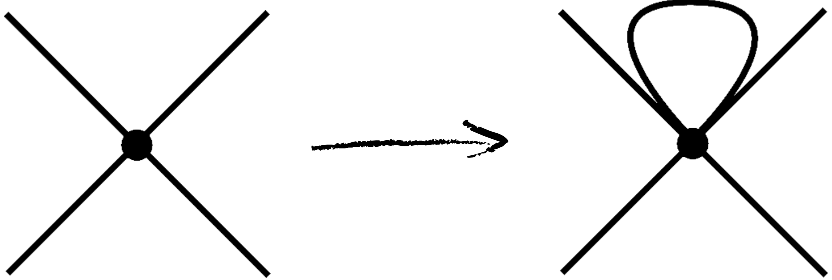
II. Graph coherent states

ii. Example 1: closed loops in the U(1) gauge theory

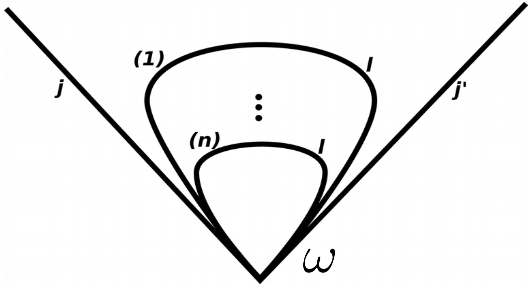
Example 1: closed loops in the U(1) gauge theory

Closed loop graph excitation

$\mathrm{Tr}[h_{\alpha_\omega}]$



$|n\rangle_\omega \equiv$



$$\mathcal{L}^2_\omega(\mathbb{R}) = \overline{\mathrm{Span}\{|n\rangle_\omega, n \in \mathbb{N}\}}$$

Example 1: closed loops in the U(1) gauge theory

Canonical structure:

⊕ Annihilation & creation operators

$$\forall\, \omega_i \in \mathcal{W}_v \;\; a_i|0_i\rangle = 0\,,\;\; a_i|n_i\rangle = \sqrt{n_i}|n_i - 1\rangle \quad , \quad a_i^\dagger|n_i\rangle = \sqrt{n_i + 1}|n_i + 1\rangle$$

$$\forall\, \omega_i, \omega_j \in \mathcal{W}_v, \quad [a_i, a_j] = [a_i^\dagger, a_j^\dagger] = 0 \quad , \quad [a_i, a_j^\dagger] = \delta_{ij}\mathbb{I}$$

⊕ Wedge coherent states

$$\forall\, \omega_i \in \mathcal{W}_v, \quad a_i|z_i\rangle = z_i|z_i\rangle \quad , \quad |z_i\rangle = e^{z_i a_i^\dagger - \bar{z}_i a_i}|0_i\rangle = \sum_{n_i} \frac{z_i^{n_i}}{\sqrt{n_i!}}|n_i\rangle$$

⊕ Graph coherent states

$$|Z_v\rangle := \bigotimes_{i=1}^{w_v} |z_i\rangle \quad , \quad Z_v := \{z_i\} \in \mathbb{C}^{w_v}$$

Example 1: closed loops in the U(1) gauge theory

Operators correspondence:

- Closed loop holonomies

$$\mathrm{Tr}[h^{(l)}_{\alpha_\omega}] =: a^\dagger_\omega \mathcal{V}_\omega \qquad \mathrm{Tr}[h^{(l)}_{\alpha_\omega}]^\dagger =: \mathcal{V}_\omega a_\omega$$

$$\mathcal{V}_{\tilde{K}\tilde{L}} := (\mathcal{N}_{\tilde{K}\tilde{L}} + \mathbb{I})^{-1/2} = (a_{\tilde{K}\tilde{L}} a^\dagger_{\tilde{K}\tilde{L}})^{-1/2}$$

- fluxes

$$X_{\tilde{I}} := \sum_{I \in \tilde{I}} X_I = \sum_{I_0 \in \tilde{I}} j_{\tilde{I}} \mathbb{I} + l \sum_{\tilde{Y}} \mathcal{N}_{\tilde{I}\tilde{Y}} - \mathcal{N}_{\tilde{Y}\tilde{I}}$$

Graph coherent states I: Maxwell Field

Coherence properties:

- Expectation values:

$$\langle h_{\alpha_\omega} \rangle = \bar{z} \sum_n \frac{|z|^{2n}}{n!\sqrt{n+1}} \qquad \langle \, (h_{\alpha_\omega})^\dagger \, \rangle = z \sum_n \frac{|z|^{2n}}{n!\sqrt{n+1}}$$

- Relative variances:

$$\Delta_r(h_{\alpha_\omega}) \xrightarrow[|z|\gg 1]{} 0 \quad , \quad \Delta_r((h_{\alpha_\omega})^\dagger) \xrightarrow[|z|\gg 1]{} 0$$

$$\boxed{\Delta_r(X_{[I]}) \xrightarrow[|z|\gg 1]{} 0}$$

Example 1: closed loops in the U(1) gauge theory

Example of an observable:

⊕ Maxwell field Hamiltonian:

$$H_{YM}(N) := \int_\Sigma N \frac{q_{ab}}{2g^2\sqrt{q}} \left(E_i^a E_i^b + B_i^a B_i^b\right)$$

$$\longrightarrow \sim \frac{\widehat{q_{IJ}}}{\sqrt{q}} \left(j_I j_J \mathbb{I} + Q_{\mathcal{M}}(v) \sum_{\omega,\omega'} (a_\omega^\dagger \mathcal{V}_\omega + \mathcal{V}_\omega a_\omega - 2\mathbb{I})(a_{\omega'}^\dagger \mathcal{V}_{\omega'} + \mathcal{V}_{\omega'} a_{\omega'} - 2\mathbb{I}) \right)$$

"The action of the Hamiltonian at a vertex is reformulated as an action in a space of a finite number of Harmonic oscillators"

II. Graph coherent states

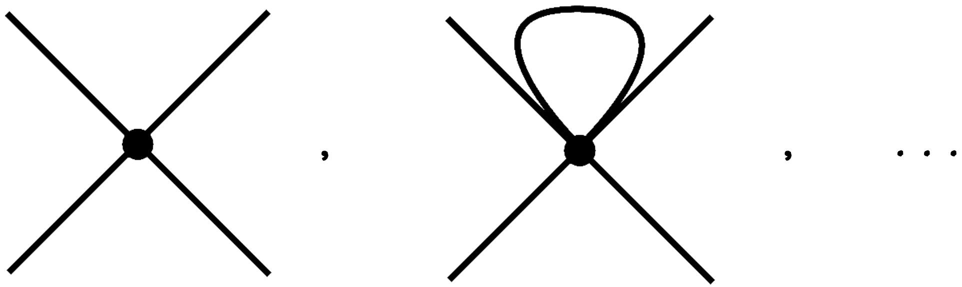
ii. Example 2: closed loops in non-Abelian gauge theories

Example 2: closed loops in non-Abelian gauge theories

Hilbert space structure:

⊕ Decomposition of the Hilbert space:

$$\mathcal{H}_v^{\Gamma^{\mathcal{A}}} := \bigoplus_{n=0}^{\infty} \mathcal{H}_{v,n}^{\Gamma^{\mathcal{A}}}$$



Example 2: closed loops in non-Abelian gauge theories

Canonical structure:

(+) Generalized annihilation & creation operators

Imposing $\forall\, \omega_i, \omega_j \in \mathcal{W}_v, \quad [a_i, a_j] = [a_i^\dagger, a_j^\dagger] = 0 \quad , \quad [a_i, a_j^\dagger] = \delta_{ij}\mathbb{I}$

$\longrightarrow \forall i \in \mathcal{W}_v, \quad a_i(\mathcal{H}_{v,0}^{\Gamma^\mathcal{A}}) = \{0\} \qquad , \qquad \forall n \geq 1, \quad a_i(\mathcal{H}_{v,n}^{\Gamma^\mathcal{A}}) = \mathcal{H}_{v,n-1}^{\Gamma^\mathcal{A}}$

$|\iota^\alpha_{\{n_i\}}\rangle \rightarrow \mathcal{H}_{v,n}^{\Gamma^\mathcal{A}}$

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$|\iota^{\alpha}_{\{n_i\}}\rangle \rightarrow \mathcal{H}_{v,n}^{\Gamma^{\mathcal{A}}}$

$$\longrightarrow \forall \omega_j \in \mathcal{W}_v,\ a_j|\iota^{\alpha}_{\{n_i\};\{m_i\}}\rangle = \sqrt{n_j - m_j}\ |\iota^{\beta}_{\{n_1,\dots,n_j-1,\dots,n_{w_v}\};\{m_i\}}\rangle$$

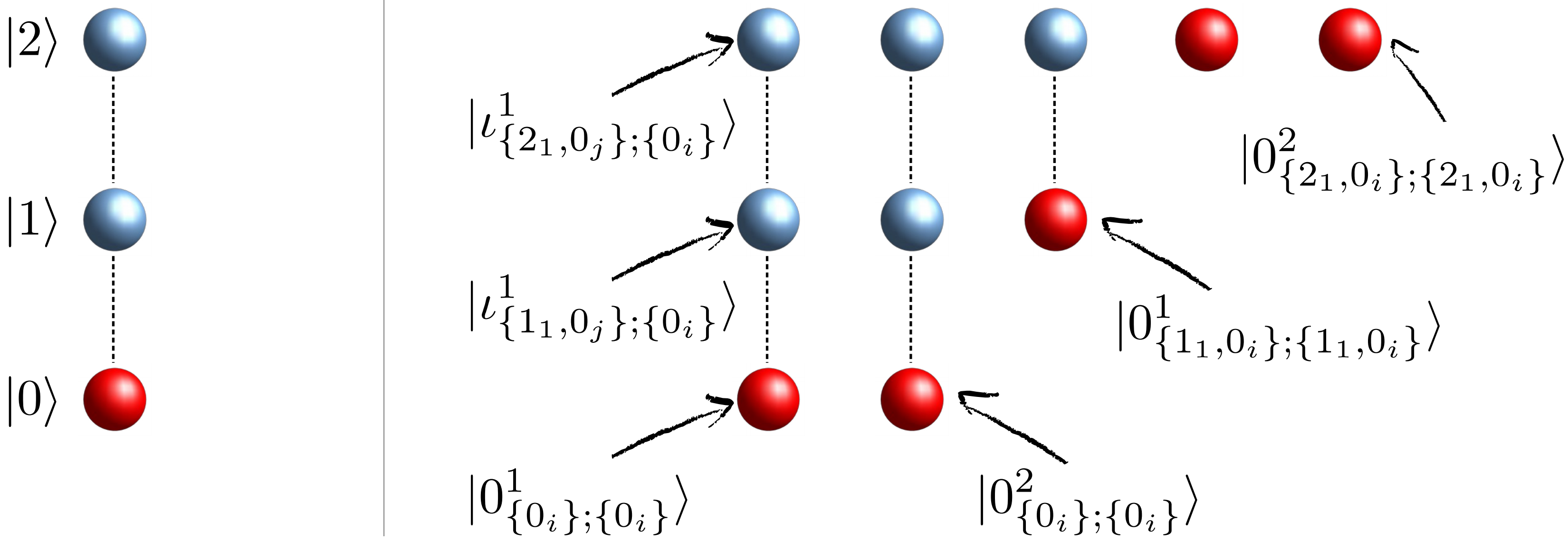
$$a_j^\dagger|\iota^{\alpha}_{\{n_i\};\{m_i\}}\rangle = \sqrt{n_j - m_j + 1}\ |\iota^{\gamma}_{\{n_1,\dots,n_j+1,\dots,n_{w_v}\};\{m_i\}}\rangle$$

Example 2: closed loops in non-Abelian gauge theories

Canonical structure:

Abelian case

Non-Abelian case



Example 2: closed loops in non-Abelian gauge theories

Choice of canonical structure:

- (+) Non-uniqueness of vacuum states
- (+) Non-uniqueness of the mapping between G-tensors

Graph coherent states:

$$|Z_v\rangle := \prod_{i=1}^{w_v} e^{z_i a_i^\dagger - \bar{z}_i a_i} |0^\alpha_{v,\{m_i\}}\rangle \;,\; |0^\alpha_{v,\{m_i\}}\rangle \in \mathcal{K}_v(\{a_i\})$$

$$\forall i \in \mathcal{W}_v, \quad a_i|Z_v\rangle = z_i|Z_v\rangle \quad , \quad Z_v = \{z_i\}$$

Example 2: closed loops in non-Abelian gauge theories

Example of canonical structure:

$$\mathcal{V}_i a_i := \mathrm{Tr}_N^{(l)}\left[\tau^k h_{\alpha_i}\right]^\dagger \text{ , or } \mathcal{V}_i a_i := \mathrm{Tr}_N^{(l)}\left[h_{\alpha_i}\right]^\dagger \text{ , } \ldots$$

$$\mathcal{V}_i := (a_i {a_i}^\dagger)^{-1/2}$$

+ Expectation values:

$$\langle \mathcal{V}_i a_i \rangle = z_i \sum_n \frac{|z_i|^{2n}}{n!\sqrt{n+1}} \qquad \left\langle \left(a_i^\dagger \mathcal{V}_i\right)^\dagger \right\rangle = \bar{z}_i \sum_n \frac{|z_i|^{2n}}{n!\sqrt{n+1}}$$

+ Relative variances:

$$\Delta_r(\mathcal{V}_i a_i) \xrightarrow[|z|\gg 1]{} 0 \quad , \quad \Delta_r(a_i^\dagger \mathcal{V}_i) \xrightarrow[|z|\gg 1]{} 0$$

III. Concluding comments

Concluding comments

Comments on the construction:

⊕ The graph change

$$\mathcal{H}^{\Gamma^{\mathcal{A}}} \cong \bigotimes_{s\in\Gamma^{\mathcal{A}}} \mathcal{H}_s^{\Gamma^{\mathcal{A}}} \qquad \mathcal{H}_s^{\Gamma^{\mathcal{A}}} \cong \bigotimes_{i=1}^{w_s} \mathcal{L}_i^2$$

→ Various valid graph changes, even "non-local" ones

⊕ Freedom in choice of vacuum states & tensor mappings

→ adapting the choice of CS to the choice of observables

⊕ Canonical structure with fluxes

$$\mathrm{V}_i a_i := \mathrm{Tr}_N^{(l)} \left[X_{I,k}\tau^k h_{\alpha_i}\right]^\dagger$$

Concluding comments

Summary:

- ✔ New coherent states which exhibit graph coherence;
- ✔ The construction method applies to various graph excitations;
- ✔ Flexibility of the mapping on the intertwiner space;

Outlook:

- 🔍 Explore the dynamics of such states;
- 🔍 Explore the possible choices of the intertwiners mapping;
- 🔍 Explore different graph excitations for the purposes of investigating semi-classical and continuum limits

Thank you & Happy birthday!!!