Compact Binary Coalescence and the BMS Group

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Applications of null geometry that would delight Jurek.
Interplay of conceptual, mathematical and phenomenological aspects of GWs.

Joint work with De Lorenzo and Khera
Inputs from Gupta, Sathyaprakash and especially Krishnan

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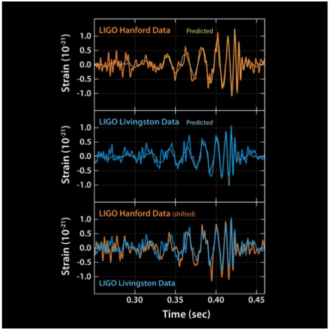
Motivation

GW150924



- Spectacular discovery of gravitational waves was made possible by matched filtering.
- Discoveries use $\sim$ hundred thousand waveforms, constructed through ingenious combinations of analytical methods (particularly PN and Effective One Body Approximation EOB) and Numerical Relativity NR.

- Grav. Radiation theory in exact GR was developed in $\sim$ 1960-80 by Trautman, Bondi, Sachs, Newman, Penrose ....

- Major surprise: Even for asymptotically Minkowski space-times, the asymptotic symmetry group is not the Poincaré group but the Bondi-Metzner-Sachs (BMS). $\Rightarrow$ Energy-momentum 4-vector replaced by the ∞-component supermomentum and angular momentum acquires a supertranslation ambiguity.

These features are largely ignored in the Waveform Community! Disconnect. Goal of this talk is to bridge this gap. Concrete lessons for compact binary coalescences (CBC) from the BMS group for both Waveform and mathematical GR communities.

Organization

1. CBC Waveforms: How they are created

A brief summary for Mathematical Relativists

2. The BMS Group and Gravitational Radiation

Change of gears: Relevant results at $\mathcal{I}^+$ from exact GR

3. Constraints on the CBC Waveforms

Bringing together the first two parts

4. Summary and Discussion

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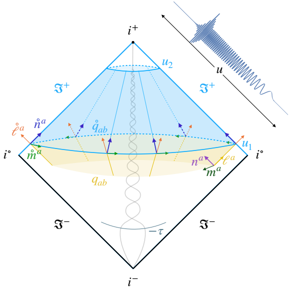
But results hold also for BH-NS and BNS-NS coalescences.

1. Wave-forms used by the LIGO-Virgo collabortion



- Need to cover up to 8 dimensional parameter space. NR simulations expensive; typically only ~ 15 cycles. So, ~ 100 cycles in the early phase of CBC evolution calculated using approximation methods.
Phenom Models: The two are then 'stitched together' and one looks for an analytical function that fits the resulting hybrid waveform.
EOB: Analytical waveform has undetermined coefficients that are calibrated against NR simulations

- External inputs are needed :
A. Analytical (PN and EOB) level; B. NR level;
C. Stitching procedure.

- A. PN Expansion: Expansion in v/c (believed to be asymptotic).
Ambiguities: (1) Choice of truncation at a PN order; (2) Choice of 'Taylor approximants': One starts with PN expansions of the energy $E(v/c)$ and flux $F(v/c)$. To obtain waveforms one needs to Taylor expand their rational functions. Ambiguity in the expansion within a PN order.

EOB: The PN trajectory mapped to that of a particle moving in a (fictitious) background space-time; corresponds to a certain resummation that yields more accurate results.
Ambiguity: Choice of the Hamiltonian of the EOB in regions of parameter space where NR simulations are sparse.

External Inputs (cont.)

B. Ambiguities in NR Require choices:
(1) Waveform $\sigma^\circ = h^\circ_+ + i h^\circ_x$ extracted at a large but finite radius, not at $\mathcal{I}^+$. Ambiguities in coordinate and null tetrad choices at a finite distance.
(2) Because of numerical errors associated with high frequency oscillations, in practice only the first few (spin weighted) spherical harmonics ($\ell = 2, 3, 4$) are calcualted.

C. Ambiguities in the stitching procedure require choices: Phenom and EOB
(1) Time during the CBC evolution at which stitching is done.
(2) PN and NR waveforms use different coordinates; matching procedure driven by intuition and past experience rather than clear cut mathematical physics.
(3) In PN, one has point particles. No horizons. In the NR initial data, one has dynamical horizons. So parameters of the two BHs determined very differently. Several ways to match the waveforms by minimizing differences over a small interval in time or frequency domain. A choice has to be made.

EOB: The way EOB waveform is joined to the quasi-normal ringing part.

For a summary addressed to Mathematical Relativists, see Appendix A of `arXiv: 1906.00913 v2`.

• Nonetheless approximate, analytical waveforms have proved to be invaluable for the first detections of gravitational waves. But already with the current LIGO-Virgo run, we entering an era of abundant event rate and greater accuracy, and with G3, LISA and Pulsar timing, we will achieve a much greater sensitivity on a significantly larger frequency band. Therefore for more accurate parameter estimation and more sensitive tests of general relativity, it is natural to ask for quantitative measures of the accuracy of waveforms relative to exact GR.

• Key problem: We do not know what the wave forms predicted by exact GR are! So, in the literature, accuracy tests involve comparing phenom and EOB waveforms with NR. But NR results themselves have ambiguities and assumptions (e.g., the final source parameters are estimated using the Isolated Horizon geometry and assumed to be the same as those at $\mathcal{I}^+$). Is there a more objective way to test for accuracy of the waveforms in the template bank, without knowing the exact waveforms themselves?

• The infinite set of balance laws at $\mathcal{I}^+$ made available at $\mathcal{I}^+$ by the BMS group provide a natural answer. Whatever the exact waveform is, it must obey these laws. Therefore their violation by any putative waveform provides an objective measure of how far the waveform in the template bank is from that of exact GR without knowing what the exact waveform is.

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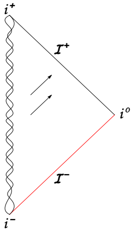
2. The BMS group

- Asymptotic flatness: Recall: $(M,\, g_{ab})$ is asymptotically Minkowski if g_{ab} approaches a Minkowski metric as $1/r$ as we recede from sources in null directions. In Bondi coordinates: $\mathrm{d}s^2 \to -\mathrm{d}u^2 - 2\mathrm{d}u\mathrm{d}r + r^2\,(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2)$

- Initial surprise: Presence of gravitational waves adds an unforeseen twist. There is no longer a canonical Minkowski metric that g_{ab} approaches! This key finding of Bondi & Sachs is still generally ignored by the waveform community.

The possible Minkowski metrics differ by angle dependent translations. (e.g. $t \to t + \xi(\theta,\varphi); \vec{x} \to \vec{x}$) The asymptotic symmetry group –the BMS Group $\mathfrak{B}$– is obtained by consistently "patching together" their Poincaré groups $\mathcal{P}$. Just as $\mathcal{P} = \mathcal{T} \rtimes \mathcal{L}$, we have $\mathfrak{B} = \mathcal{S} \rtimes \mathcal{L}$, where $\mathcal{S}$ is the infinite dimensional group of supertranslations (i.e., angle dependent translations). Just as $\mathcal{P}$ admits a 4-parameter family of Lorentz subgroups, $\mathfrak{B}$ admits an infinite parameter family, any two being related by a supertranslation.



- Generators of supertranslations : $\xi^a = \xi(\theta,\varphi)\, n^a$. $\mathcal{S}$ is an infinite dimensional Abelian normal subgroup of $\mathfrak{B}$ with $\mathfrak{B}/\mathcal{S} = \mathcal{L}$. $\mathfrak{B}$ also admit a unique 4-d Abelian normal subgroup $\mathcal{T}$ of translations: In a Bondi conformal frame, $\tau^a = \tau(\theta,\varphi) n^a$ where $\tau(\theta,\varphi) = \tau_{00} Y_{00} + \sum_m \tau_{1m} Y_{1m}$.

Gravitational Waves in Exact GR

- Radiative modes encoded in (equivalence classes of) connections $\{D\}$ at $\mathcal{I}^+$. If the curvature of $\{D\}$ is trivial, it represents a 'vacuum' $\{\mathring{D}\}$ in the YM sense. If $\{D\} = \{\mathring{D}\}$, then no gravitational radiation. Subgroup of $\mathfrak{B}$ that leaves any one 'vacuum' $\{\mathring{D}\}$ invariant is a Poincaré group: gravitational radiation is directly responsible for the enlargement of $\mathcal{P}$ to $\mathfrak{B}$. There is a natural isomorphism between the space of 'vacua' and the group $\mathcal{S}/\mathcal{T}$.

- Given a Bondi-foliation $u = \text{const}$ of $\mathcal{I}^+$ and adapted ℓ_a, m^a, we have:

(1) Radiative Information: The D on $\mathcal{I}$ is determined by the shear $\sigma^\circ(u,\theta,\phi) = -m^a m^b D_a \ell_b$ of ℓ_a. This is the 'waveform' $2\sigma^\circ = h_+^\circ + i h_x^\circ$.
Bondi news tensor N_{ab} is the conformally invariant part of the curvature of D.
$m^a m^a N_{ab} =: 2N = \dot{\sigma}^\circ$. Radiation field $\Psi_4^\circ = \ddot{\sigma}^\circ$.

(2) Coulombic Information: Ψ_2° ($= -GM$ in Kerr) and Ψ_1° ($= (3JG/2i)\sin\theta$ in Kerr) not captured in the radiative modes D or σ° on $\mathcal{I}^+$.

- Natural to assume $\{D\} \to \{\mathring{D}^\pm\}$ as $u \to \pm\infty$. Then we a acquire two preferred Poincaré subgroups $\mathcal{P}^\pm$ of $\mathfrak{B}$ adapted to i^+ and i° respectively. But the two are distinct unless gravitational memory

Balance Laws

• Standard assumption: $\dot{\sigma}^\circ = O(1/|u|^{1+\epsilon})$ as $u \to \pm\infty$. Then, in the two limits, the supermomentum is determined by $\mathrm{Re}\Psi_2^\circ$ at i^+ and i°:

$$P_\xi^+ = -\frac{1}{4\pi G}\oint_{u=\infty} \mathrm{d}^2S\ \xi(\theta,\phi)\,\mathrm{Re}\Psi_2^\circ \quad \text{and} \quad P_\xi^- = -\frac{1}{4\pi G}\oint_{u=-\infty} \mathrm{d}^2S\ \xi(\theta,\phi)\,\mathrm{Re}\Psi_2^\circ.$$

Waveforms have no knowledge of Ψ_2°. But they do determine its time derivative, and hence the supermomentum flux. we have an infinite set of balance laws on $\mathcal{I}^+$, one for each supertranslation $\xi^a = \xi(\theta,\varphi)n^a$:

$$P_\xi^- - P_\xi^+ = \frac{1}{4\pi G}\oint \mathrm{d}^2S\,\xi(\theta,\varphi)\int_{-\infty}^{\infty}\mathrm{d}u\,\left(|\dot{\sigma}^\circ|^2 - \mathrm{Re}\,\eth^2\dot{\bar{\sigma}}^\circ\right)$$

By peeling-off the arbitrary $\xi(\theta,\varphi)$ we obtain a balance Eq for each (θ,φ):

$$[\Psi_2^\circ]_{u=-\infty}^{u=\infty}(\theta,\varphi) = \int_{-\infty}^{\infty}\mathrm{d}u\,\left(|\dot{\sigma}^\circ|^2 - \mathrm{Re}\,\eth^2\dot{\bar{\sigma}}^\circ\right)$$

• Thus the waveform $\sigma^o(u,\theta,\varphi)$ in exact GR must satisfy these infinite set of equations. But not yet a constraint on the waveform because in general there is no a priori restriction on the LHS, i.e. on the angular dependence of the limiting values of Ψ_2° . But we will see that for CBC, limiting values have a very specific form. Hence the balance laws translate to constraints on waveforms.

(The SXS collaboration will soon release code to calculate Ψ_2°. Then finite (in time) versions of the balance laws can also be used to check and improve accuracy of individual seeps in the construction of the waveforms.)

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3. The Compact Binary Coalescence

- Standards assumptions made by the waveform community:

(i) NR: Space-time geometry approaches the Kerr black geometry at late times.
(ii)PN: The system is stationary in the distant past for $t < -\tau$ for some τ.

- Let us use a much weaker (and physically plausible) assumption: Asymptotic stationarity of the Weyl curvature as one approaches i^+ and i° along $\mathcal{I}^+$.

The fall -off of σ° in the limits $u \to \pm\infty$ already implies that $\partial_u \Psi_4^\circ$, $\partial_u \Psi_3^\circ$, $\partial_u \Psi_2^\circ$ all tend to zero in the limit. Let assume in addition that $\partial_u \Psi_1^\circ$ also goes to zero in the limit. Assumption trivially satisfied by waveforms in the bank.

- Then, Bianchi identities at $\mathcal{I}^+$ imply: In the Bondi conformal frame in which the system is at rest at i°, the limiting $\Psi_2^\circ|_{i^\circ}$ is real and spherically symmetric. Similarly at i^+. Note: The two Bondi-frames are in general different because of the black hole kick! Let us work in the past rest-frame at i°

- The kick velocity of the final BH is determined by the 3-momentum carried away by gravitational waves. Choose z direction along the velocity. Then

$$\gamma\,(M_{i^+})\,v \equiv P_z = -\tfrac{1}{4\pi G}\int \mathrm{d}u\,\mathrm{d}^2V\;\cos\theta\;|\dot\sigma^\circ(u,\theta,\varphi)|^2, \quad \text{with}\;\; \gamma = (1 - v^2/c^2)^{-\frac{1}{2}}$$

And in the initial rest-frame at i^o we have specific angular dependences of Ψ_2°:

$$\Psi_2^\circ\big|_{i^\circ} = GM_{i^\circ}; \quad \text{and} \quad \Psi_2^\circ\big|_{i^+} = \frac{GM_{i^+}}{\gamma^3\,\left(1 - \frac{v}{c}\cos\theta\right)^3}.$$

Constraints on the Waveforms

- Thus for CBC, our balance equation for supermomentum becomes:

$$GM_{i^\circ} - \frac{GM_{i^+}}{\gamma^3\left(1-\frac{v}{c}\,\cos\theta\right)^3} = \int \mathrm{d}u\,\left[|\dot{\sigma}^\circ|^2 - \mathrm{Re}(\eth^2\dot{\bar{\sigma}}^\circ)\right](u,\theta,\varphi)$$

The right side is completely determined by the waveform $2\sigma^\circ = h_+ + ih_-$. The kick velocity is also determined by the waveform. PN calculations provide us the initial mass M_{i° while NR calculations give us the final mass M_{i^+}. Therefore, given any waveform in the template bank we have all the ingredients to check these infinite number of constraints. Deviations provide an objective measure of the accuracy of the global procedure that went into creation of the waveform.

- Special case: No kick. Then, LHS is spherically symmetric. So GR demands that if we integrate the RHS –determined entirely by the waveform– against $Y_{\ell,m}$ with $\ell \neq 0$, we must obtain zero $\Rightarrow$ all 'pure' supermomenta vanish. Strong restriction! In practice, the kick velocity $v \sim 300\text{km/s}$ so $v/c \sim 10^{-3}$. Since

$$GM_{i^\circ} - \frac{GM_{i^+}}{\gamma^3\left(1-\frac{v}{c}\,\cos\theta\right)^3} = GM_{i^\circ} - GM_{i^+}\left(1 + 3\cos\theta\,\frac{v}{c} - \left(\tfrac{3}{2} - 6\cos^2\theta\right)\frac{v^2}{c^2} + \ldots\right),$$

to rest if the waveform is accurate to $\sim 0.3\%$ we need to keep only the first term in v/c and then integration of the RHS against $Y_{\ell,m}$ with $\ell \neq 0, 1$, we must obtain zero, etc.

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4. Discussion

Goal:To bridge the gap between mathematical GR and waveform communities.

• Mathematical GR: Presence of supertranslations came as a surprise initially but their inevitability was quickly recognized. Because the Poincaré groups $\mathcal{P}^{\pm}$ of $\mathfrak{B}$ selected by $\{D\} \to \{\mathring{D}^{\pm}\}$ are generically distinct and differ by a supertranslation, there is a supertranslation ambiguity in the notion of angular momentum. Subtracting $\vec{J}^{+}$ from $\vec{J}^{-}$ (to get radiated 'angular momentum') is like subtracting apples and oranges!
However, there was considerable confusion about the expression of supermomentum till 1980s when it was resolved via phase space of radiative modes (AA+Streubel). Early expressions gave nontrivial fluxes of supermomentum and angular momentum even in Minkowski space-time! This was cleared up. But general viewpoint has been that the waveforms are unrestricted –radiative modes are 'free data' at $\mathcal{I}^{+}$.

• Waveform community: Supertranslations largely ignored. One typically works with a fixed Minkowski metric η_{ab} to which g_{ab} approaches, and uses its Poincaré group. But in presence of radiation, this Poincaré group is adapted either to i^{o} or i^{+}; not to both if gravitational memory is non-trivial, as is generically the case. $\vec{J}^{+}$ is subtracted from $\vec{J}^{-}$ to get radiated angular momentum!

Lessons for both communities

• Mathematical GR: 1. For CBC, radiative modes $\{D\} \leftrightarrow \sigma^\circ$ cannot be freely specified on $\mathcal{I}^+$ because the boundary conditions on sources in the asymptotic past and asymptotic future introduce unforeseen global constraints. 2. It is true that $\mathcal{P}^+ \neq \mathcal{P}^-$ if there is non-trivial memory (i.e. $\int \mathrm{d}u\, \sigma^\circ \neq 0$) so comparing $\vec{J}^\pm$ is like comparing apples and oranges. Still, if the final BH does not receive a kick –i.e., if the radiated 3-momentum is zero– all (pure) supermomenta vanish! Hence, because ${}^+R^a = {}^-R^a + \xi(\theta,\varphi)n^a$ and , the naive subtraction $\vec{J}^+ - \vec{J}^-$ gives the correct radiated angular momentum w.r.t. both $\mathcal{P}^\pm$. Surprise!

• Waveform community: Supertranslations not a nuisance! Supermomentum balance laws provide an infinite family of constraints, that can be used as objective measures of the accuracy of waveforms in the template banks. Such measures are needed: (i) Because there are 'external' inputs that are not derived from first principles, we need sharper measures of the accuracy of candidate waveforms vis a vis exact GR; and, (ii) We are now entering an era when higher precision will be needed both for source characterization and tests of GR.
In the generic case when there is a BH kick, the naive subtraction $\vec{J}^+ - \vec{J}^-$ is incorrect both in principle and in practice. In accuracies also in the angular momentum estimates; need to revisit carefully.