Superradiance of quantum fields: From dry friction to black hole radiation

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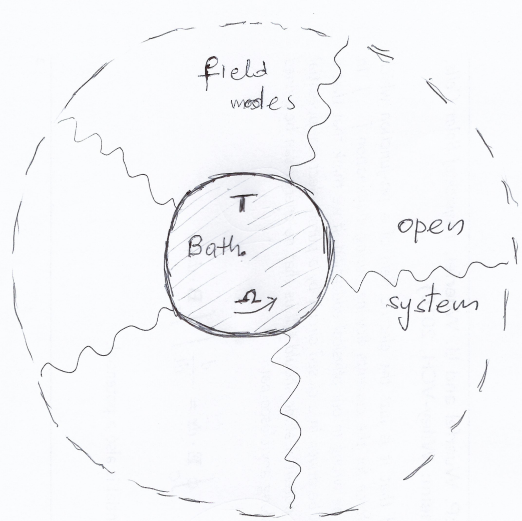
The Model (R. A. and A. Jenkins, Ann. Phys. (NY) **395**, 69 (2018))



Bosonic or fermionic quantum field modes (open system) interacting with rotating heat bath at the temperature T

$$[a_k, a^\dagger_{k'}]_\pm = \delta_{kk'}$$

k - quantum numbers of the mode

Quantum field Hamiltonian and angular momentum (z-component)

$$H_f = \hbar \sum_k \omega_k \, a^\dagger_k a_k, \quad L^z_f = \hbar \sum_k m(k) \, a^\dagger_k a_k$$

$m(k)$ - magnetic quantum number

Linear in fields and symmetric field-bath interaction

$$H_{\rm int} = \sum_k \left(a_k \otimes B^\dagger_k + a^\dagger_k \otimes B_k \right) \ ,$$

$$[L^z_{\rm b}, B_k] = -\hbar m(k) B_k,$$

Effective Hamiltonian for the rotating bath (Ω - angular frequency of rotation)

$$H^{\rm eff}_{\rm b} = H_{\rm b} - \Omega L^z_{\rm b} \ ,$$

Markovian Master equation for density matrix of the field

$$\frac{d\rho(t)}{dt} = -\frac{i}{\hbar}\left[H_{\mathrm{f}}, \rho(t)\right] + \mathcal{L}\rho(t) = -\frac{i}{\hbar}\left[H_{\mathrm{f}}, \rho(t)\right]$$
$$+\frac{1}{2}\sum_{k}\gamma_{\downarrow}(k)\left(\left[a_k, \rho(t)a_k^{\dagger}\right] + \left[a_k\rho(t), a_k^{\dagger}\right]]\right)$$
$$+\gamma_{\uparrow}(k)\left(\left[a_k^{\dagger}, \rho(t)a_k\right] + \left[a_k^{\dagger}\rho(t), a_k\right]\right)\ .$$

$\gamma_{\downarrow}(k)$ - annihilation rate

$\gamma_{\uparrow}(k) = \gamma_{\downarrow}(k)e^{-\hbar\beta(\omega_k - m(k)\Omega)}$ - creation rate, $e^{-\hbar\beta(\omega - m\Omega)}$ - modified Boltzmann factor, $\beta = 1/k_B T$,

$\gamma_{\uparrow}(k) \equiv \gamma_{\uparrow}(k)[\omega_k] \mapsto \gamma_{\uparrow}(k)[\omega_k + m\Omega] \geq 0$

Reduced description

Averaged quantum field in terms of averaged quantum mode amplitudes

$$\alpha \equiv \{\alpha_k\}, \quad \alpha_k(t) = \mathrm{Tr}\,(\rho(t) a_k)$$

corresponds to classical field description for bosonic fields.

(Quasi)particle population numbers

$$\bar{n}_k(t) = \mathrm{Tr}\,(\rho(t) a_k^\dagger a_k)$$

are used to compute average energy, z- component of angular momentum.

More general reduced description involves single-particle density matrices (R.A. , Entropy **21**, 705 (2019))

$$\sigma_{kl}(t) = \mathrm{Tr}\,(\rho(t) a_l^\dagger a_k)$$

Field equations and kinetic equations

The Master equation leads to the following evolution equations for the averaged field

$$\frac{d}{dt}\alpha_k(t) = \{-i\omega_k - \frac{1}{2}[\gamma_\downarrow(k) - (\pm)\gamma_\uparrow(k)]\}\alpha_k(t)$$

and to the kinetic equation for the average occupation number of a single mode

$$\frac{d}{dt}\bar{n}_k(t) = -[\gamma_\downarrow(k) - (\pm)\gamma_\uparrow(k)]\bar{n}_k(t) + \gamma_\uparrow(k)$$

where $(+)$ – bosons $(-)$ – fermions.

The validity of classical field description for bosons

Only for the zero-temperature bath at rest the following conditions holds:

1) The coherent states of the field - $|\alpha\rangle$, $a_k|\alpha\rangle = \alpha_k|\alpha\rangle$ evolve into coherent states $|\alpha(t)\rangle$ such that

$$\frac{d}{dt}\alpha_k(t) = \{-i\omega_k - \frac{1}{2}\gamma_{\downarrow}(k)\}\alpha_k(t), \quad \alpha_k(t) = e^{\{-i\omega_k - \frac{1}{2}\gamma_{\downarrow}(k)\}t}\alpha_k$$

2) For the initial coherent state populations are completely determined by the "classical field"

$$\bar{n}_k(t) = |\alpha_k(t)|^2$$

Superradiance for rotating heat baths

Bosonic modes satisfying the condition $\omega_k < m(k)\Omega$ are unstable – Zel'dovich's rotational superradiance in"Amplification of Cylindrical Electromagnetic Waves Reflected from a Rotating Body", Sov. Phys. JETP **35**, 1085 (1972) .

Exponential increase of particle number

$$\bar{n}_k(t) = \exp\left\{\gamma_\downarrow(k)\left[e^{\hbar\beta(m(k)\Omega-\omega_k)} - 1\right]t\right\}\bar{n}_k(0)$$

$$+\left(\exp\left\{\gamma_\downarrow(k)\left[e^{\hbar\beta(m(k)\Omega-\omega_k)} - 1\right]t\right\} - 1\right)\frac{1}{e^{\hbar\beta(m(k)\Omega-\omega_k)} - 1},$$

Amplification of the incident field (laser action)

$$\alpha_k(t) = \exp\left\{\frac{1}{2}\gamma_\downarrow(k)\left[e^{\hbar\beta(m(k)\Omega-\omega_k)} - 1\right]t\right\}\alpha_k(0)$$

Rotational energy produces particles and heats the bath.

Black hole radiation

Outer modes – the open system, – $a_k, a_k^\dagger$

Inner modes – the bath – $b_{k'}, b_{k'}^\dagger$, at the vacuum state

Tunneling Hamiltonian

$$H_{int} = \sum_k (a_k \otimes B_k^\dagger + a_k^\dagger \otimes B_k)$$

$$B_k = \sum_{k'} \left[f_{kk'} b_{k'} + g_{kk'} b^\dagger_{-k'} \right] \quad -k' = \text{time reversal of } k'$$

Hawking : strong gravity of BH creates indeterminacy between $b_{k'}$ and $b_{k'}^\dagger$.

The key result of Hawking

$$\frac{|g_{kk'}|^2}{|f_{kk'}|^2} \simeq e^{-\hbar\beta_{\rm H}\omega(k)}, \quad \text{for} \quad \omega(k) = \omega(k')$$

where

$$\beta_{\rm H} = \frac{1}{k_B T_H}, \quad T_H = \frac{\hbar c^3}{8\pi G M_{BH} k_B} = 6.2\,10^{-8} K \times \frac{M_{Sun}}{M_{BH}}$$

The results imply that:

a) inner modes at the vacuum state act as a heat bath at T_H,

b) rotating BH will superradiate bosons obeying the condition $\omega_k < m(k)\Omega$,

c) incident gravitation waves with $\omega_k < m(k)\Omega$ will be amplified by a rotating BH.

Quantum origin of shock waves

A shock wave is a propagating disturbance that moves faster than the local wave phase speed and is characterized by an abrupt, nearly discontinuous, change in pressure, temperature, and density of the medium.

Despite the long history the theoretical description of shock waves is rather poor, they are treated as singularities in the solutions.

Steven Hawking:

It seems to be a good principle that the prediction of a singularity by a physical theory indicates that the theory has broken down, i.e. it no longer provides a correct description of observations .

From rotational superradiance to shock waves

$$\bar{n}_k(t) = \exp\left\{\gamma_{\downarrow}(k)\left[e^{\hbar\beta(m(k)\Omega-\omega_k)} - 1\right] t\right\}\bar{n}_k(0)$$
$$+\left[\exp\left\{\gamma_{\downarrow}(k)\left[e^{\beta(m(k)\Omega-\omega_k)}\right] t\right\} - 1\right]\frac{1}{e^{\hbar\beta(m(k)\Omega-\omega_k)} - 1}\,,$$

with thermal equilibrium initial populations

$$\bar{n}_k(0) = \frac{1}{e^{\hbar\beta\omega_k} - 1}$$

For slow, macroscopic modes satisfying $|\hbar(\omega_k - m\Omega)/k_B T| << 1$ (for typical acoustic waves at room temperature $\hbar\omega/k_B T \sim 10^{-10}$) the energy of an unstable or close to instability mode increases linearly in time

$$E_k(t) = \hbar\omega_k n_k(t) = k_B T + [\gamma_{\downarrow}(k)\hbar m(k)\Omega]\, t$$

for $m(k)\Omega > 0$

Generic moving bath

Introducing the wave vector of the mode $\vec{k}$ and the dispersion law $\omega(\vec{k})$ we have

$$m(k)\Omega \mapsto \vec{k} \cdot \vec{V}$$

where $\vec{V}$ is a local velocity of the bath

Dissipated power by a single mode

$$P(\vec{k}) = \gamma_{\downarrow}(\vec{k})\hbar\vec{k} \cdot \vec{V} > 0.$$

The superradiance condition reads now

$$|\vec{V}| > \frac{\omega(\vec{k})}{|\vec{k}|} = v(\vec{k}) - -\text{local phase velocity}$$

Compare with the model of ocean wave generation by wind in P. Paradoksov (Zeldovich), "How quantum mechanics helps us understand classical mechanics", Sov. Phys. Usp. **9**, 618 (1967)

Further applications – work in progress

1) **Coronal heating**

The detailed physical processes that heat the outer atmosphere of the Sun and of solar-like stars to millions of degrees are still poorly understood, and remain a major open issue in astrophysics. (P. Testa et. al., Philos Trans A Math Phys Eng Sci. 373(2015)).

Plausible model: The motion of convective cells in solar atmosphere generates, by superradiation mechanism , magnetohydrodynamic Alfven shock waves which transport energy upwards. This energy is dissipated in the outer atmosphere. The velocity of plasma in a convective cell at the surface must be higher than the local Alfven wave speed.

2) **Dry friction and triboelectricity**

A generally accepted microscopic theory of dry friction does not exist (similarly for triboelectricity).

Plausible model: The relative motion of two solid surfaces generates, by superradiation mechanism (shock waves), low frequency surface phonons .

Similarly, the analog of superradiance for electrons (population inversion) generates triboelectric current . (R. A. and A. Jenkins , https://arxiv.org/abs/1904.11997v1)

3) **Singularities in General Relativity**

Gravity shock waves (superradiance) – transition from classical field-theoretical description to graviton gas model ?

Concluding remarks

Assume that the complete description of matter is given by a certain Effective Quantum Field Theory (e.g. bosonic excitations of ground state). Then:

1) shock waves are related to purely quantum superradiance phenomena, present in linearized models and caused by stimulated emission (positive feedback),

2) creation of shock waves can be treated as a “phase transition" between two regimes - wave or (quasi)particle dominated.

Two complementary classical approximation (wave-particle duality):

I) Classical Field Theory - valid for reversible coherent processes

II) Kinetic Theory of quasi-particle gas - valid for irreversible random processes

Formation of shock waves is accompanied by the transition from I to II