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Cosmic Cable

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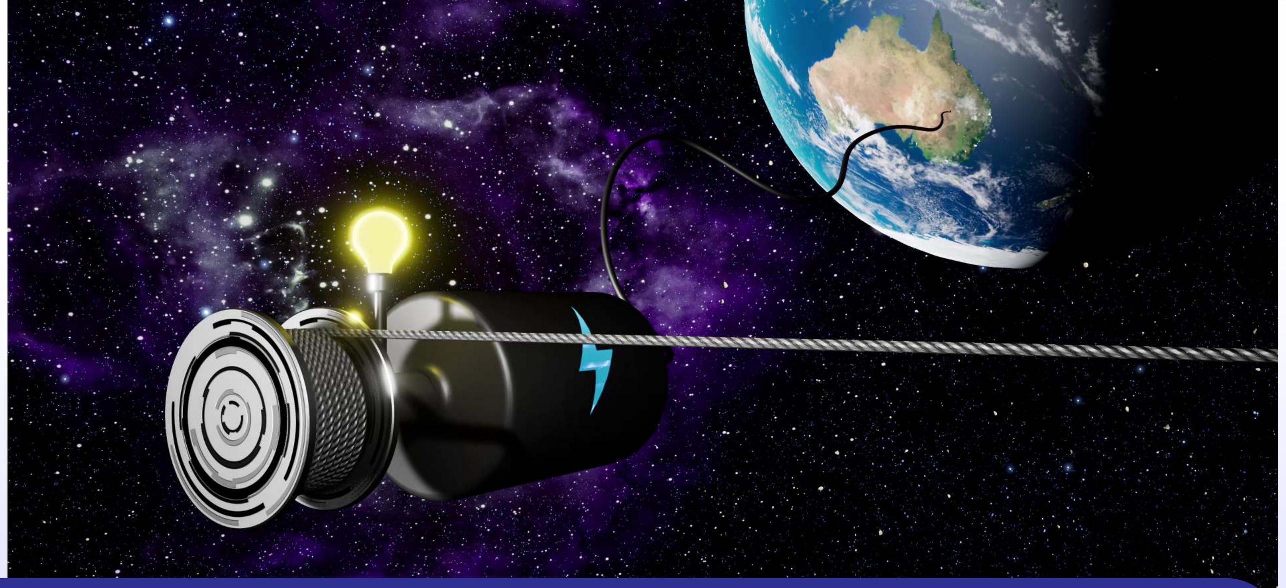


Introduction

We examine the relativistic mechanics of a rigid cable in curved spacetime. Such ropes have been applied to black hole thermodynamics, and energy in an expanding universe.

Spool one end, and attach it to a winch or turbine at fixed location. By allowing the cable to lower under its own weight (this depends on the spacetime), energy is harvested, which loosely speaking is converted from gravitational potential. We generalise the quasi-static case to a moving cable.

Background literature

The cable is distinct from Nambu-Goto relativistic strings, superstrings, and typical cosmic strings. It obeys ordinary macroscopic energy conditions, is classical not quantised, and is manufactured rather than naturally occurring.

Black holes: Penrose (1969) considered lowering an object on a rope towards a black hole, to extract 100% of the object's rest energy. A talk by Geroch inspired Bekenstein (1972) to consider lowering a box of thermal radiation towards a black hole, to break the 2nd law of thermodynamics. (Not his famous 1973 paper.)

Gibbons (1972) was the first to analyse the cable's tension. Unruh & Wald (1982) resolved Bekenstein's paradox via a buoyancy effect due to Hawking radiation. Redmount (1984) examined an exact solution, a case of Weyl's axisymmetric metric.

Expanding universes: Davies (1984) showed no energy is gained by hauling up quantum radiation from the de Sitter horizon. Harrison (1995) proposed latching a rope onto a distant receding body in FLRW spacetime.

Other: More peripheral sources include Szekeres' (1965) "gravitational compass". Maugin (1978), Wernig-Pichler (2006), and others studied relativistic elasticity in 1 dimension.

Conclusions

- Generalisation to a moving cable.
- Up to 100% of the cable's rest energy can be extracted, concurs with the static case.
- Relativity pedagogy: extended objects, length measurement, forces. There are many wrong approaches to this problem!
- Application to open research e.g. energy of spacetime, or black hole thermodynamics.

References

- Gibbons (1972), "On lowering a rope into a black hole", *Nature Physical Science*
- Unruh & Wald (1982), "Acceleration radiation and the generalized second law of thermodynamics", *Physical Review D*
- Redmount (1984), "Topics in black hole physics", PhD thesis, supervisor Thorne
- Wernig-Pichler (2006), "Relativistic elastodynamics", PhD thesis, supervisor Beig
- MacLaurin (2019a), "Cosmic cable", Proceedings of Marcel Grossmann conference 15 (submitted)
- MacLaurin (2019b), "Spatial measurement in curved spacetime", Proceedings as above

Stationary cable

This section is based on Gibbons, as corrected by Unruh & Wald; Redmount; and Fouxon+ (2008). Consider firstly an arbitrary spacetime. A cable with no tangential rigidity has stress-energy tensor:

$$\mathbf{T} = \frac{\mu}{A}\mathbf{u}\otimes\mathbf{u} + \frac{T}{A}\mathbf{q}\otimes\mathbf{q}.$$

Label	Usage
μ	linear mass density
T	tension
$\mathbf{u}$	4-velocity
$\mathbf{q}$	unit spatial vector along cable
τ	proper time
L	proper length
$\mathbf{T}$	stress-energy
A	cross section area
$\boldsymbol{\xi}$	timelike Killing vector
V	$\sqrt{-\boldsymbol{\xi}\cdot\boldsymbol{\xi}}$, redshift
β	speed relative to static
γ	Lorentz factor

These all depend on position. If stress-energy is conserved then $\operatorname{div}\mathbf{T} = \mathbf{0}$, so contract this vector with $\mathbf{u}$ and $\boldsymbol{\xi}$ respectively:

$$\frac{d\mu}{d\tau} = -\mu\operatorname{div}\mathbf{u} + T\mathbf{u}\cdot\nabla_{\mathbf{q}}\mathbf{q}, \qquad \frac{dT}{dL} = -T\operatorname{div}\mathbf{q} - \mu\mathbf{q}\cdot\nabla_{\mathbf{u}}\mathbf{u}.$$

But typically $\operatorname{div}\mathbf{T} \neq \mathbf{0}$ for test particles on a fixed background spacetime, as $\mathbf{T}$ doesn't satisfy the Einstein field equation. Still, $\mathbf{u}\cdot\operatorname{div}\mathbf{T} = 0$ is guaranteed (Wernig-Pichler).

Now suppose the spacetime is stationary. A stationary cable has $\mathbf{u} = \boldsymbol{\xi}/V$. It follows $d\mu/d\tau = 0$, and if μ is also constant over space the tension varies with location as:

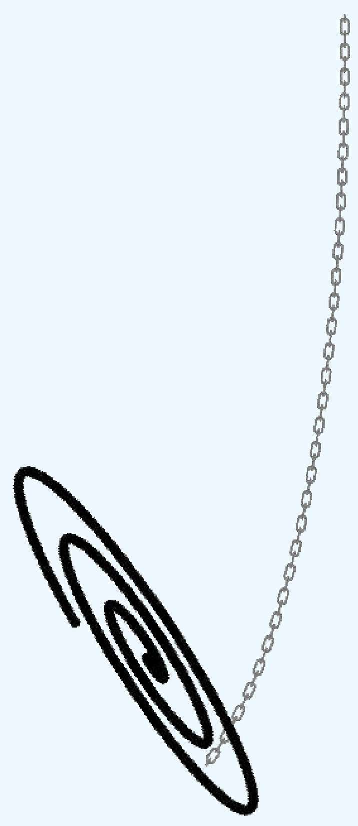
$$T = W\frac{V_{\text{end}}}{V} + \mu\Big(1 - \frac{V_{\text{end}}}{V}\Big).$$

Here W is a weight hanging at the end, and V_{end} is evaluated at the lower end of the cable. Curiously, "in curved spacetime the weight of the rope redshifts away" (Brown 2013). The $\mu \equiv 0$, $W = 1$ case is well known in *surface gravity* (Wald 1984).

Moving cable: Kinematics

An arbitrary static, spherically symmetric spacetime has metric:

$$ds^2 = -e^{2\alpha_r}dt^2 + e^{2\zeta_r}dr^2 + r^2(d\theta^2\sin^2\theta\, d\phi^2).$$



Take $\boldsymbol{\xi} := \partial_t$. Set the turbine at r_0, and assume no redshift there: $e^{\alpha_{r_0}} = 1$. For simplicity assume Born-rigidity, and that the cable's motion is symmetric in time (the field $\mathbf{u}$ is unchanged by translation in $\boldsymbol{\xi}$). Along the cable, conservation of particle number and zero expansion:

$$^{(1)}\operatorname{div}(n\mathbf{u}) = 0, \qquad ^{(1)}\operatorname{div}\mathbf{u} = 0,$$

imply the density μ is an overall constant, and the 4-velocity is:

$$u^\mu = e^{-\alpha}\big(\sqrt{1 + K^2e^{-2\alpha}}, Ke^{-\zeta}, 0, 0\big).$$

Brotas (2006) has a similar result for Schwarzschild spacetime. $K \in \mathbb{R}$ is an overall constant we term the "redshifted proper speed", since at each location $K = \beta\gamma e^\alpha$. Kinematics are not trivial, and authors including Harrison do not account for the frame-dependence of distance in general relativity.

Moving cable: Dynamics

The 4-acceleration $\mathbf{a} = \nabla_{\mathbf{u}}\mathbf{u}$ of a cable particle has magnitude $a := \sqrt{\mathbf{a}\cdot\mathbf{a}} = |\alpha'|e^{-\zeta}/\sqrt{1 + K^2e^{-2\alpha}}$. We compute the overall energy entering and exiting the system. Firstly, to a static observer at r, the local segment of cable transfers energy at the rates in the table below.

Energy	Flux rate
mass	$\mu K e^{-\alpha_r}$
kinetic	$\mu K(\gamma_r - 1)e^{-\alpha_r}$
total	$\mu K\gamma_r e^{-\alpha_r}$

If a local energy rate is transferred to another static frame at r_1 say, the received power is redshifted by $e^{\alpha_r - \alpha_1}$ twice (think of photons: both the wavelength and number rate are changed). The overall profit is the incoming power at the turbine; minus the outgoing power at the far end — but redshifted twice to the turbine frame:

$$\mu K\big(1 - \sqrt{K^2 + e^{2\alpha_{\text{end}}}}\big).$$

(The calculation invokes the time symmetry. We assume the cable always ends at the constant r_{end}, say due to a robot repeatedly cutting it. The incoming mass is treated as *gratis*, but the incoming kinetic energy must be provided, in our mathematical accounting.) The $1 - \sqrt{\cdots}$ term is the energy harvest per mass, which $\to 1$ in the limit of a slow cable: $K \to 0$, ending near a Killing horizon: $e^{\alpha_{\text{end}}} \to 0$. If the outgoing kinetic energy can be recovered, the profit becomes $\mu K(1 - e^{\alpha_{\text{end}}})$.

The tension at some r_1 must support the cable beneath it. Consider a location r_1 somewhere below. An interval dr contains a proper length of cable $\gamma e^{\xi}dr$, as measured in a static frame (MacLaurin 2019b). Locally this contributes force $\mu\alpha' dr$, which gets redshifted once to r_1. Integrating:

$$T = \mu\big(1 - e^{\alpha_{\text{end}} - \alpha_1}\big).$$

We focused on static frames because their measurements are sustainable over time, and they are more familiar in the literature.