(Anti)evaporation of Schwarzschild–de Sitter black holes revisited

Maciej Kolanowski

Institute of Theoretical Physics, Faculty of Physics,
University of Warsaw, Pasteura 5, 02-093 Warsaw, Poland

Abstract

It is widely believed that in the presence of a positive cosmological constant, heavy black holes can exhibit non-standard behaviour, namely there is a possibility that such objects would grow instead of evaporating. We point out that all those results (obtained in different frameworks) rely heavily upon the identification of the Nariai spacetime with the Schwarzschild–de Sitter (Kottler) black hole. We argue that it is an incorrect assumption. As a result, previous treatments need revisiting. In particular, we show that within effective action approach, there is no solution corresponding to the Schwarzschild–de Sitter black hole.

Effective action

In [1] Bousso and Hawking considered N quantum scalar fields coupled to the gravity. One can restrict themselves to the spherically symmetric configurations and the large N approximation. In a conformal gauge metric takes the form:

$$ds^2 = e^{2\rho}\left(-dt^2 + dx^2\right) + e^{-2\phi}d\Omega^2, \tag{1}$$

where ρ, ϕ are functions of t, x only. In aforementioned approximations, one can derive effective (1-loop) equations of motion – they are non-linear second order differential equations for ρ, ϕ and some auxiliary field Z which is needed to make action local.

Schwarzschild–de Sitter black hole

Spherically symmetric solutions of Einstein equations with $\Lambda > 0$ are Schwarzschild–de Sitter black holes:

$$ds^2 = -\left(1 - \frac{2M}{r} - \frac{r^2}{\alpha^2}\right)dt^2 + \frac{dr^2}{1 - \frac{2M}{r} - \frac{r^2}{\alpha^2}} + r^2 d\Omega^2, \tag{2}$$

where $\alpha^2 = \frac{3}{\Lambda}$ and M is a mass of the black hole. If $9M^2\Lambda < 1$, this solutions enjoys two horizons which coincides when $9M^2\Lambda = 1$ when our black holes becomes extremal. We could also take its Near Horizon limit, which reads the Nariai spacetime:

$$ds^2 = -\frac{1}{\Lambda}\sin^2\chi d\psi^2 + \frac{1}{\Lambda}d\chi^2 + \frac{1}{\Lambda}d\Omega^2 \tag{3}$$

with is simply a product $dS^2 \times S^2$ [3].

(In)stability of the Nariai solution

One can find Nariai-like solutions to... which reads:

$$ds^2 = \frac{1}{\Lambda_1 cos^2 t}\left(-dt^2 + dx^2\right) + \frac{1}{\Lambda_2}d\Omega^2, \tag{4}$$

where radii are given by:

$$\Lambda_1 \approx \frac{\Lambda}{1 - \frac{\omega b}{4}} \tag{5}$$

$$\Lambda_2 \approx \Lambda\left(1 - \frac{b}{2}\right) \tag{6}$$

One can linearize equations around this solutions. It happens to be unstable, in particular there is mode in which horizon area radius grows. Such black holes could be produced during inflation and this mechanism would make their lifetime much longer [5].

Physical solutions?

The Nariai spacetime does not describe physical spacetime with an extremal black hole. In particular, it has incorrect asymptotics at large r - it does not go the de Sitter spacetime in the limit $r \to \infty$. Thus, any 'small' perturbations considered before are small only near the horizon and they grow dramatically far away.

One could try to look at more physical solutions. We look for spacetimes with Schwarzschild–de Sitter asymptotics equipped with an additional Killing vector ∂_x. We can formulate now:

Theorem *There is no solution to effective equations of motion with a Killing vector ∂_x and an asymptotic behaviour:*

$$ds^2 = -A^2\frac{dr^2}{r^2} + \frac{r^2}{B^2}dx^2 + r^2 d\Omega^2 \tag{7}$$

for large $2-$ sphere radius r where A, B are constants.

Thus, we find that one cannot describe the usual black holes in this framework.

Conclusions

We have shown that the presence of anti-evaporation mechanism for heavy black holes in the presence of $\Lambda > 0$ is a mathematical artifact – it relies heavily upon an incorrect identification of extremal Schwarzschild–de Sitter black hole and its Near Horizon Geometry. One can expect thus no deviation from the usual Hawking radiation scenario also for primordial black holes and thus their quick evaporation. However, further analysis is needed in some other approaches, e.g. in $f(R)$ theories [4].

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